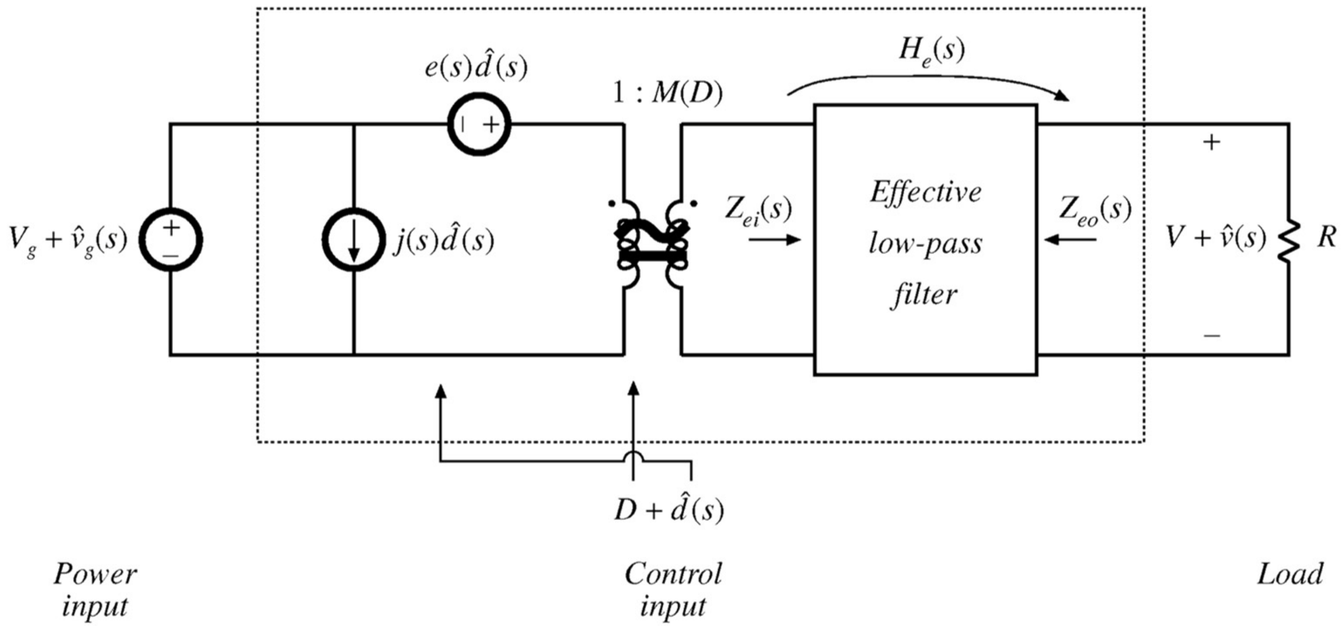
Section 7.4

CANONICAL CIRCUIT MODEL

Canonical Circuit Model



Line-to-output transfer function: $G_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)} = M(D)\, H_e(s)$

Control-to-output transfer function: $G_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)} = e(s)\, M(D)\, H_e(s)$

Canonical Form of Basic Converters

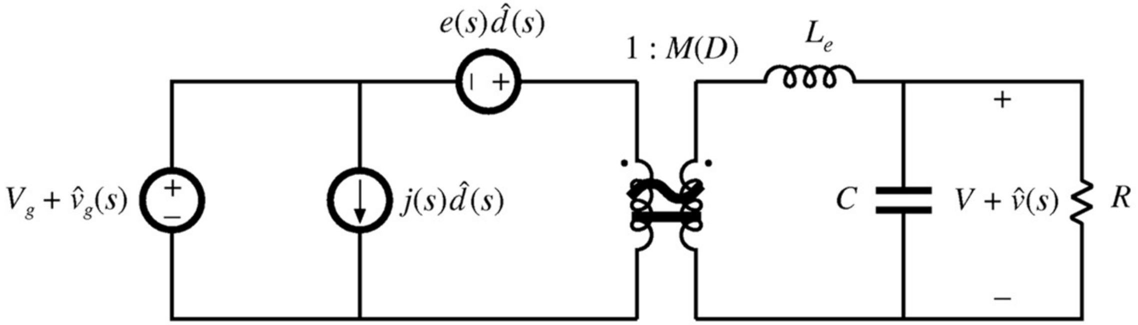


Table 7.1. Canonical model parameters for the ideal buck, boost, and buck-boost converters

Converter	$M(D)$	L_e	$e(s)$	$j(s)$
Buck	D	L	$\frac{V}{D^2}$	$\frac{V}{R}$
Boost	$\frac{1}{D'}$	$\frac{L}{D'^2}$	$V\left(1 - \frac{sL}{D'^2 R}\right)$	$\frac{V}{D'^2 R}$
Buck-boost	$-\frac{D}{D'}$	$\frac{L}{D'^2}$	$-\frac{V}{D^2}\left(1 - \frac{sDL}{D'^2 R}\right)$	$-\frac{V}{D'^2 R}$

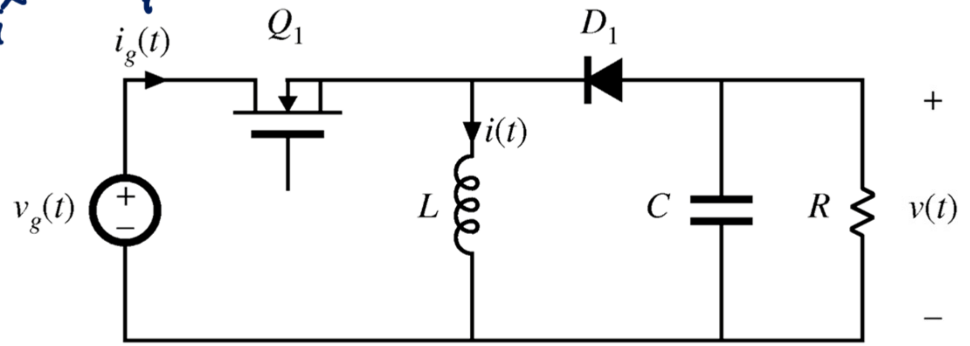
Section 7.5

STATE SPACE AVERAGING

7.3: State Space Modeling of Buck Boost

$\frac{dx}{dt} = \dot{x} = A_i x + B_i u \qquad i \in 1,2$

$y = C_i x + D_i u$



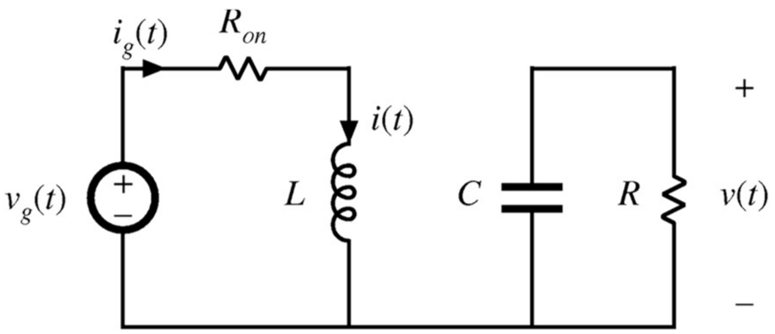
Model nonidealities:

