Bode Plot Review

For any s-domain expression (Transfer Function)

$G(s) \rightarrow G(j\omega) \cong$ frequency response (if no RHP poles)

Bode plot: straight line approx to plots of

$\| G(j\omega) \|_{dB}$ & $\measuredangle G(j\omega)$ -vs- $\log(f)$

For LTI circuits, any TF between v/i is a ratio of polynomials

$$G(s) = \frac{\sum_{i=0}^{\wedge} a_i s^i}{\sum_{i=0}^{\wedge} b_i s^i}$$

factor & manipulate into standard form to find poles & zeros

then, for any term $\| s^i \|_{dB} = 20 \log(|s^i|) = 20 \cdot i \log(|j\omega|) = 20 i \log(\omega)$

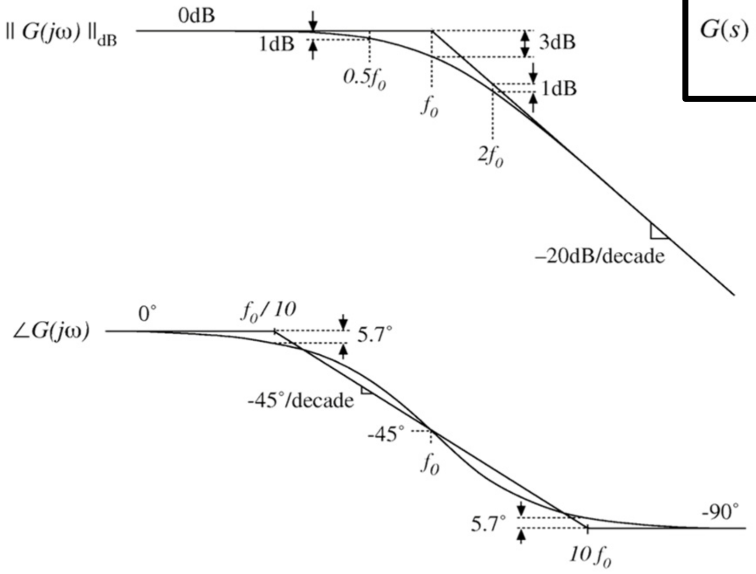
$\hookrightarrow 20 \cdot i$ dB/dec straight line

on a log scale "everything" is far apart, so approximations can be made to neglect non-dominant terms

$$\| 1 + s^i \|_{dB} \approx \begin{cases} \| 1 \|_{dB} & |s| < 1 \\ \| s^i \|_{dB} & |s| > 1 \end{cases}$$

