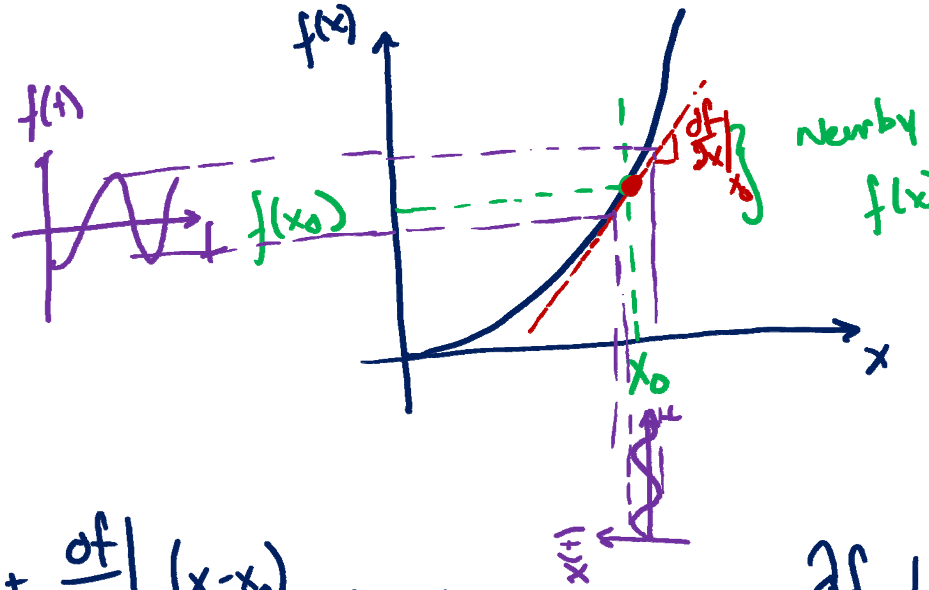
Small Signal Modeling: Linearization



Nearby

$$f(x) \approx f(x_0) + \left.\frac{df}{dx}\right|_{x_0}(x-x_0) + \underbrace{\frac{1}{2!}\frac{d^2f}{dx^2}(x-x_0)^2}_{\text{neglect higher order terms}}$$

if we have a multivariate function

linearize

$$f(x,y,z) \approx \underbrace{f(x_0,y_0,z_0)}_{F_0} + \left.\frac{\partial f}{\partial x}\right|_{F_0}(x-x_0) + \left.\frac{\partial f}{\partial y}\right|_{F_0}(y-y_0) + \left.\frac{\partial f}{\partial z}\right|_{F_0}(z-z_0) + \underbrace{}_{\text{higher order terms}}$$

Notation

$(x-x_0) \longrightarrow \hat{x}$

small-signal

$\hat{x} \ll x_0 = X$ (capital)

(1) Perturb and Linearize

$d'(t) \to D' - \hat{d}$

$L \frac{d\langle i_L \rangle}{dt} = \langle v_g \rangle - d'(t) \langle v \rangle$

$(1-d(t))$

with DC + small signal

① Replace all time-varying terms ($\langle x \rangle$ & $d(t)$)

$\langle x \rangle \to X + \hat{x}$

② Neglect higher order terms → anything with a product of two small-signal variables

& seperate DC & AC equations

$L \frac{d}{dt}(I_L + \hat{i}_L) = V_g + \hat{v}_g - (1-D-\hat{d})(V+\hat{v})$

$+ \hat{v}_g + \hat{d}V - D'\hat{v} + \cancel{\hat{d}\hat{v}}$ neglect (linearization)

$L\frac{d}{dt}I_L + L\frac{d}{dt}\hat{i}_L = V_g - D'V$

$0 = L \frac{dI_L}{dt} = V_g - D'V$ → DC (same as steady-state model)