- MOSFET on-resistance R_{on}
- Diode forward voltage drop V_D

state vector	input vector	output vector
$\mathbf{x}(t) = \begin{bmatrix} i(t) \\ v(t) \end{bmatrix}$	$\mathbf{u}(t) = \begin{bmatrix} v_g(t) \\ V_D \end{bmatrix}$	$\mathbf{y}(t) = \begin{bmatrix} i_g(t) \end{bmatrix}$

Model in Subinterval 1

$$L\frac{di(t)}{dt} = v_g(t) - i(t)\,R_{on}$$
$$C\frac{dv(t)}{dt} = -\frac{v(t)}{R}$$
$$i_g(t) = i(t)$$



$$\underbrace{\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}}_{\mathbf{K}} \underbrace{\frac{d}{dt}\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\frac{d\mathbf{x}(t)}{dt}} = \underbrace{\begin{bmatrix} -R_{on} & 0 \\ 0 & -\frac{1}{R} \end{bmatrix}}_{\mathbf{A}_1} \underbrace{\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\mathbf{x}(t)} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{B}_1} \underbrace{\begin{bmatrix} v_g(t) \\ V_D \end{bmatrix}}_{\mathbf{u}(t)}$$

$$\underbrace{\begin{bmatrix} i_g(t) \end{bmatrix}}_{\mathbf{y}(t)} = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathbf{C}_1} \underbrace{\begin{bmatrix} i(t) \\ v(t) \end{bmatrix}}_{\mathbf{x}(t)} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{\mathbf{E}_1} \underbrace{\begin{bmatrix} v_g(t) \\ V_D \end{bmatrix}}_{\mathbf{u}(t)}$$

State Space Model

Given: a PWM converter, operating in continuous conduction mode, with two subintervals during each switching period.

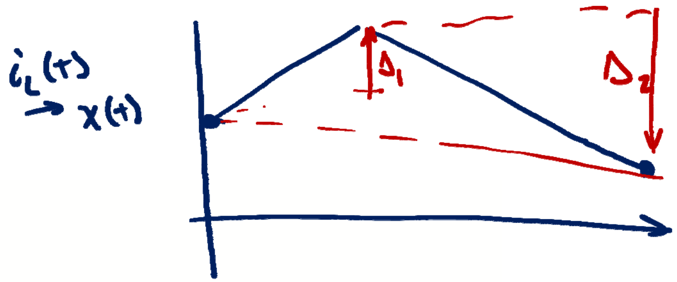
During subinterval 1, when the switches are in position 1, the converter reduces to a linear circuit that can be described by the following state equations:

$$\mathbf{K}\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_1\,\mathbf{x}(t) + \mathbf{B}_1\,\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}_1\,\mathbf{x}(t) + \mathbf{E}_1\,\mathbf{u}(t)$$

During subinterval 2, when the switches are in position 2, the converter reduces to another linear circuit, that can be described by the following state equations:

$$\mathbf{K}\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_2\,\mathbf{x}(t) + \mathbf{B}_2\,\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}_2\,\mathbf{x}(t) + \mathbf{E}_2\,\mathbf{u}(t)$$

State Space Averaging



The averaged (nonlinear) state equations:

$$\mathbf{K}\frac{d\left\langle \mathbf{x}(t)\right\rangle_{T_s}}{dt} = \left(d(t)\,\mathbf{A}_1 + d'(t)\,\mathbf{A}_2\right)\left\langle \mathbf{x}(t)\right\rangle_{T_s} + \left(d(t)\,\mathbf{B}_1 + d'(t)\,\mathbf{B}_2\right)\left\langle \mathbf{u}(t)\right\rangle_{T_s}$$

$$\left\langle \mathbf{y}(t)\right\rangle_{T_s} = \left(d(t)\,\mathbf{C}_1 + d'(t)\,\mathbf{C}_2\right)\left\langle \mathbf{x}(t)\right\rangle_{T_s} + \left(d(t)\,\mathbf{E}_1 + d'(t)\,\mathbf{E}_2\right)\left\langle \mathbf{u}(t)\right\rangle_{T_s}$$

The converter operates in equilibrium when the derivatives of all elements of $\langle \mathbf{x}(t) \rangle_{T_s}$ are zero. Hence, the converter quiescent operating point is the solution of

$$\mathbf{0} = \mathbf{A}\,\mathbf{X} + \mathbf{B}\,\mathbf{U}$$

$$\mathbf{Y} = \mathbf{C}\,\mathbf{X} + \mathbf{E}\,\mathbf{U}$$

where

$$\mathbf{A} = D\,\mathbf{A}_1 + D'\,\mathbf{A}_2$$
$$\mathbf{B} = D\,\mathbf{B}_1 + D'\,\mathbf{B}_2$$
$$\mathbf{C} = D\,\mathbf{C}_1 + D'\,\mathbf{C}_2$$
$$\mathbf{E} = D\,\mathbf{E}_1 + D'\,\mathbf{E}_2$$

and

$$\mathbf{X} = \textit{equilibrium (dc) state vector}$$
$$\mathbf{U} = \textit{equilibrium (dc) input vector}$$
$$\mathbf{Y} = \textit{equilibrium (dc) output vector}$$
$$D = \textit{equilibrium (dc) duty cycle}$$