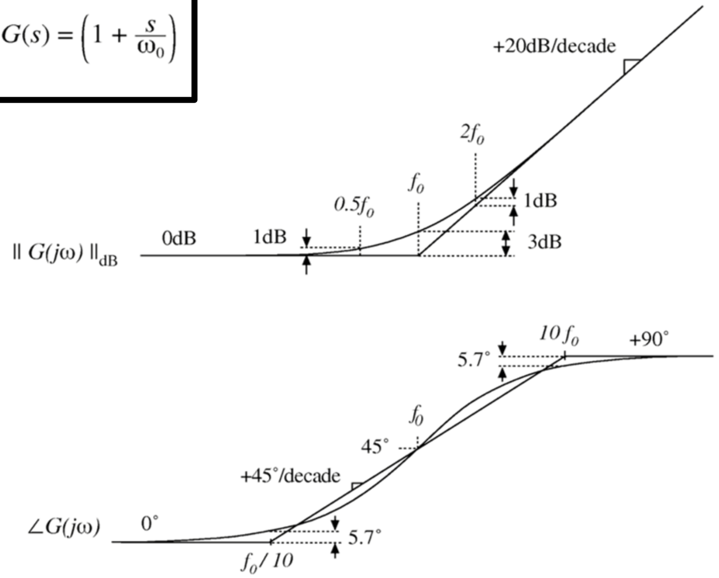
Real Pole

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)}$$



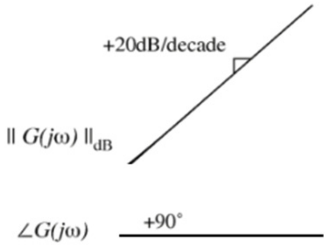
Real Zero

$$G(s) = \left(1 + \frac{s}{\omega_0}\right)$$



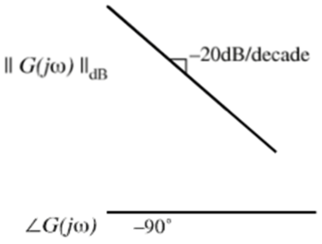
$$G(s) = s$$

LF Zero



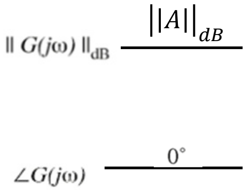
$$G(s) = \frac{1}{s}$$

LF Pole



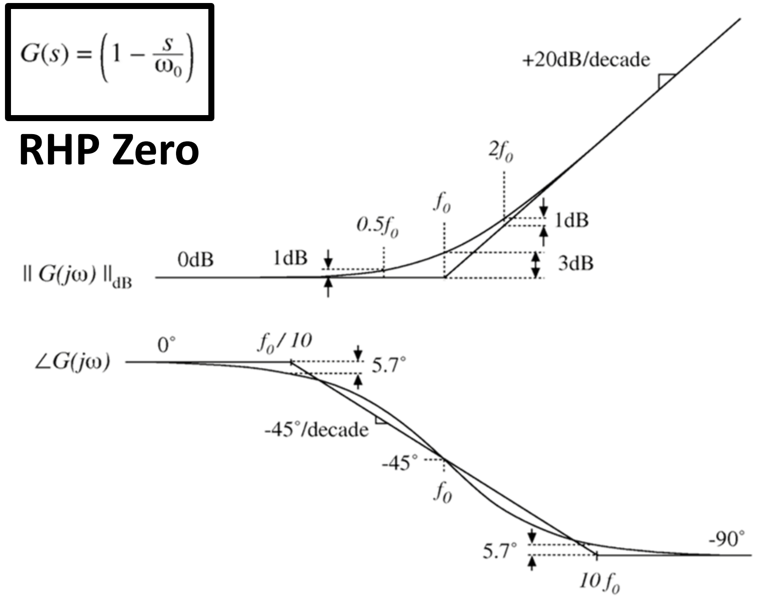
$$G(s) = A$$

Constant



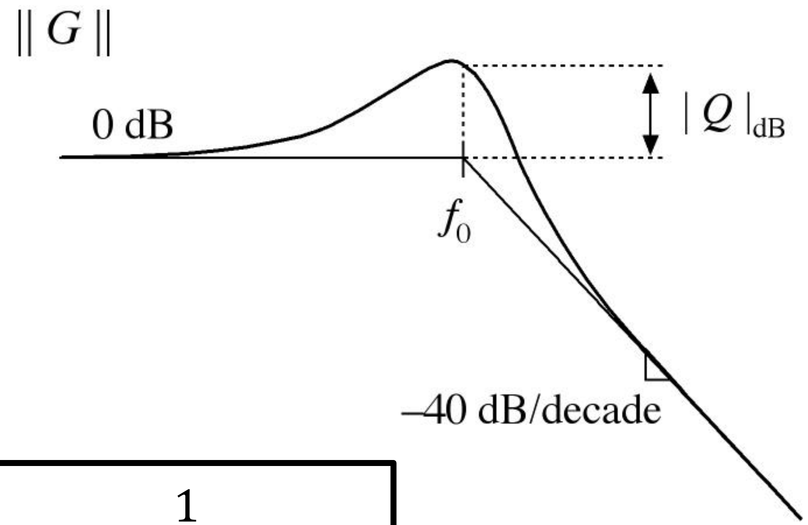
$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$