$\boxed{L \frac{d}{dt}\hat{i}_L = \hat{v}_g + \hat{d}V - \hat{v}D'}$ → AC

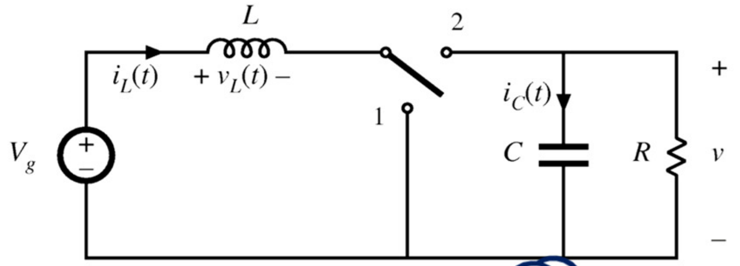
(2) 1st order Taylor Series Expansion

$$L\frac{d\langle i_L\rangle}{dt} = \underbrace{\langle v_g\rangle - d'(t)\langle v\rangle}_{f(v_g,\, d,\, v)}$$

$$L\frac{d\hat{i}_L}{dt} = \left.\frac{\partial f}{\partial v_g}\right|_{\substack{DC\\ v_g \to V_g\\ d \to D\\ v \to V}} \hat{v}_g + \left.\frac{\partial f}{\partial d}\right|_{DC} \hat{d} + \left.\frac{\partial f}{\partial v}\right|_{DC} \hat{v}$$

$$\boxed{L\frac{d\hat{i}_L}{dt} = 1\cdot \hat{v}_g + V\hat{d} - D'\hat{v}}$$

Equivalent Circuit Modeling: Boost Example



① $v_L(t) = V_g(t)$

$i_C(t) = -\dfrac{v(t)}{R}$

② $v_L(t) = V_g(t) - v(t)$

$i_C(t) = i_L(t) - \dfrac{v(t)}{R}$

Average

$$\langle v_L \rangle = L \frac{d\langle i_L \rangle}{dt} = \langle V_g \rangle - d'(t)\langle v \rangle$$

$$\langle i_C \rangle = C \frac{d\langle v \rangle}{dt} = d'(t)\langle i_L \rangle - \frac{\langle v \rangle}{R}$$

Process:

solve converter equations for all v_{Li}, i_{Ci} & (if necessary) $i_g(t)$ in each subinterval

↓

① Average over one switching period (w/out assuming/enforcing steady-state)

↓ Averaged, nonlinear circuit

② Linearize all equations by { Perturb & linearize / 1st or Taylor Series

↓ Averaged, linearized equations

③ Sketch & solve ac-equivalent averaged circuit

Linearization

Volt-second balance

$$\boxed{L\frac{d\hat{i}_L}{dt} = \hat{v}_g - D'\hat{v} + V\hat{d}}$$

Cap-charge balance :

