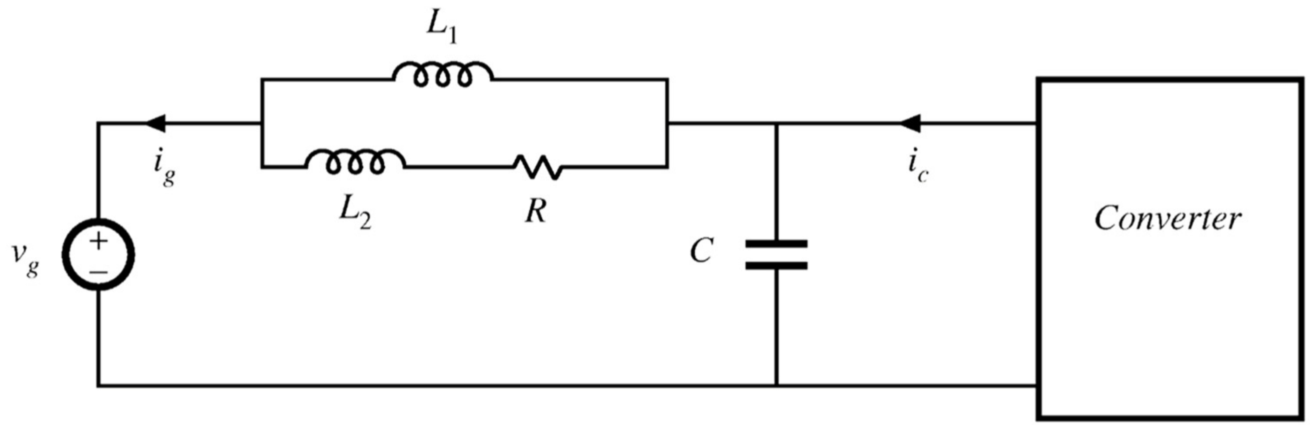
Example: Damped Input EMI Filter



$$G(s) = \frac{i_g(s)}{i_c(s)} = \frac{1 + s\,\dfrac{L_1 + L_2}{R}}{1 + s\,\dfrac{L_1 + L_2}{R} + s^2 L_1 C + s^3\,\dfrac{L_1 L_2 C}{R}}$$

Approximate Roots of High-Order Polynomial

$a_0 = 1$

$$P(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots$$

let's say the factorization is

$$P(s) = (1 + \tau_1 s)(1 + \tau_2 s)(1 + \tau_3 s) \cdots$$

& $|\tau_i| \gg |\tau_{i+1}|$

if we multiply back out

$$P(s) = 1 + (\tau_1 + \tau_2 + \tau_3 + \cdots) s + [\tau_1(\tau_2 + \tau_3 + \cdots) + \tau_2(\tau_3 + \cdots) + \tau_3(\cdots)] s^2 + [\tau_1\tau_2(\tau_3 + \cdots) + \tau_1\tau_3(\cdots) + \cdots] s^3$$

$$P(s) \approx 1 + \tau_1 s + \tau_1\tau_2 s^2 + \tau_1\tau_2\tau_3 s^3$$

Then,

$$\left.\begin{aligned} \tau_1 &= a_1 & \tau_3 &= \frac{a_3}{a_2} \\ \tau_2 &= \frac{a_2}{a_1} & \tau_k &= \frac{a_n}{a_{k-1}} \end{aligned}\right\} \text{ if } \quad \begin{aligned} |\tau_i| &\gg |\tau_{i+1}| \\ \left|\frac{a_i}{a_{i-1}}\right| &\gg \left|\frac{a_{i+1}}{a_i}\right| \end{aligned}$$

Results

$$P(s) = 1 + a_1 s + a_2 s^2 + \cdots + a_n s^n$$

If the following inequalities are satisfied

$$|a_1| \gg \left|\frac{a_2}{a_1}\right| \gg \left|\frac{a_3}{a_2}\right| \gg \cdots \gg \left|\frac{a_n}{a_{n-1}}\right|$$

Then the polynomial $P(s)$ has the following approximate factorization

$$P(s) \approx \left(1 + a_1 s\right)\left(1 + \frac{a_2}{a_1} s\right)\left(1 + \frac{a_3}{a_2} s\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

If any inequality not satisfied

$$|a_1| \gg \left|\frac{a_2}{a_1}\right| \gg \cdots \gg \left|\frac{a_k}{a_{k-1}}\right| \quad \not\gg \quad \left|\frac{a_{k+1}}{a_k}\right| \gg \cdots \gg \left|\frac{a_n}{a_{n-1}}\right|$$

