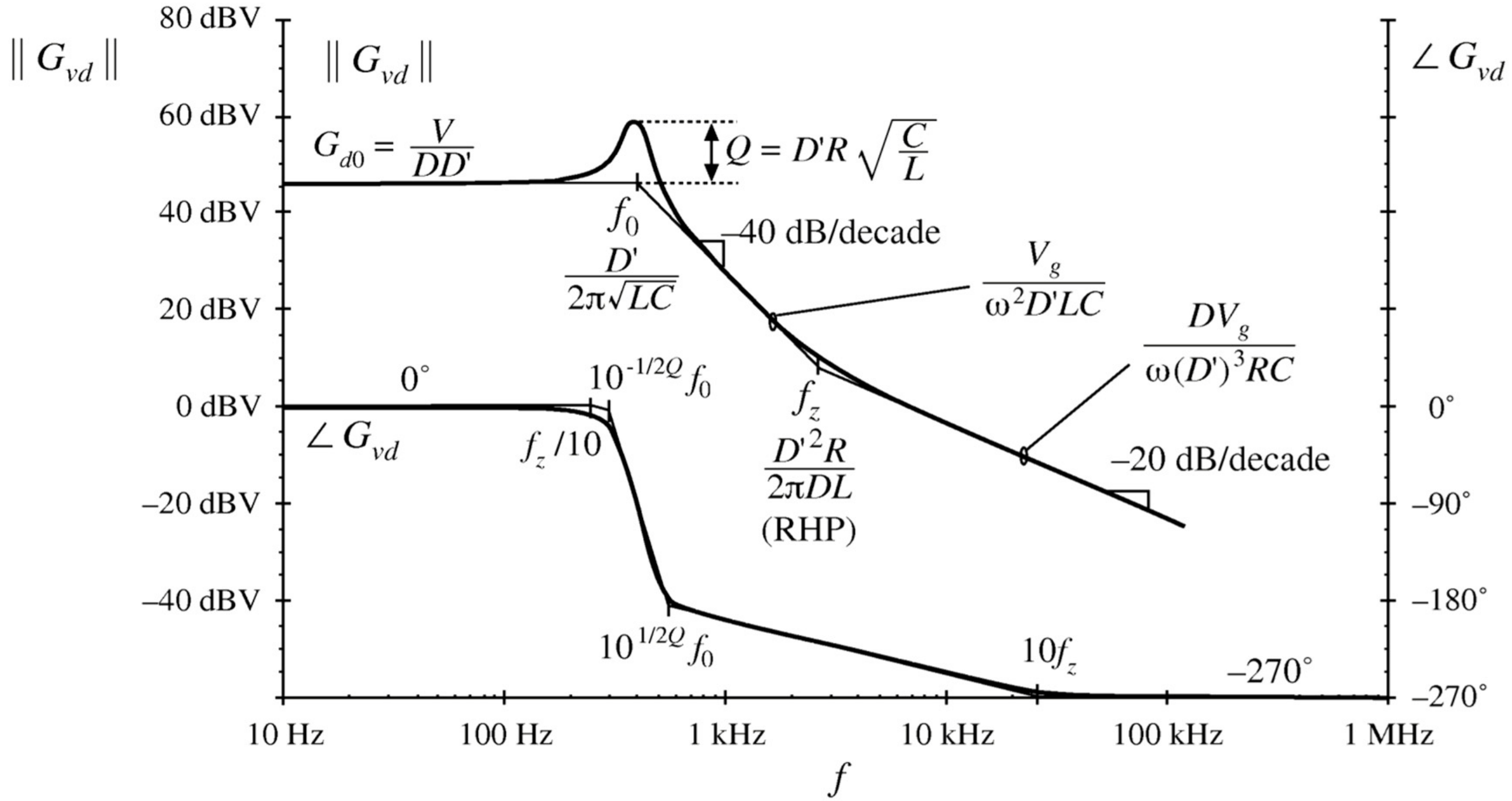
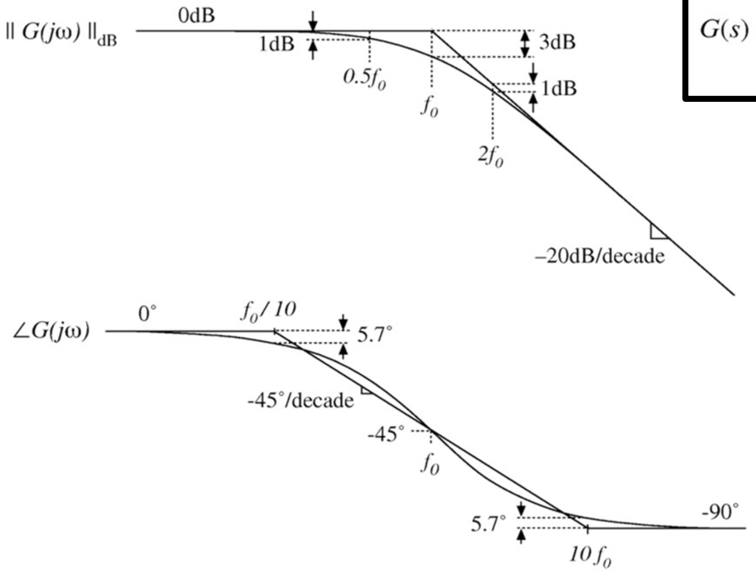
Control-to-output Transfer Function