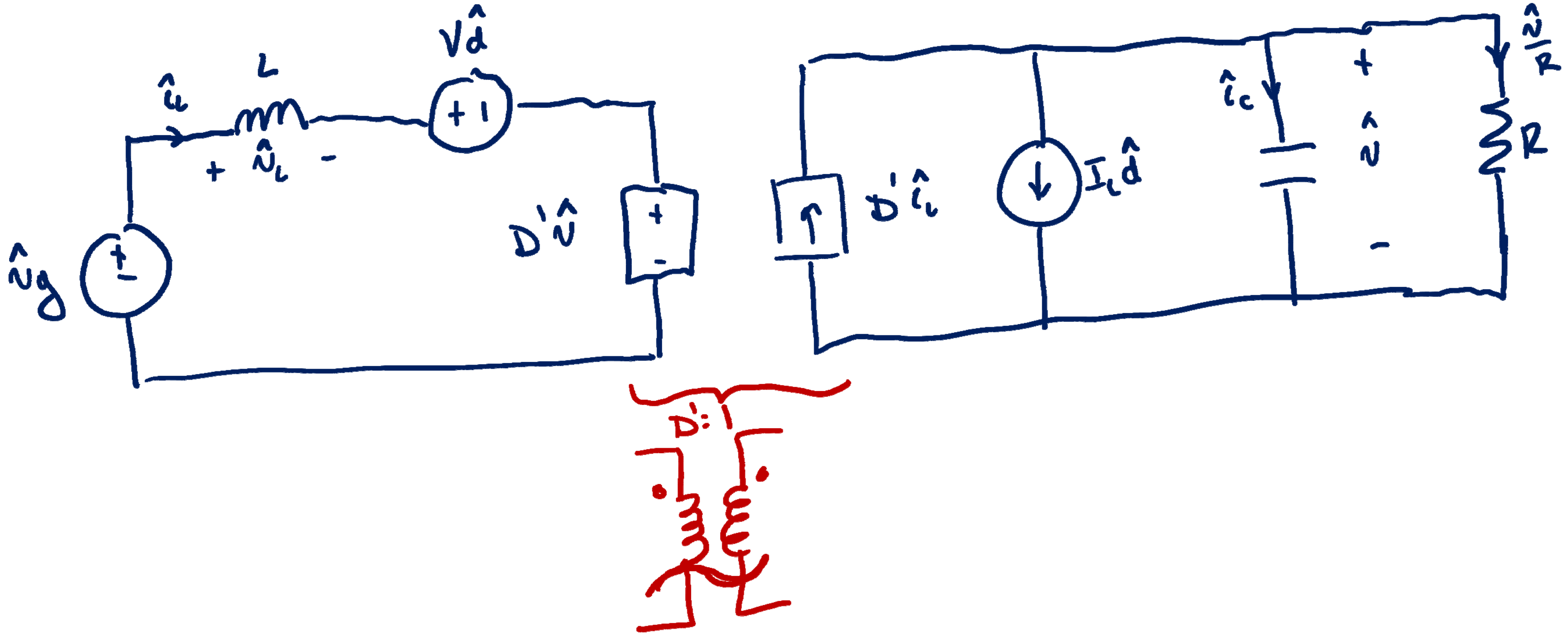
$$C\frac{d\langle v\rangle}{dt} = d'(t)\langle i_L\rangle - \frac{\langle v\rangle}{R}$$