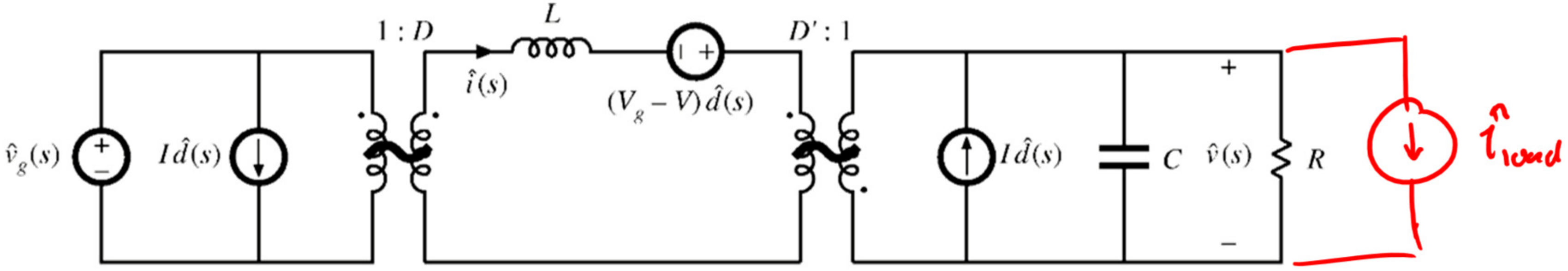
Then leave the terms corresponding to roots k and $(k+1)$ in quadratic form, as follows:

$$P(s) \approx \left(1 + a_1 s\right)\left(1 + \frac{a_2}{a_1} s\right) \cdots \left(1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_{k-1}} s^2\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

This approximation is accurate provided

$$|a_1| \gg \left|\frac{a_2}{a_1}\right| \gg \cdots \gg \left|\frac{a_k}{a_{k-1}}\right| \gg \left|\frac{a_{k-2}\, a_{k+1}}{a_{k-1}^2}\right| \gg \left|\frac{a_{k+2}}{a_{k+1}}\right| \gg \cdots \gg \left|\frac{a_n}{a_{n-1}}\right|$$

Graphical Construction: Output Impedance



$$G_{vd}(s) = \left(-\frac{V_g - V}{D'}\right)\frac{1 - sL\frac{I}{D'(V_g - V)}}{1 + \frac{sL}{D'^2 R} + s^2\frac{LC}{D'^2}}$$

$$\hat{v}(s) = G_{vd}(s)\,\hat{d}(s) + \underbrace{\hat{v}_g\left(-\frac{D}{D'}\right)\frac{R\|\frac{1}{sC}}{R\|\frac{1}{sC} + \frac{sL}{D'^2}}}_{G_{vg}(s)} - \hat{i}_{load}\underbrace{\left(\frac{sL}{D'^2}\,\Big\|\,\frac{1}{sC}\,\Big\|\,R\right)}_{Z_{out}(s)}$$

Numerical Example

$L = 160\ \mu\text{H}$

$D = 0.6$

$R = 10\ \Omega$

$C = 160\ \mu\text{F}$

$V_g = 30\ \text{V}$

$V = -45\ \text{V}$

$L/_{D'^2} = 1\text{mH}$

$\omega_0 = \sqrt{\frac{D'^2}{LC}} = 2.5\ \text{krad/s} \quad \rightarrow \quad f_0 = 400\ \text{Hz}$