Bode Plot Review

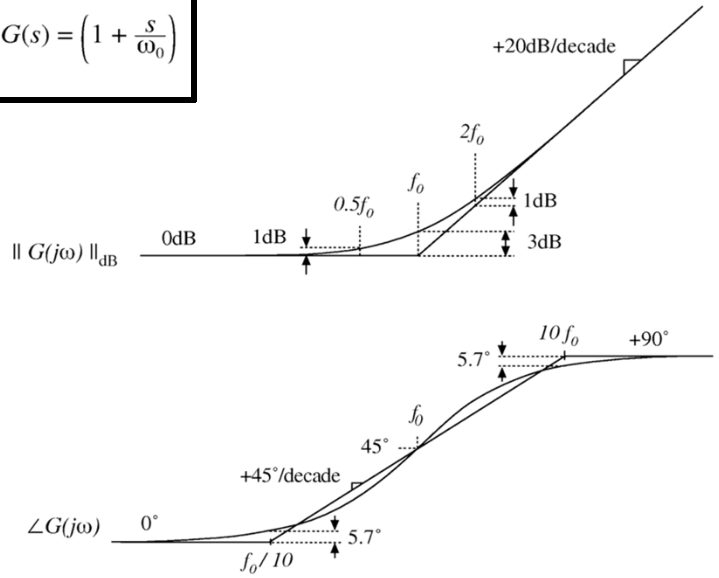
Real Pole

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)}$$



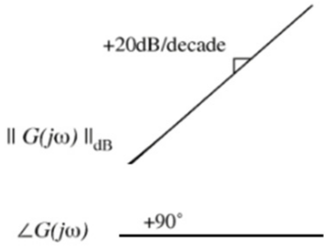
Real Zero

$$G(s) = \left(1 + \frac{s}{\omega_0}\right)$$



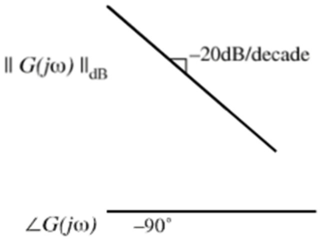
$$G(s) = s$$

LF Zero



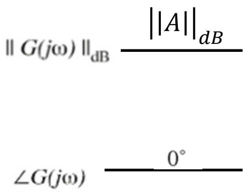
$$G(s) = \frac{1}{s}$$

LF Pole



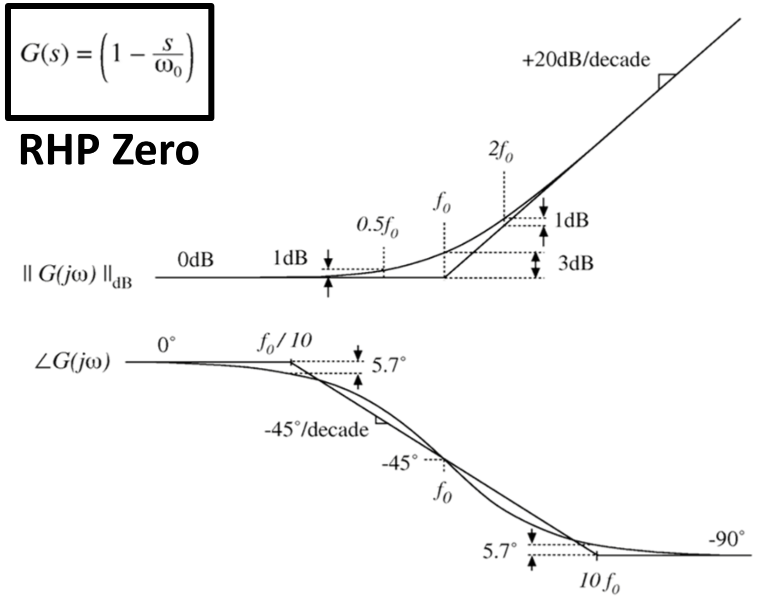
$$G(s) = A$$

Constant



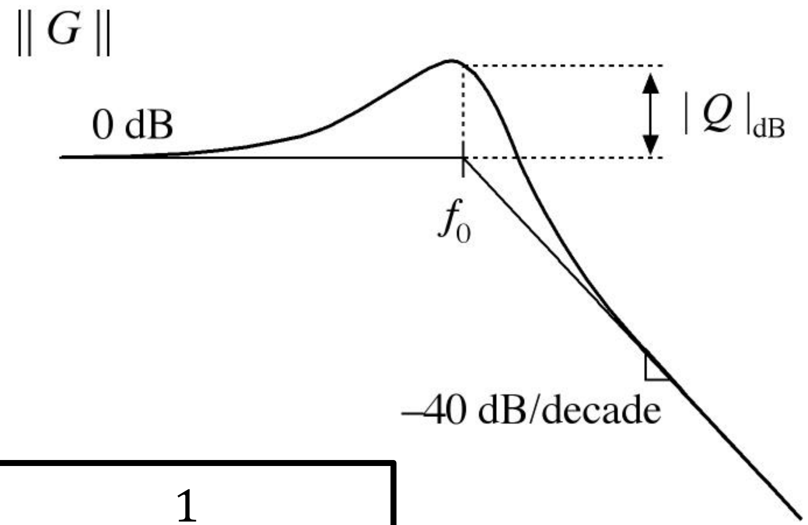
$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$