RHP Zero



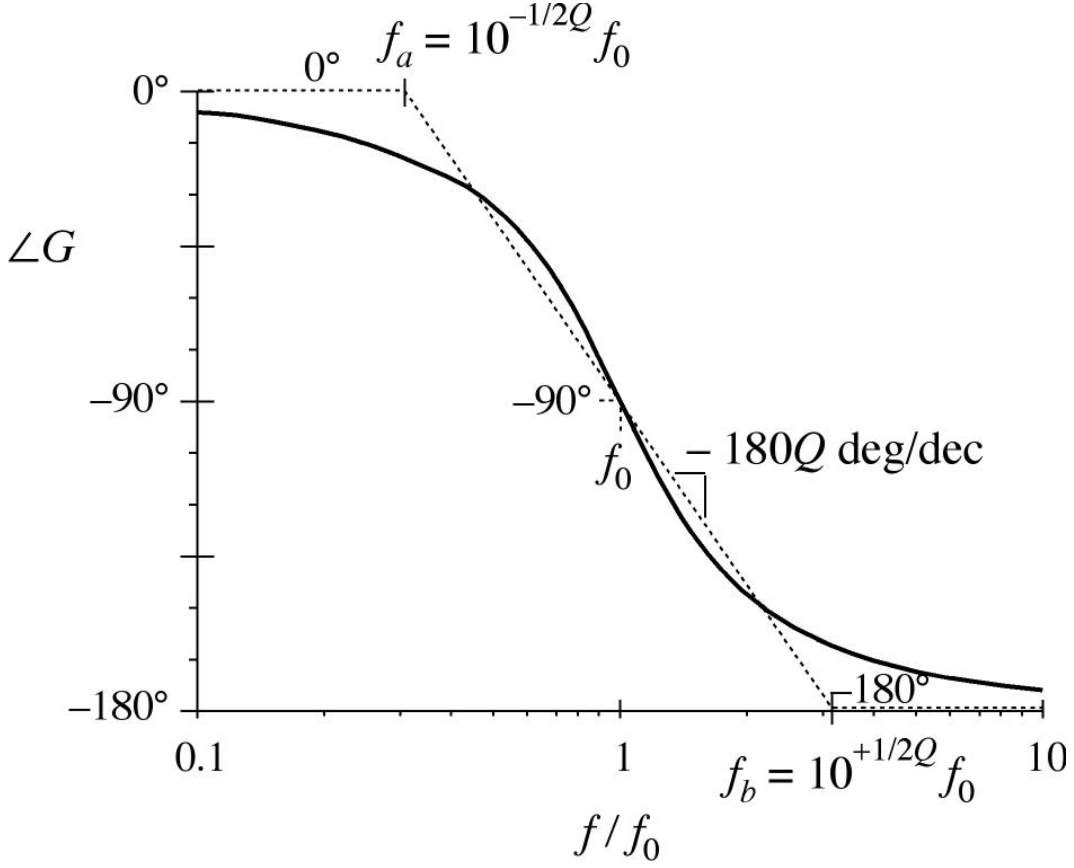
Asymptotes for Complex Poles, Q>0.5

Magnitude

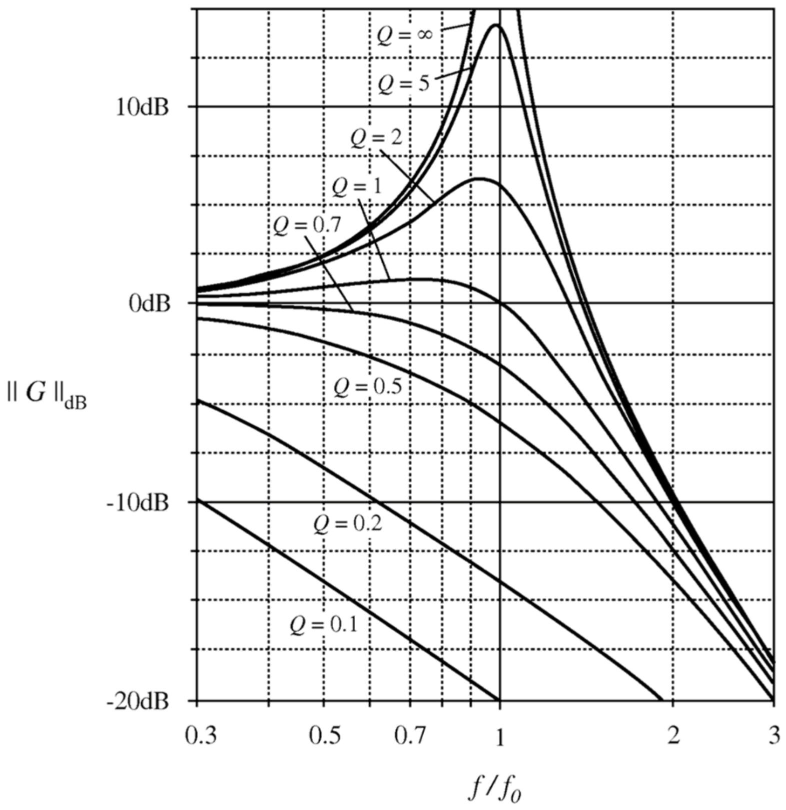


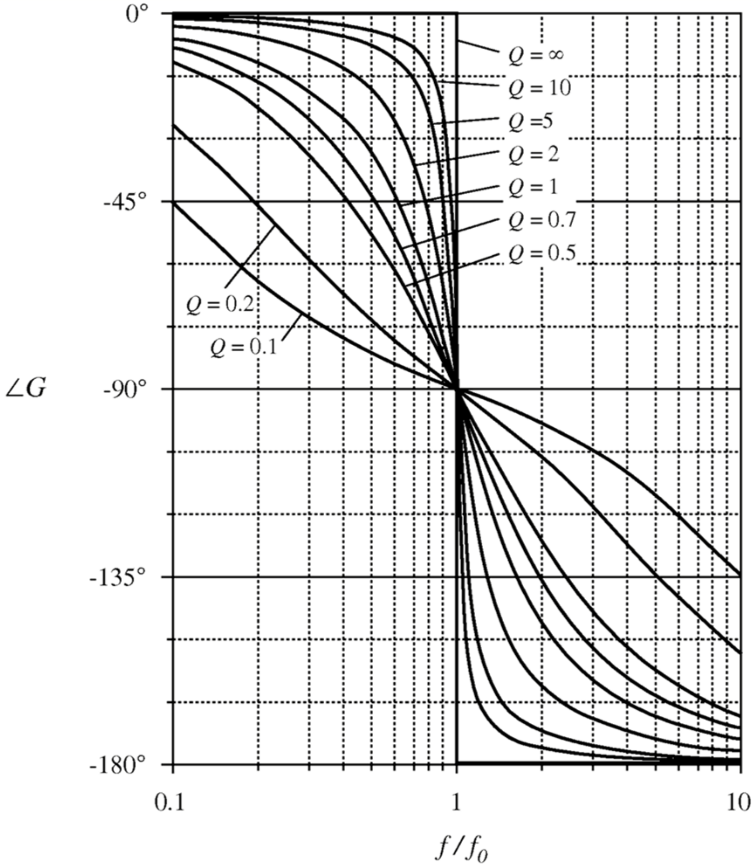
$$G(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

Phase



Curves for Varying Q





Combinations

for some transfer function

$$G_1(s) \Rightarrow G_1(j\omega) = R_1(\omega)\, e^{j\theta_1(\omega)}$$

where $R_1(\omega) = |G_1(j\omega)|$
$\theta_1(\omega) = \angle G_1(j\omega)$

then, if we have

$$G_3(s) = G_1(s) \cdot G_2(s)$$

$$G_3(j\omega) = R_1(\omega)\, e^{j\theta_1(\omega)} \cdot R_2(\omega)\, e^{j\theta_2(\omega)}$$