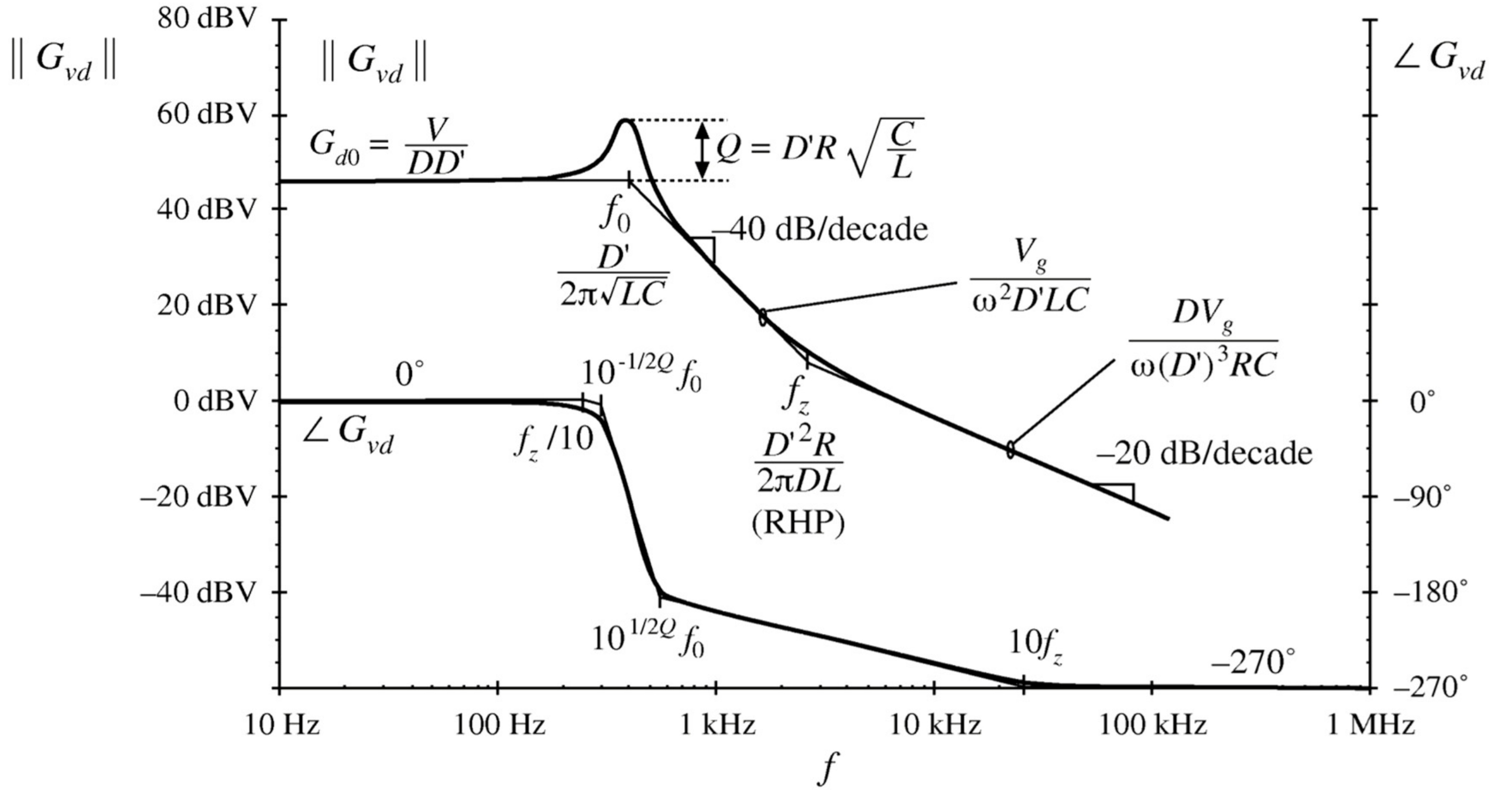
$Q = \frac{D'R}{\sqrt{L/_C}} = 4 \quad \rightarrow \quad \|Q\|_{\text{dB}} = 12\ \text{dB}$

$\omega_z = \frac{LI}{(Vg-V)} = 16.7\ \text{krad/s} \quad \rightarrow \quad f_z = 2.7\ \text{kHz}$

$G_{d0} = \frac{V_g - V}{D'} = 187.5\ \text{V} \quad \rightarrow \quad \| G_{d0} \|_{\text{dB}} = 45.5\ \text{dBV}$

$G_{g0} = \frac{-D}{D'} = 1.5 \quad \rightarrow \quad \| G_{g0} \|_{\text{dB}} = 3.5\ \text{dBV}$

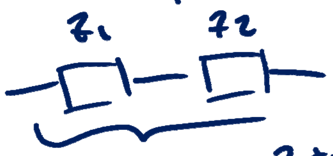
Control-to-output Transfer Function



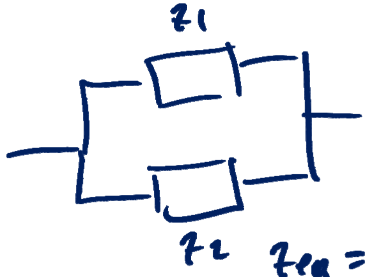
Graphical Construction

Looking at impedances on the bode plot

on Bode plot



$$Z_{eq} = Z_1 + Z_2 \approx \begin{cases} Z_1, & |Z_1| \gg |Z_2| \\ Z_2, & |Z_2| \gg |Z_1| \end{cases} \qquad Z_1 + Z_2 \approx \max(Z_1, Z_2)$$