RHP Zero

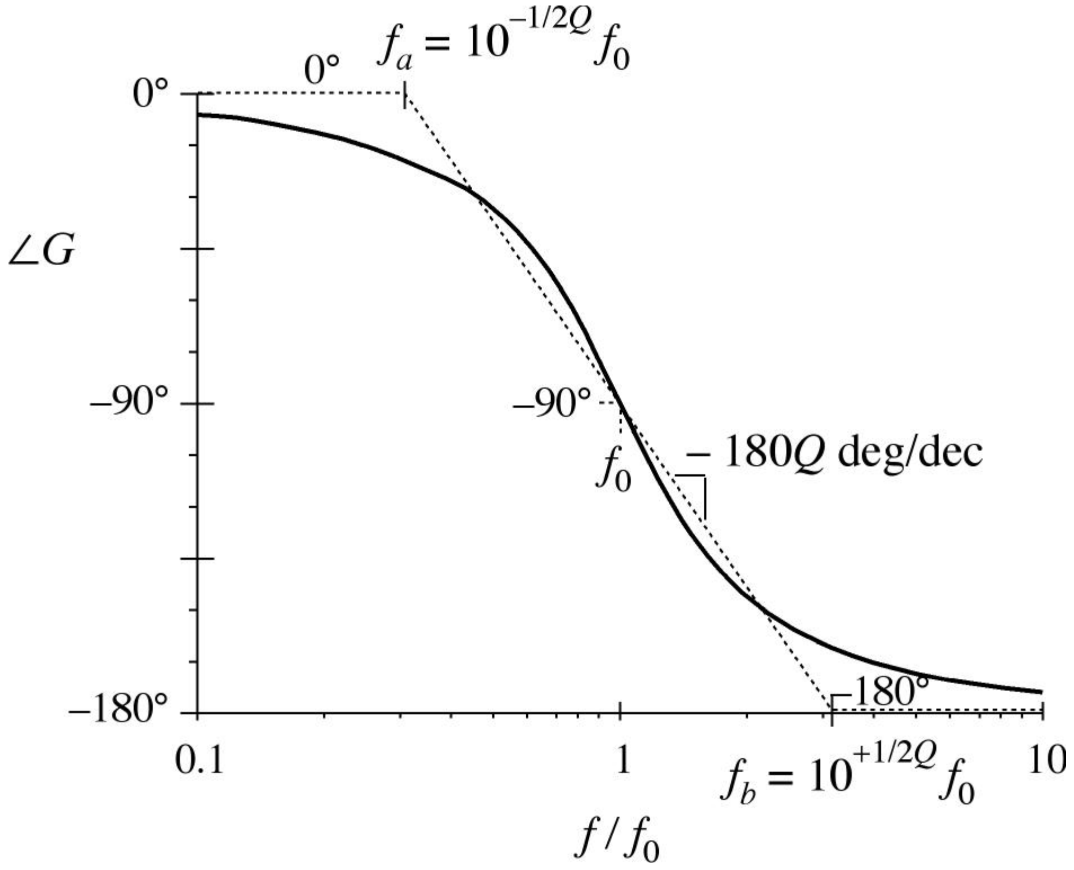


Asymptotes for Complex Poles, *Q*>0.5

Magnitude

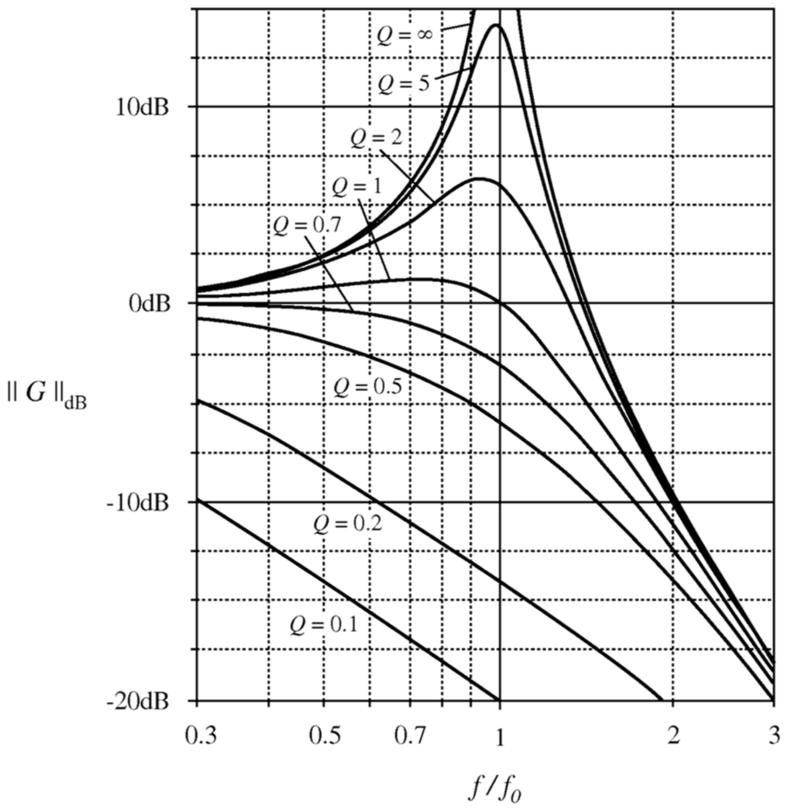


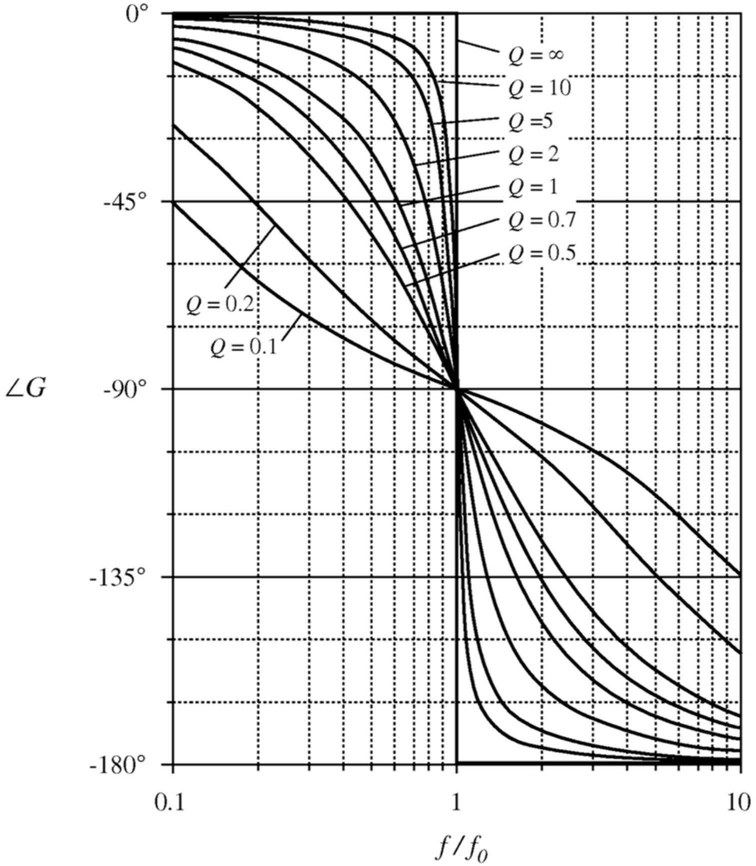