$$\boxed{C\frac{d\hat{v}}{dt} = D'\hat{i}_L - I_L\hat{d} - \frac{\hat{v}}{R}}$$

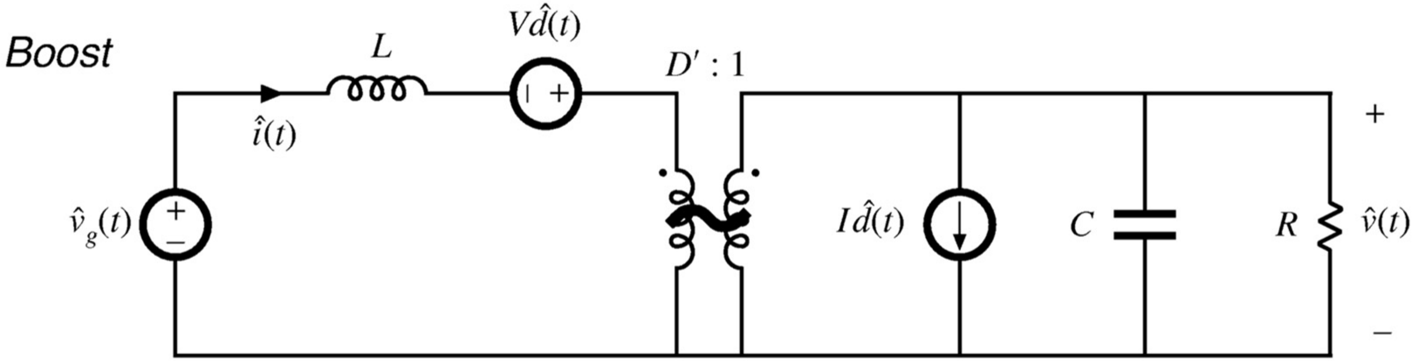
Equivalent Circuit Model

$$\hat{v}_L = L\frac{d\hat{i}_L}{dt} = \hat{v}_g - D'\hat{v} + V\hat{d}$$

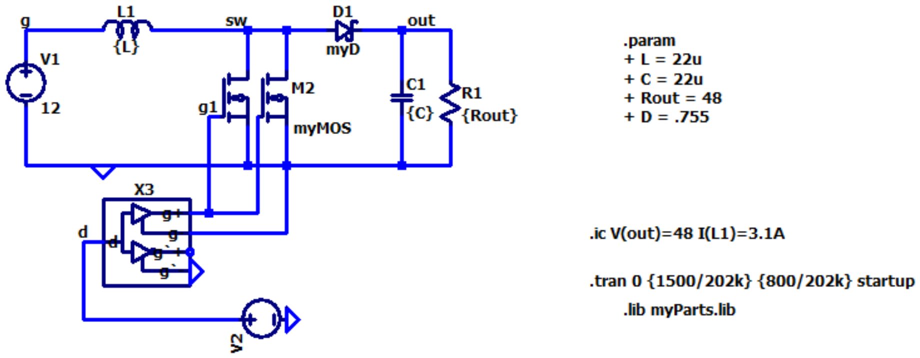
$$\hat{i}_C = C\frac{d\hat{v}}{dt} = D'\hat{i}_L - I_L\hat{d} - \frac{\hat{v}}{R}$$



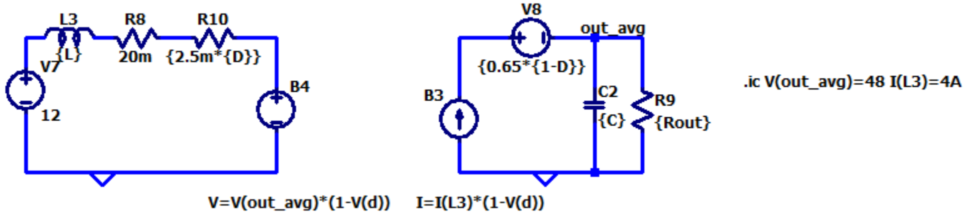
Boost Converter Averaged, AC, Linear Circuit Model