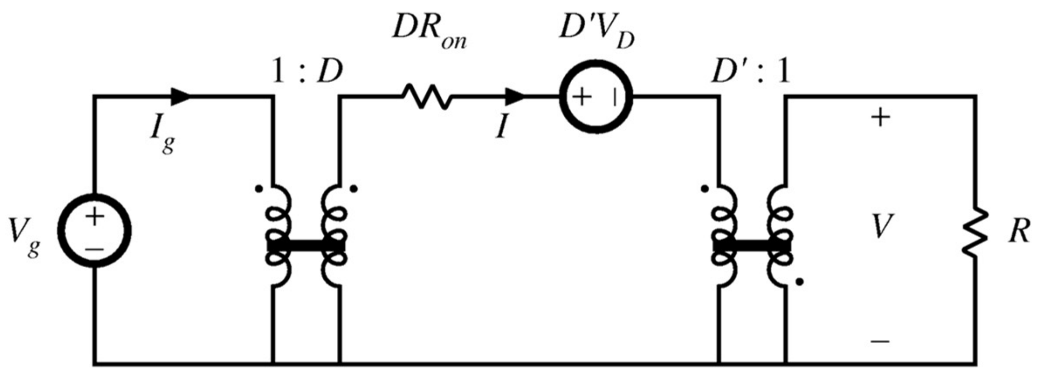
DC Solution

DC state equations:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -DR_{on} & D' \\ -D' & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} + \begin{bmatrix} D & -D' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_D \end{bmatrix}$$

$$\begin{bmatrix} I_g \end{bmatrix} = \begin{bmatrix} D & 0 \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_D \end{bmatrix}$$

Corresponding equivalent circuit:



Linearization of Averaged State Space Equations

Let $\left\langle \mathbf{x}(t) \right\rangle_{T_s} = \mathbf{X} + \hat{\mathbf{x}}(t)$ with $\| \mathbf{U} \| >> \| \hat{\mathbf{u}}(t) \|$

$\left\langle \mathbf{u}(t) \right\rangle_{T_s} = \mathbf{U} + \hat{\mathbf{u}}(t)$ $\qquad D >> \left| \hat{d}(t) \right|$

$\left\langle \mathbf{y}(t) \right\rangle_{T_s} = \mathbf{Y} + \hat{\mathbf{y}}(t)$ $\qquad \| \mathbf{X} \| >> \| \hat{\mathbf{x}}(t) \|$

$d(t) = D + \hat{d}(t) \Rightarrow d'(t) = D' - \hat{d}(t)$ $\qquad \| \mathbf{Y} \| >> \| \hat{\mathbf{y}}(t) \|$

Substitute into averaged state equations:

$$\mathbf{K} \frac{d\left(\mathbf{X}+\hat{\mathbf{x}}(t)\right)}{dt} = \left(\left(D+\hat{d}(t)\right) \mathbf{A}_1 + \left(D'-\hat{d}(t)\right) \mathbf{A}_2 \right) \left(\mathbf{X}+\hat{\mathbf{x}}(t)\right) + \left(\left(D+\hat{d}(t)\right) \mathbf{B}_1 + \left(D'-\hat{d}(t)\right) \mathbf{B}_2 \right) \left(\mathbf{U}+\hat{\mathbf{u}}(t)\right)$$

$$\left(\mathbf{Y}+\hat{\mathbf{y}}(t)\right) = \left(\left(D+\hat{d}(t)\right) \mathbf{C}_1 + \left(D'-\hat{d}(t)\right) \mathbf{C}_2 \right) \left(\mathbf{X}+\hat{\mathbf{x}}(t)\right) + \left(\left(D+\hat{d}(t)\right) \mathbf{E}_1 + \left(D'-\hat{d}(t)\right) \mathbf{E}_2 \right) \left(\mathbf{U}+\hat{\mathbf{u}}(t)\right)$$

AC Solution

Evaluate matrices in small-signal model:

$$\left(\mathbf{A}_1 - \mathbf{A}_2\right)\mathbf{X} + \left(\mathbf{B}_1 - \mathbf{B}_2\right)\mathbf{U} = \begin{bmatrix} -V \\ I \end{bmatrix} + \begin{bmatrix} V_g - IR_{on} + V_D \\ 0 \end{bmatrix} = \begin{bmatrix} V_g - V - IR_{on} + V_D \\ I \end{bmatrix}$$

$$\left(\mathbf{C}_1 - \mathbf{C}_2\right)\mathbf{X} + \left(\mathbf{E}_1 - \mathbf{E}_2\right)\mathbf{U} = \begin{bmatrix} I \end{bmatrix}$$

Small-signal ac state equations:

$$\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \hat{i}(t) \\ \hat{v}(t) \end{bmatrix} = \begin{bmatrix} -DR_{on} & D' \\ -D' & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} \hat{i}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} D & -D' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_g(t) \\ \hat{v}_D(t) \overset{0}{\nearrow} \end{bmatrix} + \begin{bmatrix} V_g - V - IR_{on} + V_D \\ I \end{bmatrix} \hat{d}(t)$$

$$\begin{bmatrix} \hat{i}_g(t) \end{bmatrix} = \begin{bmatrix} D & 0 \end{bmatrix} \begin{bmatrix} \hat{i}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_g(t) \\ \hat{v}_D(t) \overset{0}{\nearrow} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \hat{d}(t)$$

Resulting AC Equations

Small-signal ac equations, in scalar form:

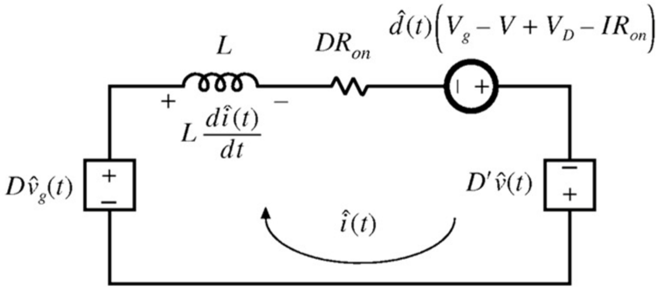
$$L\frac{d\hat{i}(t)}{dt} = D'\,\hat{v}(t) - DR_{on}\,\hat{i}(t) + D\,\hat{v}_g(t) + \left(V_g - V - IR_{on} + V_D\right)\hat{d}(t)$$

$$C\frac{d\hat{v}(t)}{dt} = -D'\,\hat{i}(t) - \frac{\hat{v}(t)}{R} + I\,\hat{d}(t)$$

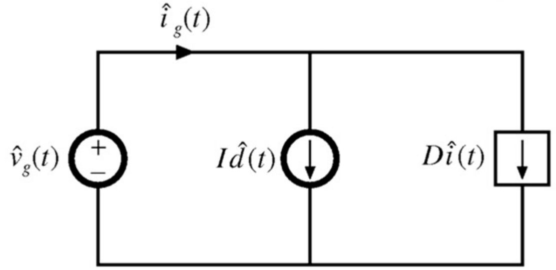
$$\hat{i}_g(t) = D\,\hat{i}(t) + I\,\hat{d}(t)$$