Phase



$$G(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

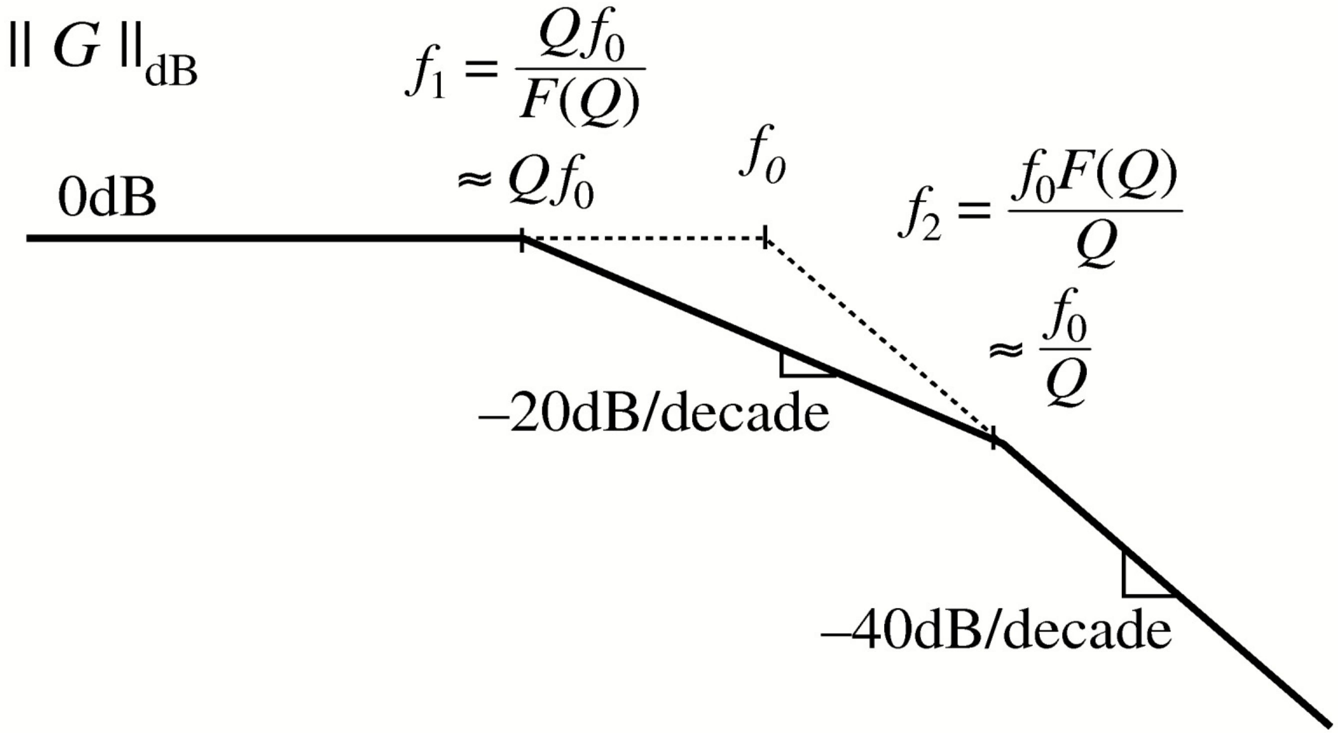
Curves for Varying Q





Combinations