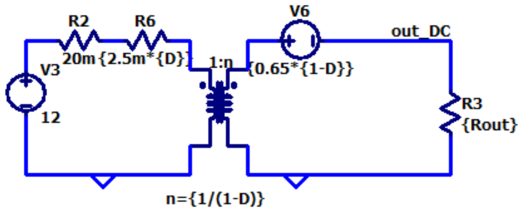
Model Simulation



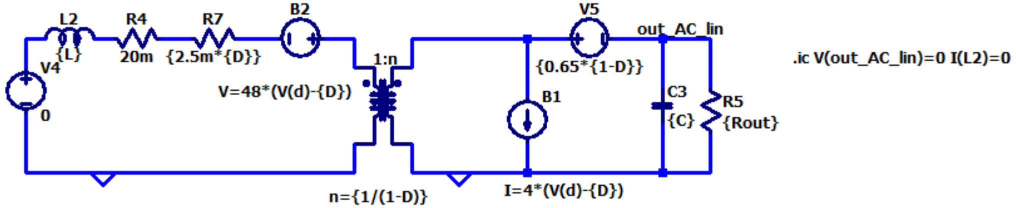
Full Switching Model



Averaged, Nonlinear

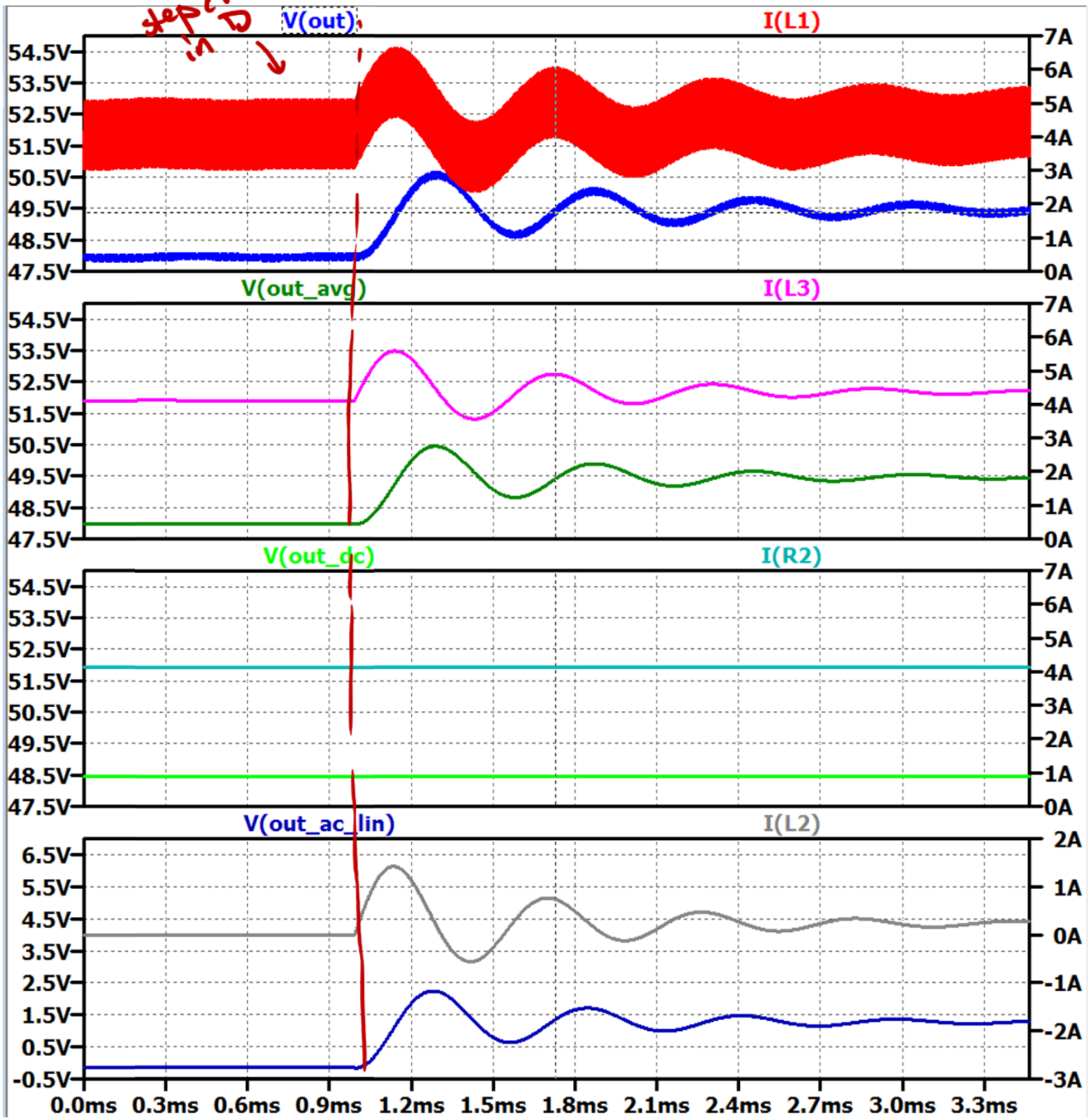


DC Averaged Model



AC Averaged,
Linearized Model

Model Comparison



Full Switching Model