$$= \underbrace{R_1(\omega) \cdot R_2(\omega)}_{\text{magnitudes multiply}}\, e^{j\underbrace{(\theta_1(\omega) + \theta_2(s))}_{\text{phases add}}}$$

phases add $\theta_3(\omega) = \theta_1(\omega) + \theta_2(\omega)$

magnitudes multiply $R_3(\omega) = R_1(\omega) \cdot R_2(\omega)$

$$\|G_3(j\omega)\|_{dB} = 20 \log(|R_1(\omega) \cdot R_2(\omega)|)$$

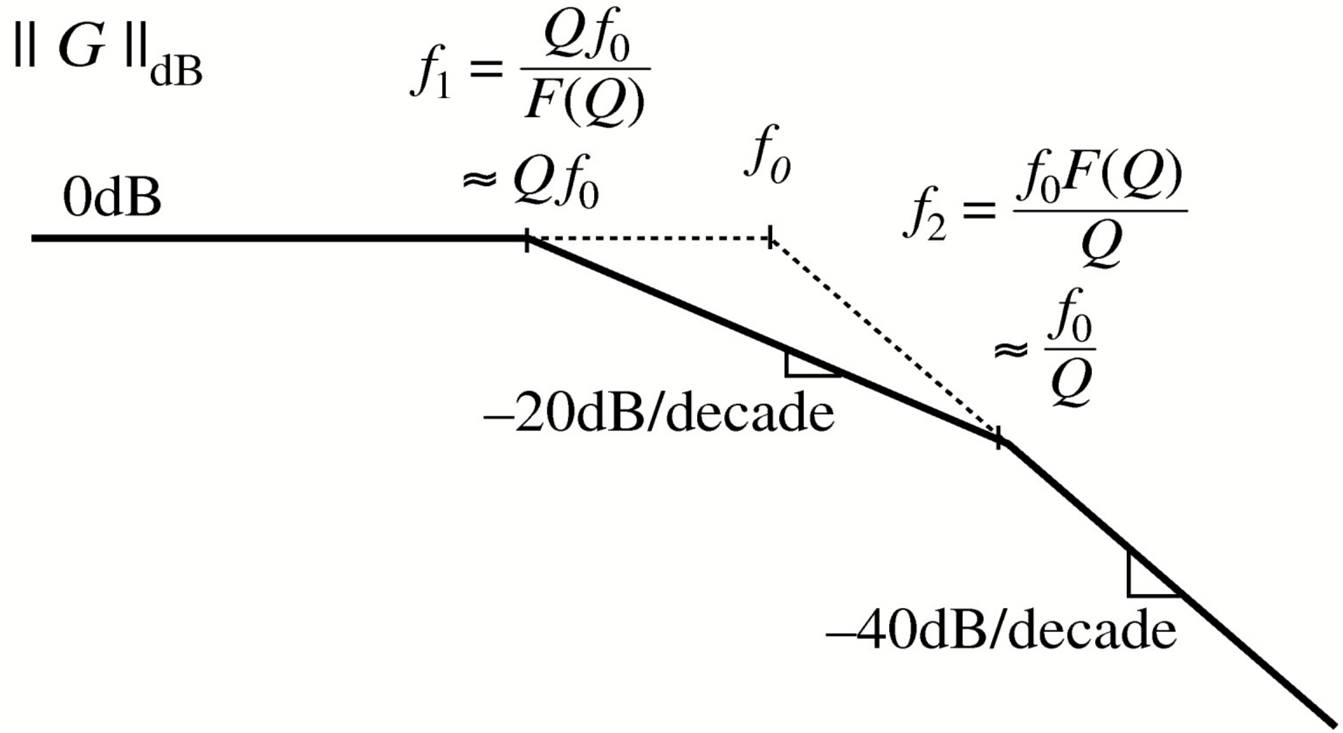
$$= 20 \log(|R_1(\omega)|) + 20 \log(|R_2(\omega)|) = \|R_1(\omega)\|_{dB} + \|R_2(\omega)\|_{dB}$$