Corresponding equivalent circuits:

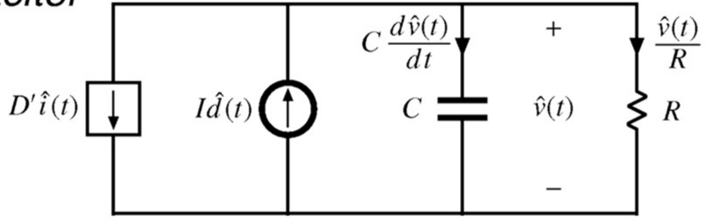
inductor equation



input eqn



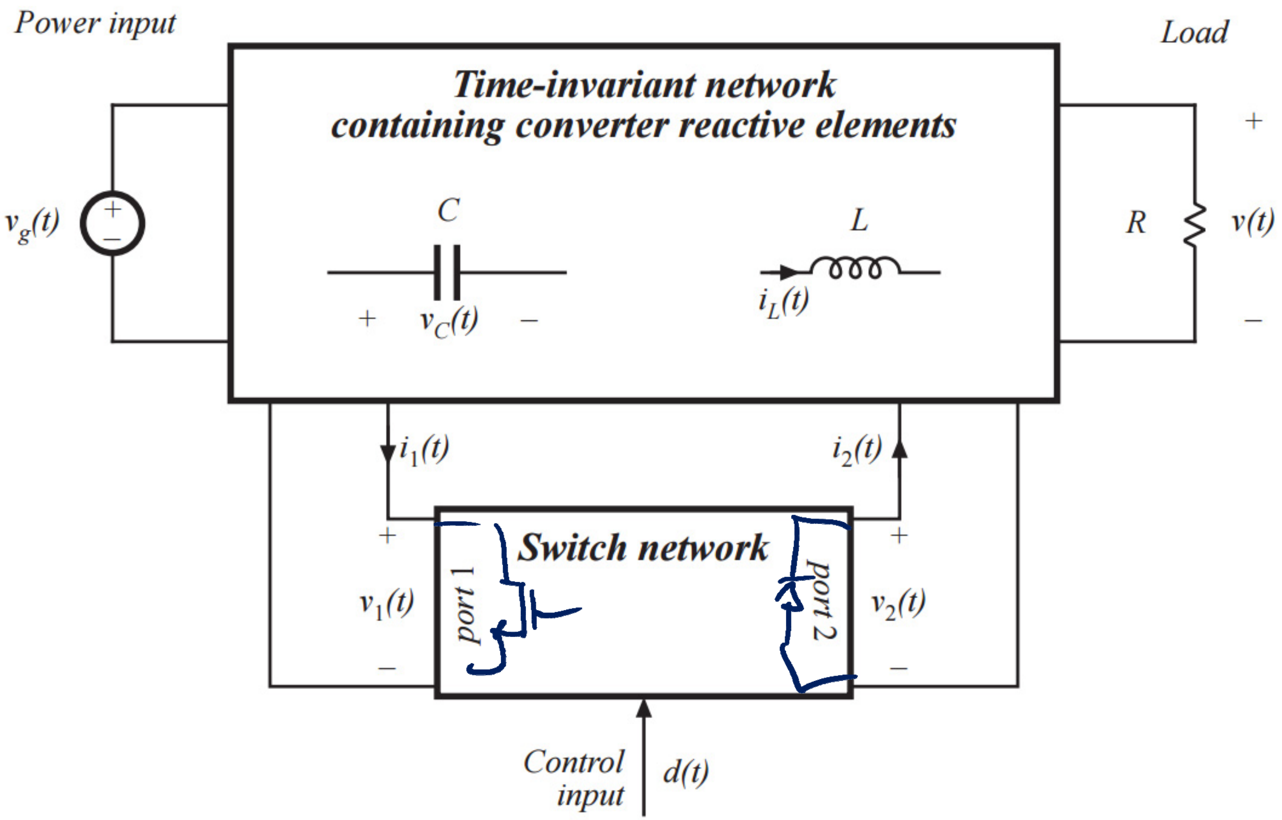
capacitor eqn



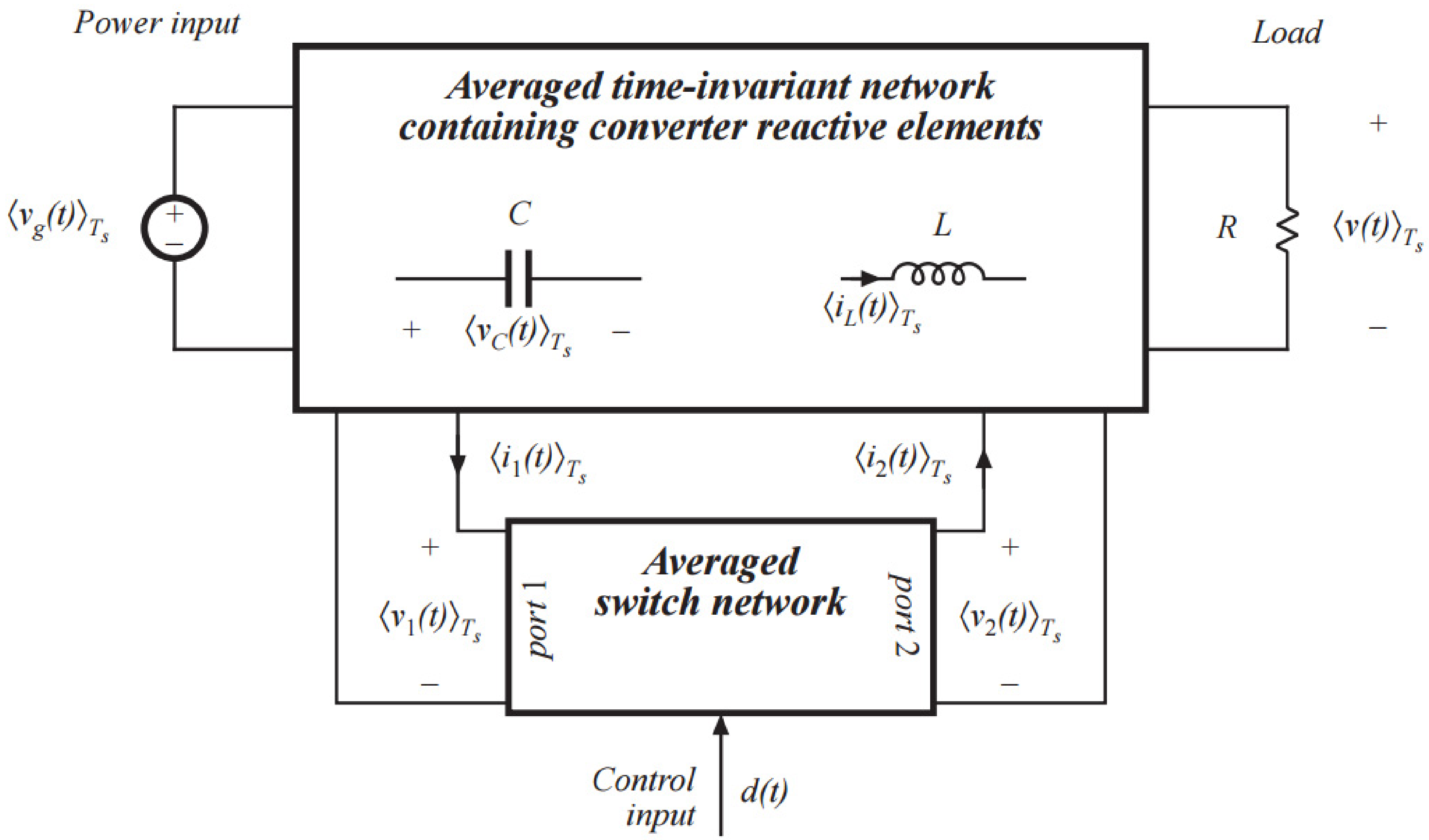
Section 14.1

AVERAGE SWITCH MODELING