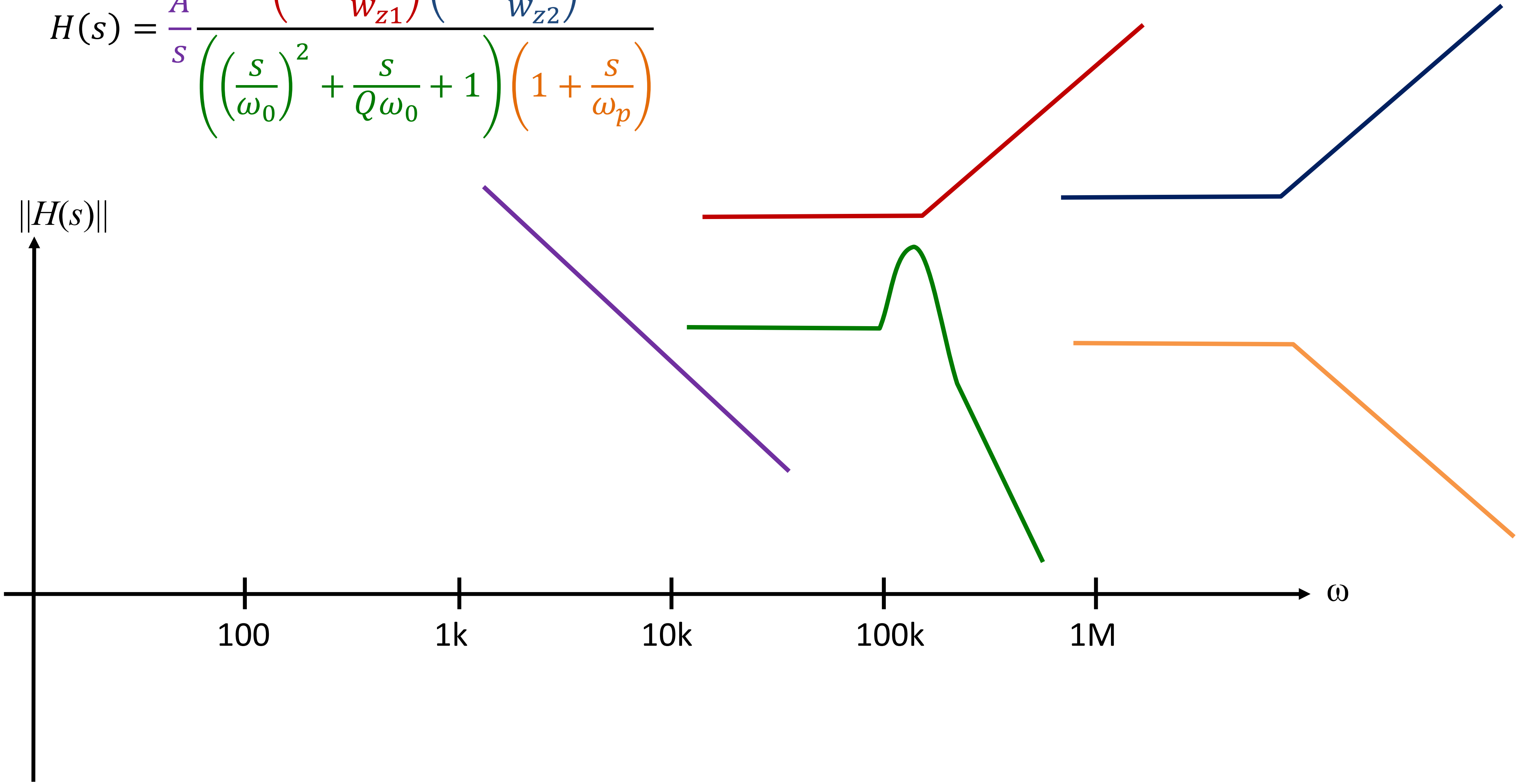
The Low-Q Approximation



Example

A	w_{z1}	w_{z2}	ω_0	Q	ω_p
1000	100	10k	1k	10	100k

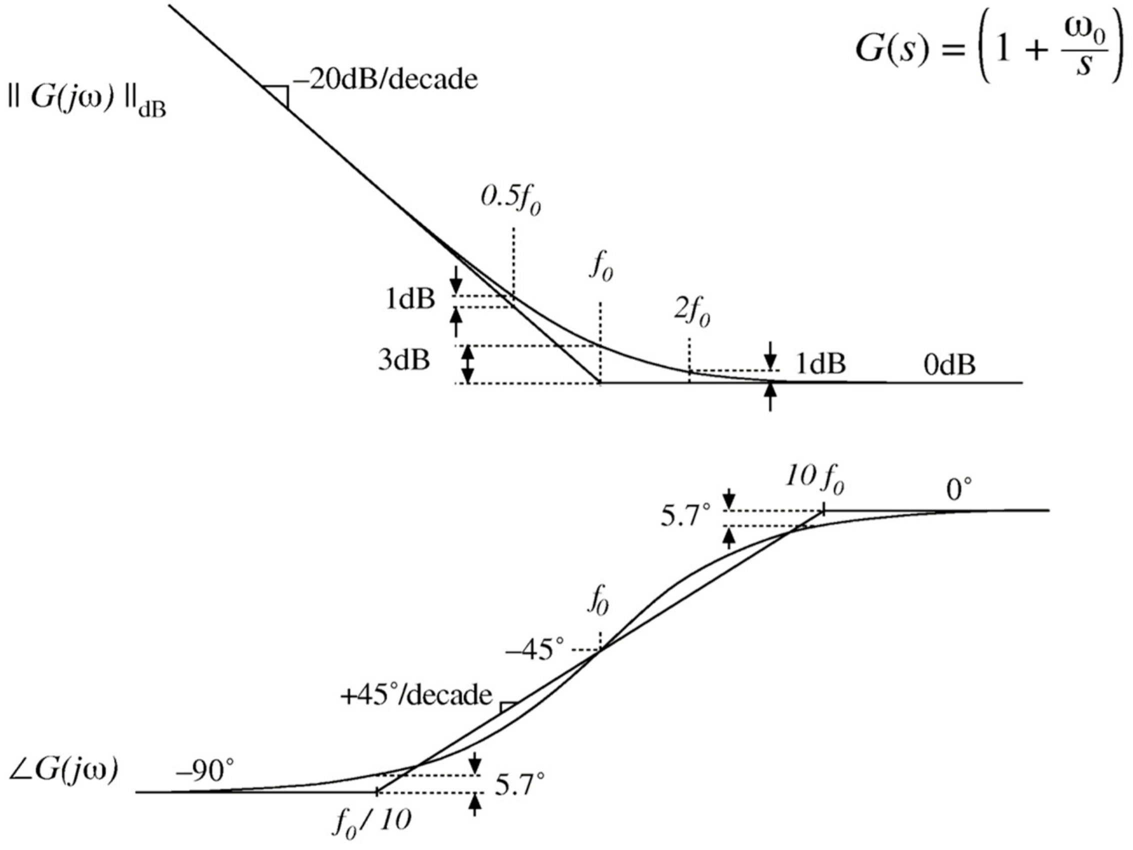
$$H(s) = \frac{A}{s} \frac{\left(1 + \frac{s}{w_{z1}}\right)\left(1 + \frac{s}{w_{z2}}\right)}{\left(\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1\right)\left(1 + \frac{s}{\omega_p}\right)}$$