Averaged, Nonlinear

DC Averaged Model

AC Averaged, Linearized Model

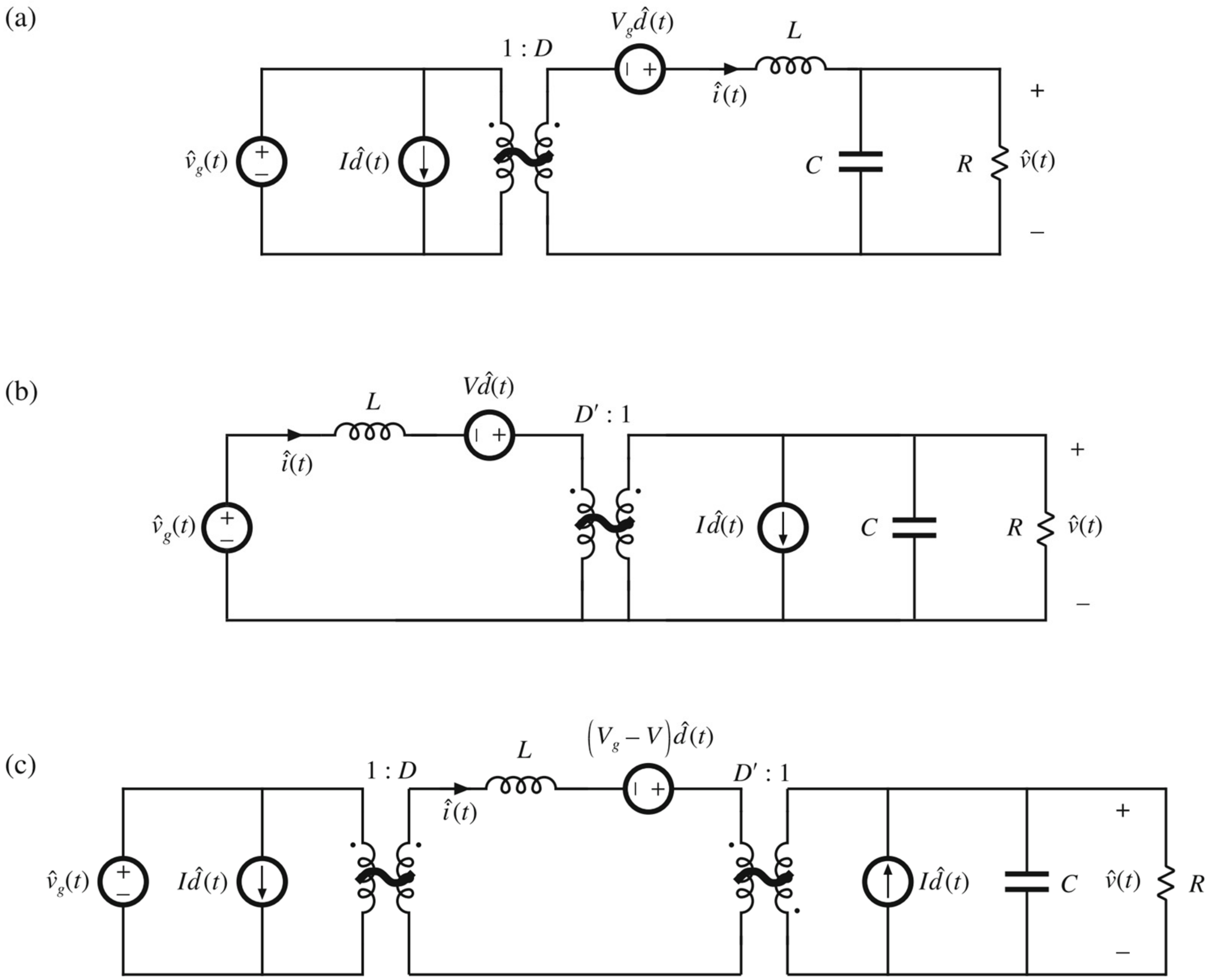


Fig. 7.18 Averaged small-signal ac models for several basic converters operating in continuous conduction mode: (**a**) buck, (**b**) boost, (**c**) buck–boost