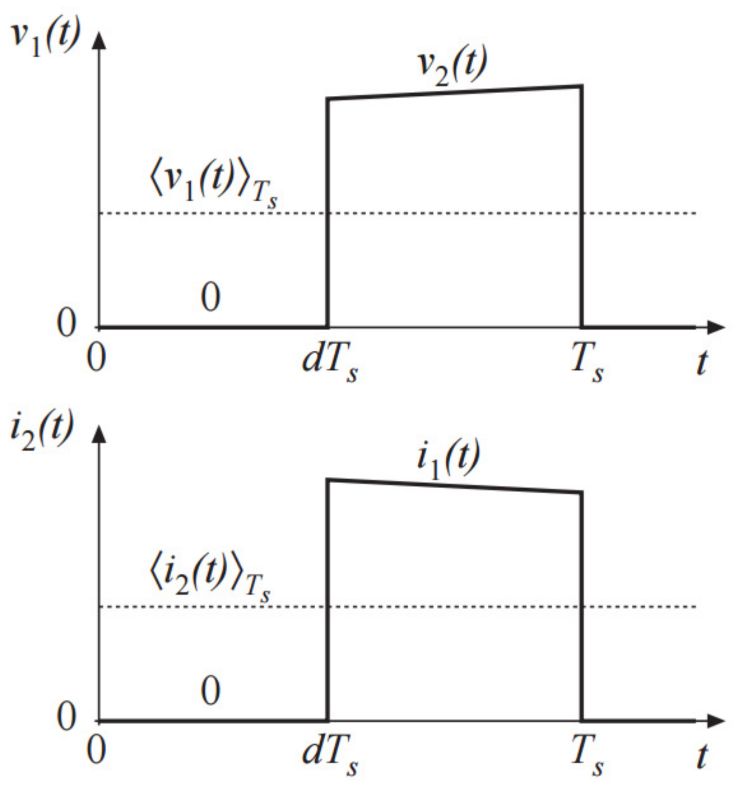
Removing Switch Network

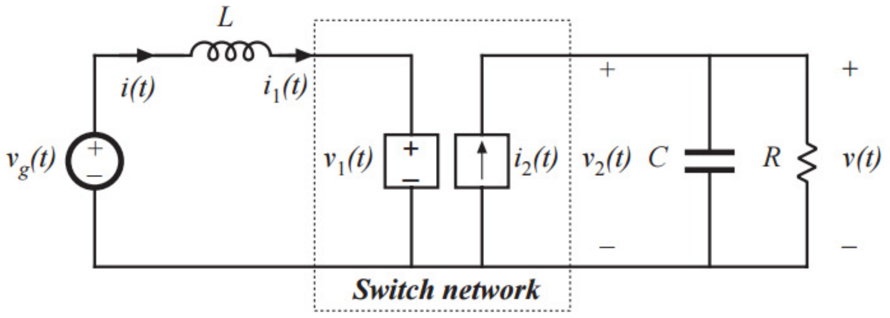


Averaged Switch Network



Definition of Equivalent Sources



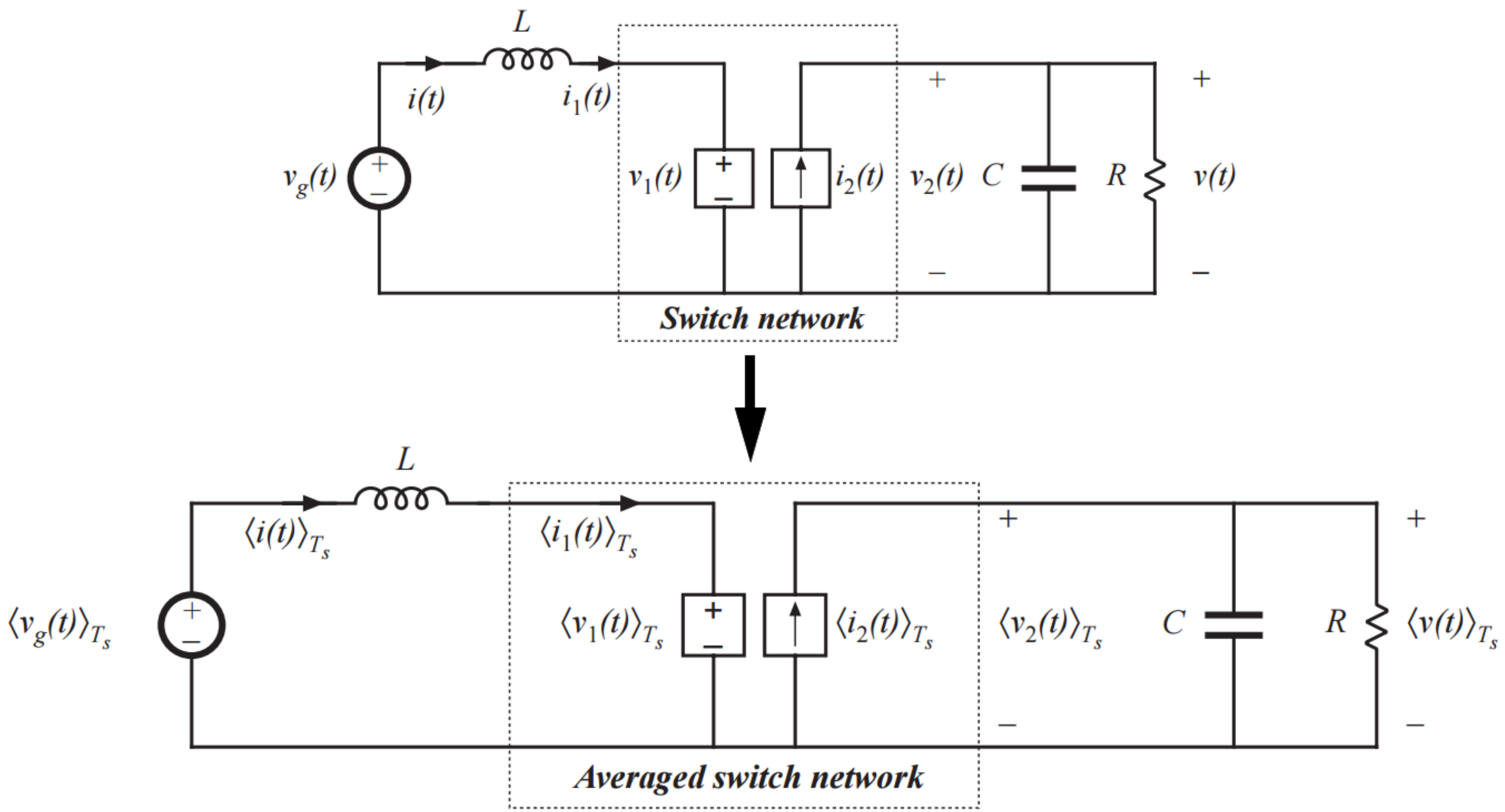