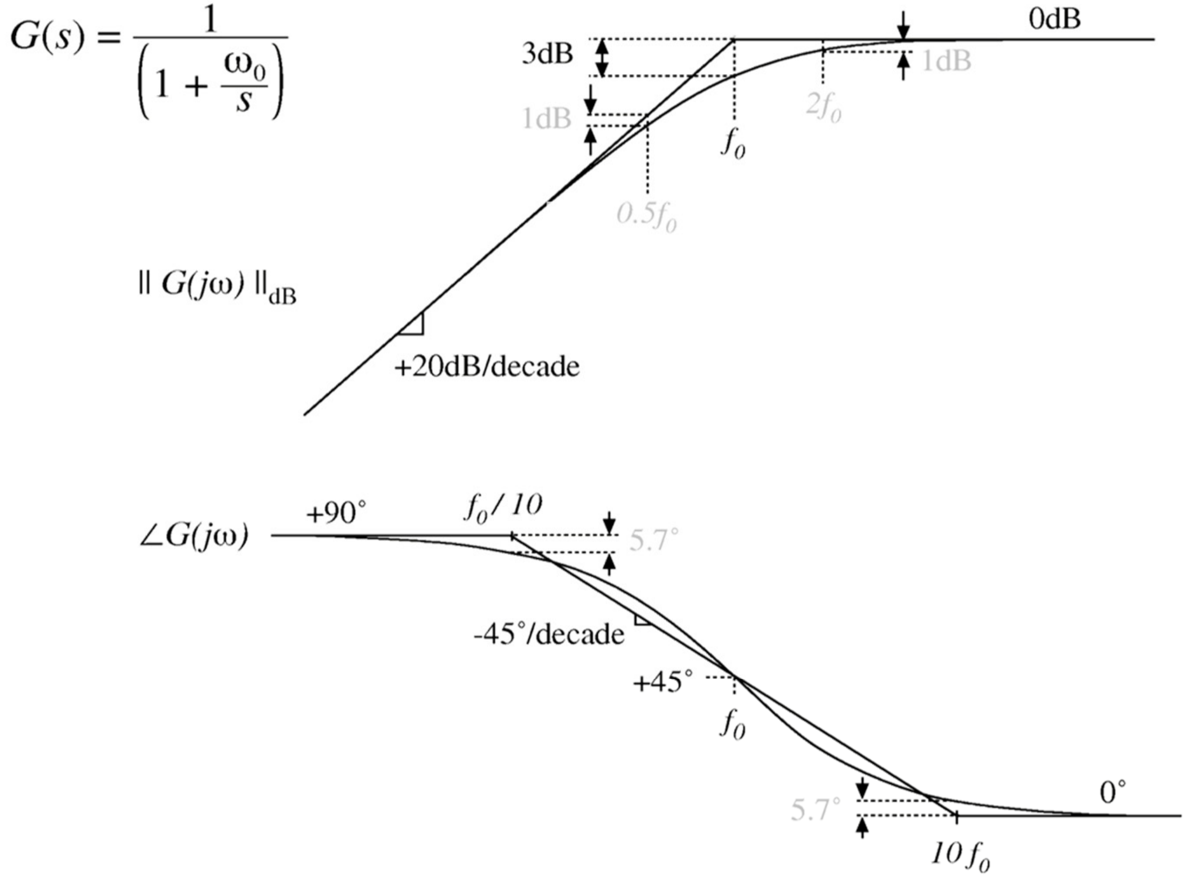
Additional Design-Oriented Techniques

- 8.1.4: Frequency Inversion
- 8.1.9: Approximate Roots of Arbitrary-Degree Polynomial
- 8.3: Graphical Construction of Impedances and Transfer Functions

Inverted Zero



Inverted Pole



Approximate Roots of High-Order Polynomial

Quadratic Roots: Not Well Separated

Suppose inequality k is not satisfied:

$$\left| a_1 \right| >> \left| \frac{a_2}{a_1} \right| >> \cdots >> \left| \frac{a_k}{a_{k-1}} \right| \quad \not\gg \quad \left| \frac{a_{k+1}}{a_k} \right| >> \cdots >> \left| \frac{a_n}{a_{n-1}} \right|$$

↑
not
satisfied

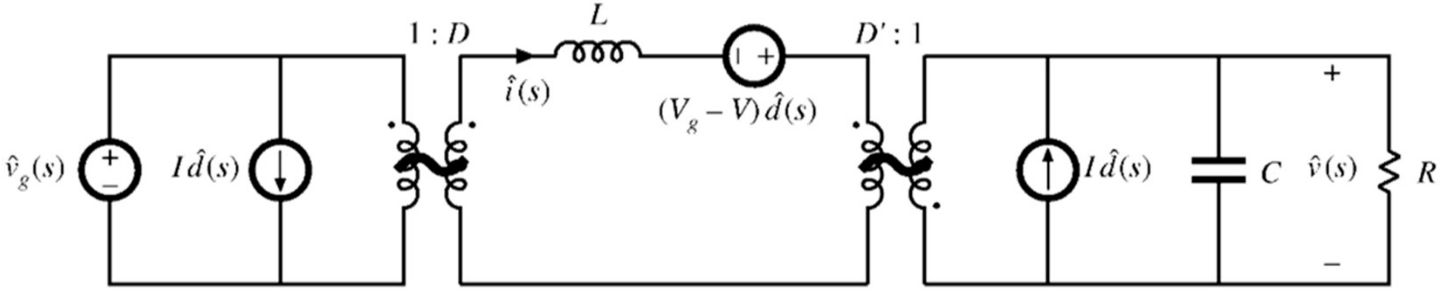
Then leave the terms corresponding to roots k and $(k+1)$ in quadratic form, as follows:

$$P(s) \approx \left(1 + a_1 s\right) \left(1 + \frac{a_2}{a_1} s\right) \cdots \left(1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_{k-1}} s^2\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

This approximation is accurate provided

$$\left| a_1 \right| >> \left| \frac{a_2}{a_1} \right| >> \cdots >> \left| \frac{a_k}{a_{k-1}} \right| >> \left| \frac{a_{k-2}\, a_{k+1}}{a_{k-1}^2} \right| >> \left| \frac{a_{k+2}}{a_{k+1}} \right| >> \cdots >> \left| \frac{a_n}{a_{n-1}} \right|$$

Output Impedance



Graphical Construction

Reactance Paper