$$\|G_3(j\omega)\|_{dB} = \|G_1(j\omega)\|_{dB} + \|G_2(j\omega)\|_{dB}$$

But, on log scale, magnitudes add on log scale

The Low-Q Approximation

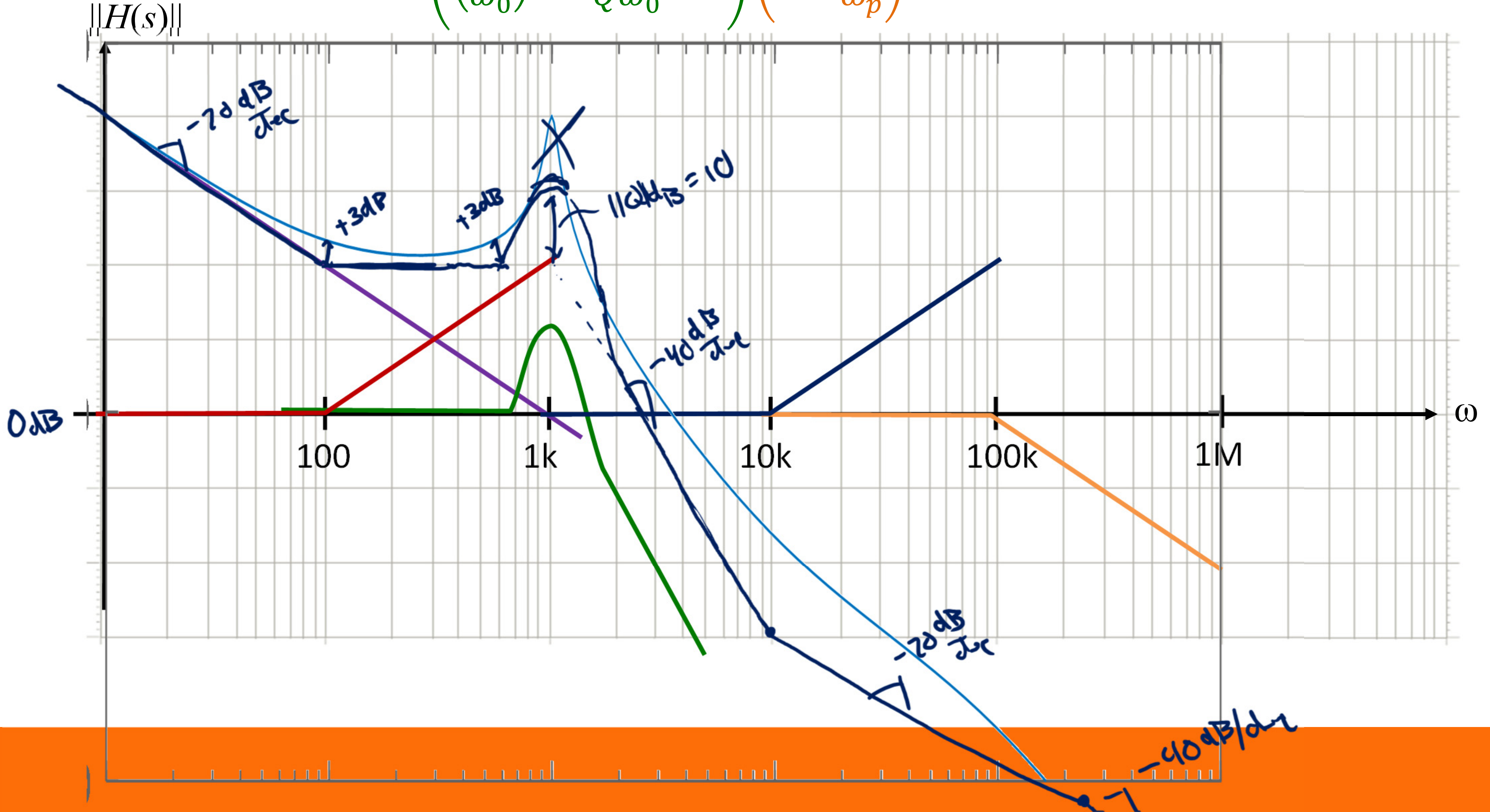
if Q << 0.5



Example

A	w_{z1}	w_{z2}	ω_0	Q	ω_p
1000	100	10k	1k	10	100k

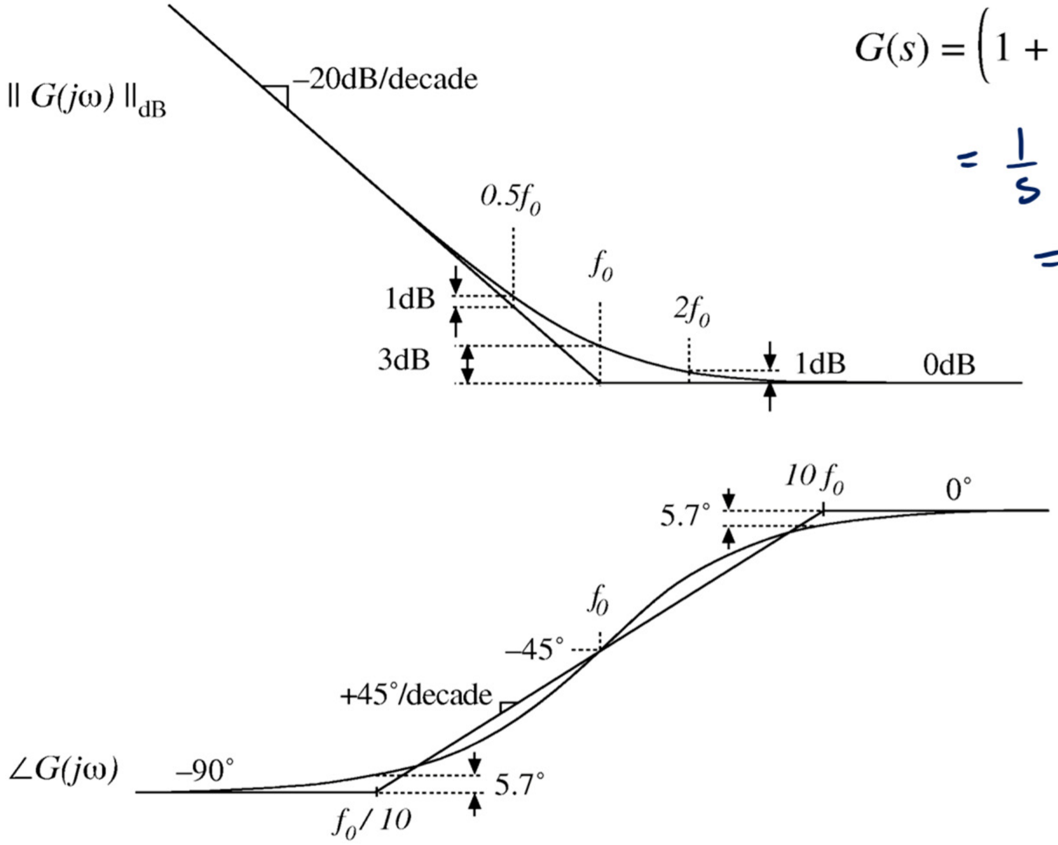
$$H(s) = \frac{A}{s}\frac{\left(1+\frac{s}{w_{z1}}\right)\left(1+\frac{s}{w_{z2}}\right)}{\left(\left(\frac{s}{\omega_0}\right)^2+\frac{s}{Q\omega_0}+1\right)\left(1+\frac{s}{\omega_p}\right)}$$



Additional Design-Oriented Techniques

- 8.1.4: Frequency Inversion
- 8.1.9: Approximate Roots of Arbitrary-Degree Polynomial
- 8.3: Graphical Construction of Impedances and Transfer Functions