The waveforms of the dependent generators are defined to be identical to the actual terminal waveforms of the switch network.

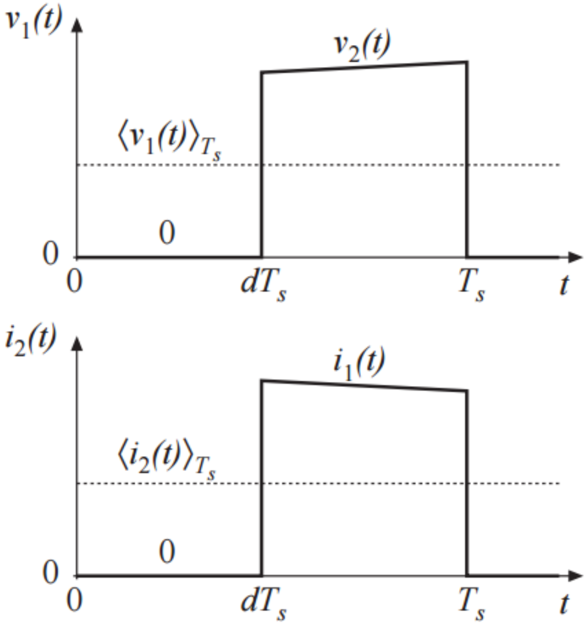
The circuit is therefore electrical identical to the original converter.

So far, no approximations have been made.

Switch Averaging

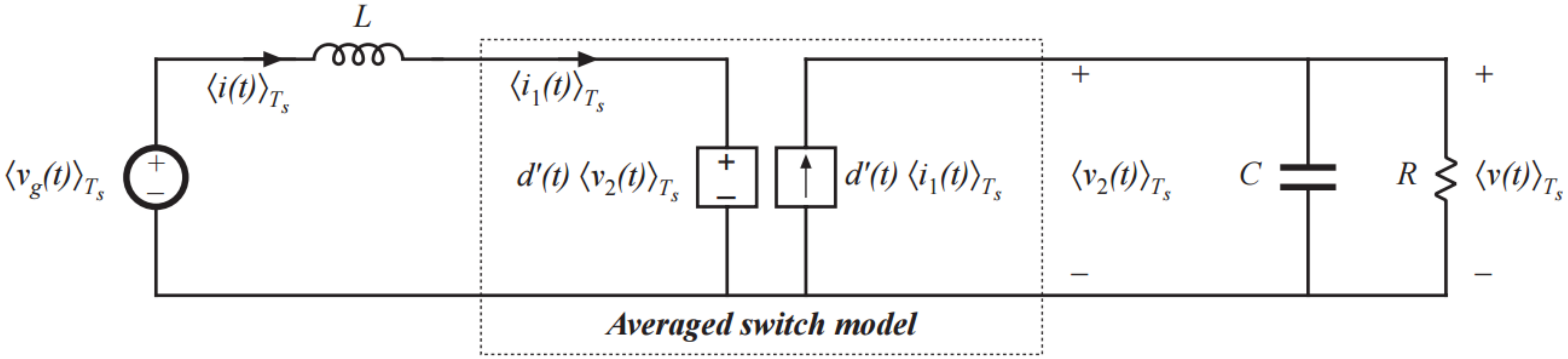


Computation of Average Values

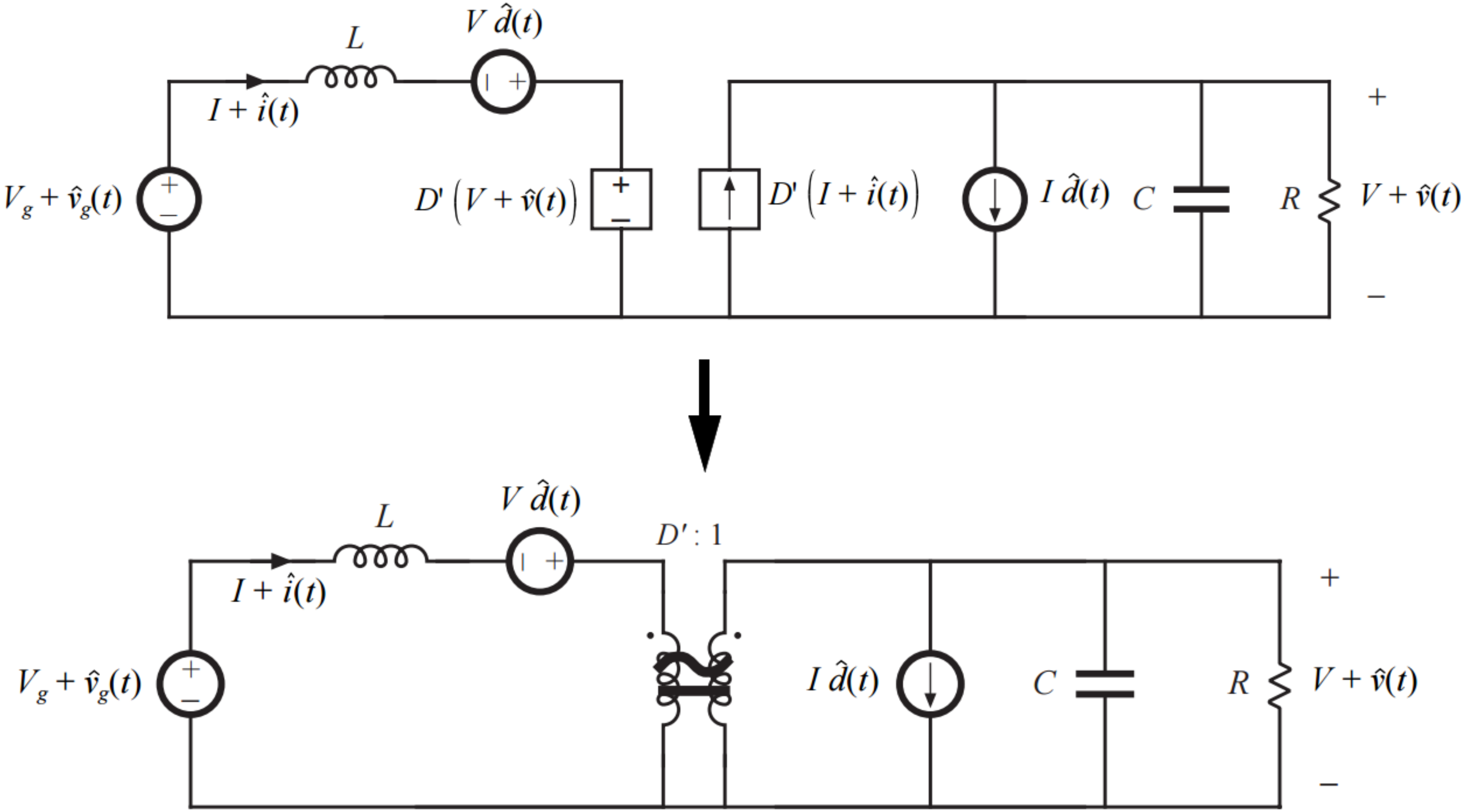


Average the waveforms of the dependent sources:

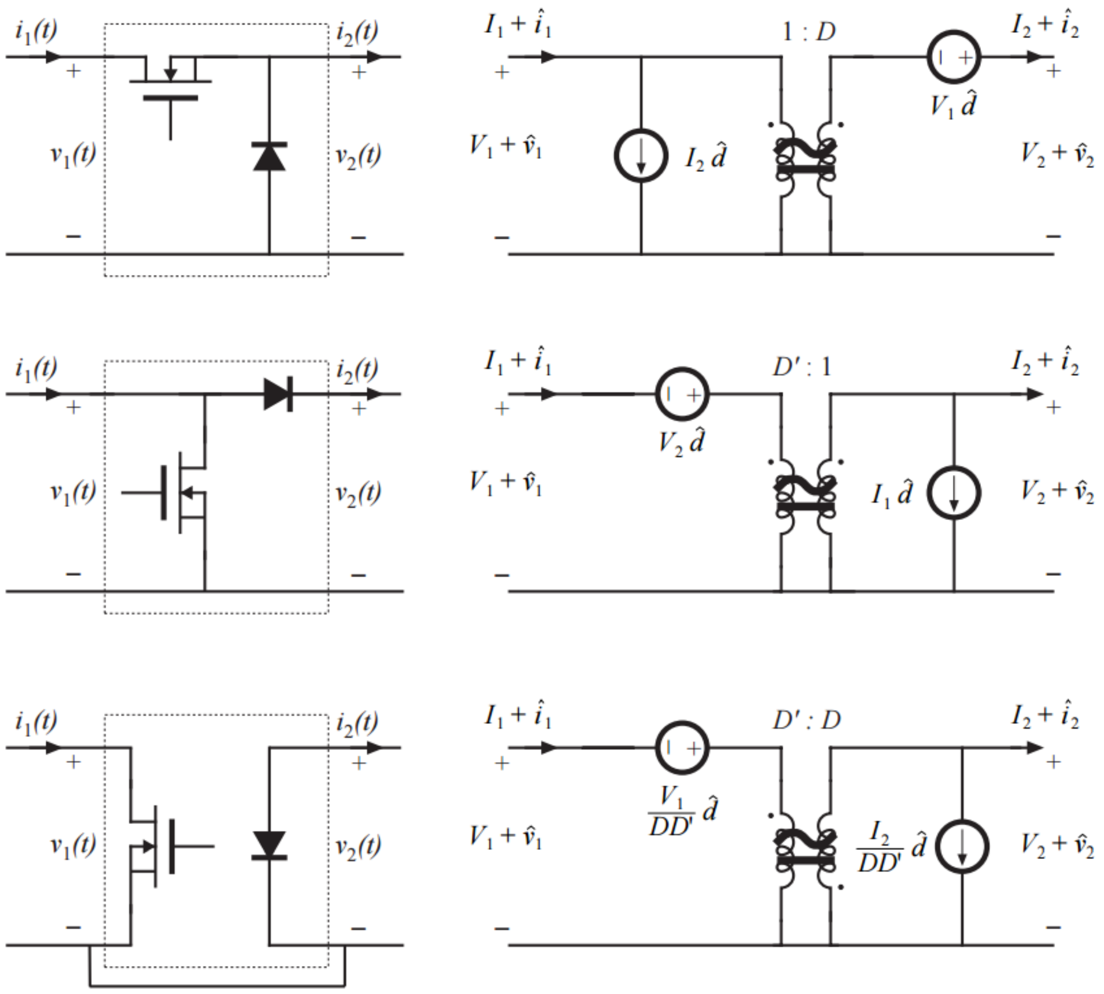
$$\left\langle v_1(t)\right\rangle_{T_s} = d'(t)\left\langle v_2(t)\right\rangle_{T_s}$$
$$\left\langle i_2(t)\right\rangle_{T_s} = d'(t)\left\langle i_1(t)\right\rangle_{T_s}$$