Inverted Zero



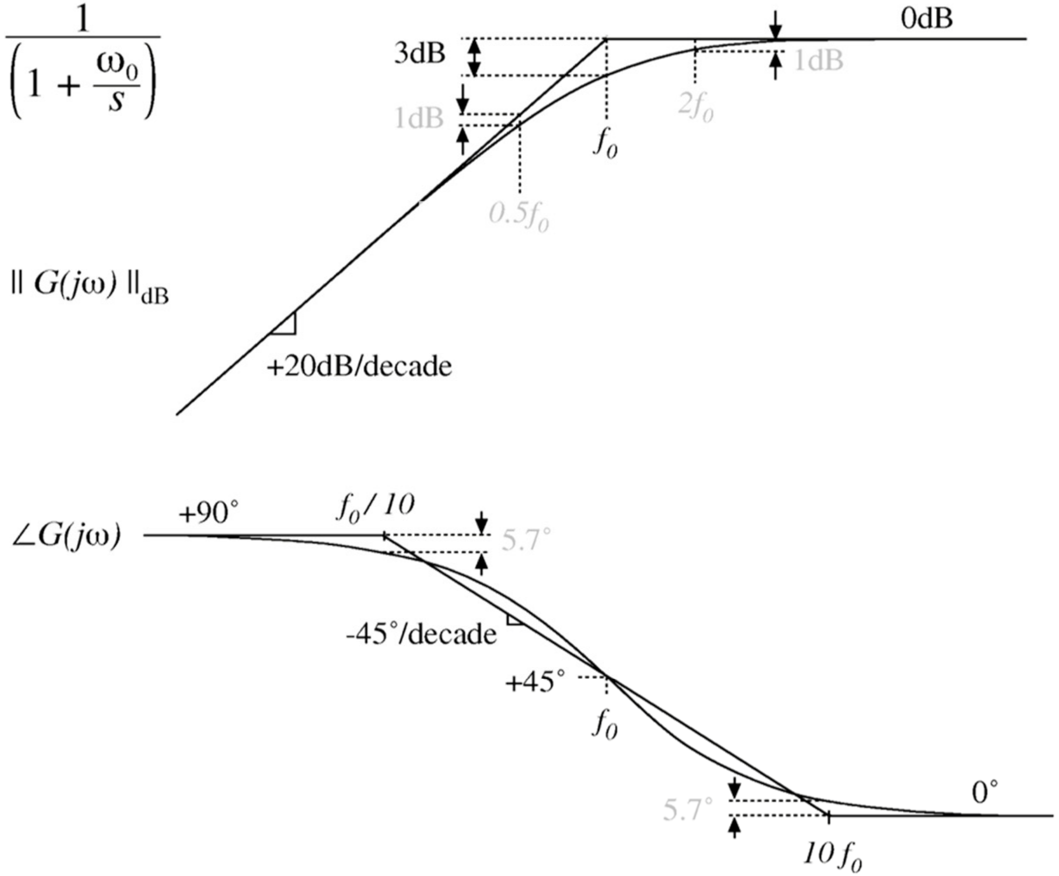
$$G(s) = \left(1 + \frac{\omega_0}{s}\right)$$

$$= \frac{1}{s}\left(s + \omega_0\right)$$

$$= \frac{1}{\omega_0} \cdot \frac{1}{s}\left(\frac{s}{\omega_0} + 1\right)$$

Inverted Pole

$$G(s) = \frac{1}{\left(1 + \frac{\omega_0}{s}\right)}$$



Example

$$H(s) = \frac{\frac{A}{s}\left(1+\frac{s}{w_{z1}}\right)\left(1+\frac{s}{w_{z2}}\right)}{\left(\left(\frac{s}{\omega_0}\right)^2+\frac{s}{Q\omega_0}+1\right)\left(1+\frac{s}{\omega_p}\right)}$$

$$\frac{A}{s}\left(1+\frac{s}{\omega_z}\right)$$
$$= \frac{A}{\omega_{z1}}\left(1+\frac{\omega_{z1}}{s}\right)$$

A	w_{z1}	w_{z2}	ω_0	Q	ω_p
1000	100	10k	1k	10	100k