Linearization of Model



Averaged Switch Cells



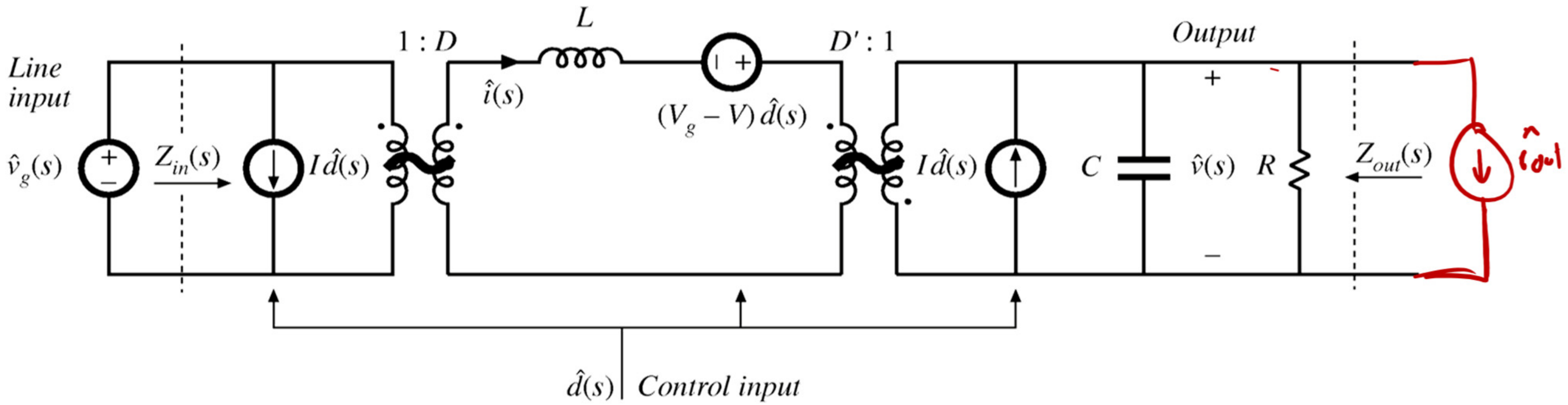
see also

Appendix 3
Averaged switch modeling of a CCM SEPIC

Chapter 8

CONVERTER TRANSFER FUNCTIONS

Buck Boost Model



$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{d}(s)=0,\ \hat{i}_{out}(s)=0}$$

Line-to-output

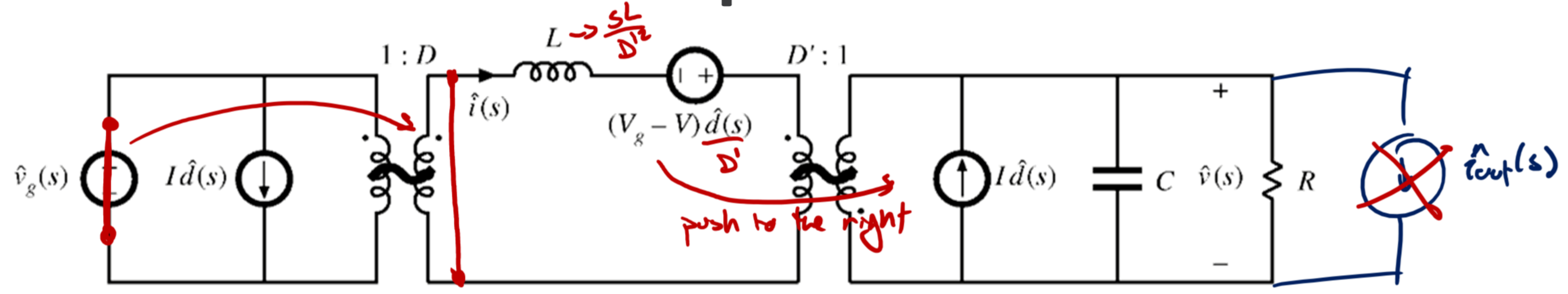
$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_g(s)=0,\ \hat{i}_{out}=0}$$

Control-to-output

$$Z_{out}(s) = \left.\frac{\hat{v}(s)}{\hat{i}_{out}(s)}\right|_{\hat{d}(s)=0,\ \hat{v}_g(s)=0}$$

load-to-output

Buck-Boost Control-to-Output TF



Solve $G_{vd}(s) = \left.\dfrac{\hat{v}(s)}{\hat{d}(s)}\right|_{\substack{\hat{v}_g(s)=0 \\ \hat{i}_{out}(s)=0}}$

$$\hat{v}(s) = \frac{(V_g - V)}{D'}\,\hat{d}(s)\,\frac{R \| \frac{1}{sC}}{R \| \frac{1}{sC} + \frac{sL}{D'^2}} + I\hat{d}(s)\left(R \,\|\, \frac{1}{sC} \,\|\, \frac{sL}{D'^2}\right)$$

$$G_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)} = -\frac{V_g - V}{D'}\,\frac{\frac{R}{1+sCR}\left(\frac{1+sCR}{R}\right)}{\left(\frac{R}{1+sCR} + \frac{sL}{D'^2}\right)\left(\frac{1+sCR}{R}\right)} + I\left(\frac{1}{\frac{1}{R} + sC + \frac{D'^2}{sL}}\right)\left(\frac{\frac{sL}{D'^2}}{\frac{sL}{D'^2}}\right)$$

$$G_{vd}(s) = -\left(\frac{V_g - V}{D'}\right)\frac{1}{1 + \frac{sL}{D'^2 R} + s^2\frac{LC}{D'^2}} + I\left(\frac{sL/D'^2}{1 + \frac{sL}{D'^2 R} + \frac{s^2 LC}{D'^2}}\right)$$

Buck-Boost Control-to-Output TF

$$G_{vd}(s) = \frac{-\left(\frac{V_g - V}{D'}\right) + I\frac{sL}{D'^2}}{1 + \frac{sL}{D'^2 R} + s^2\frac{LC}{D'^2}}$$

$$G_{vd}(s) = \left(-\frac{V_g - V}{D'}\right)\frac{1 - sL\frac{I}{D'(V_g - V)}}{1 + \frac{sL}{D'^2 R} + s^2\frac{LC}{D'^2}}$$

$$G_{vd}(s) = G_{d0}\frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$ → standard form

Control-to-output Transfer Function