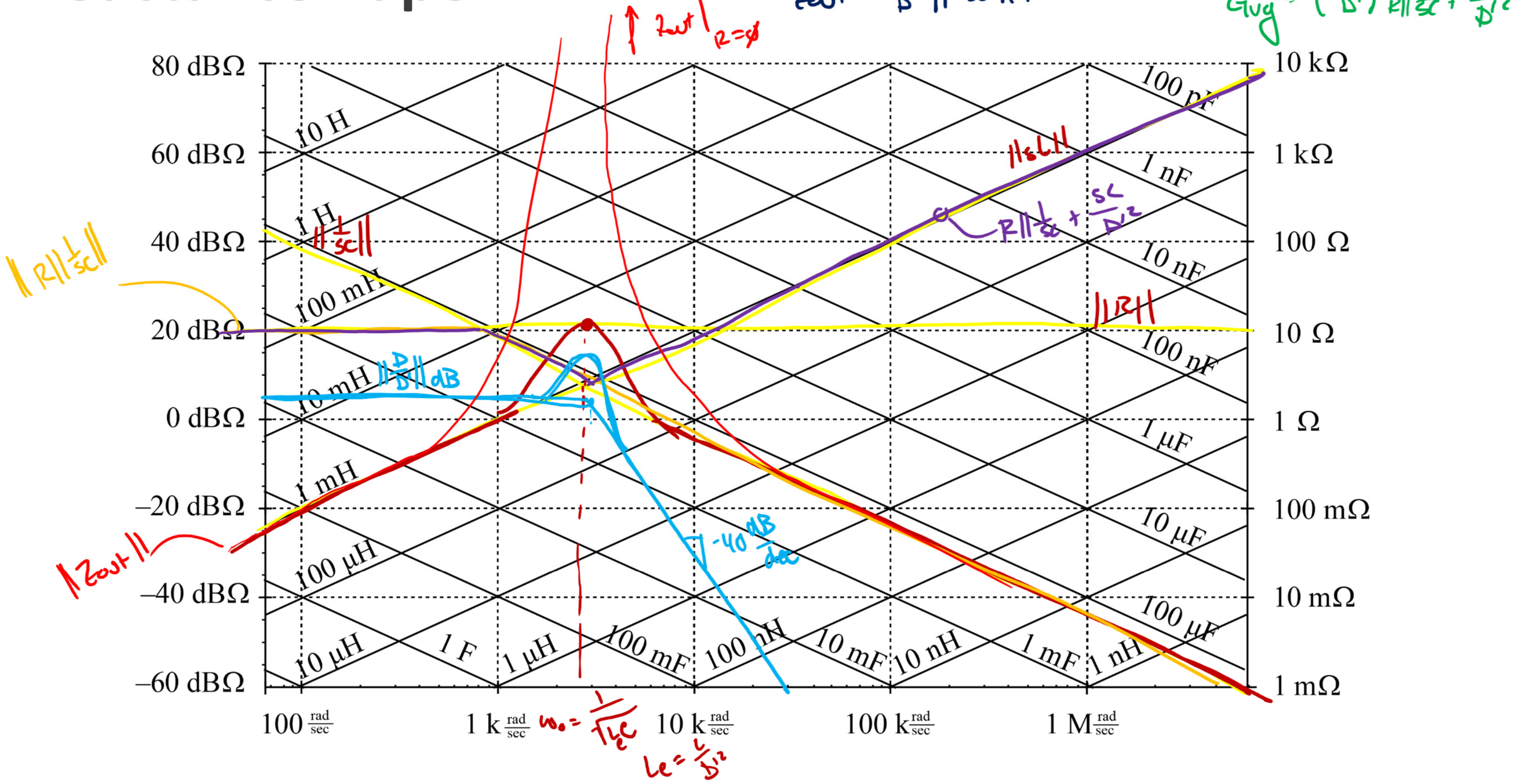


$$Z_{eq} = Z_1 \| Z_2 = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} \approx \begin{cases} Z_2, & |Z_1| \gg |Z_2| \\ Z_1, & |Z_2| \gg |Z_1| \end{cases} \qquad Z_1 \| Z_2 \approx \min(Z_1, Z_2)$$

Reactance Paper

$$Z_{out} = \frac{sL}{D'^2} \,\|\, \frac{1}{sC} \,\|\, R$$

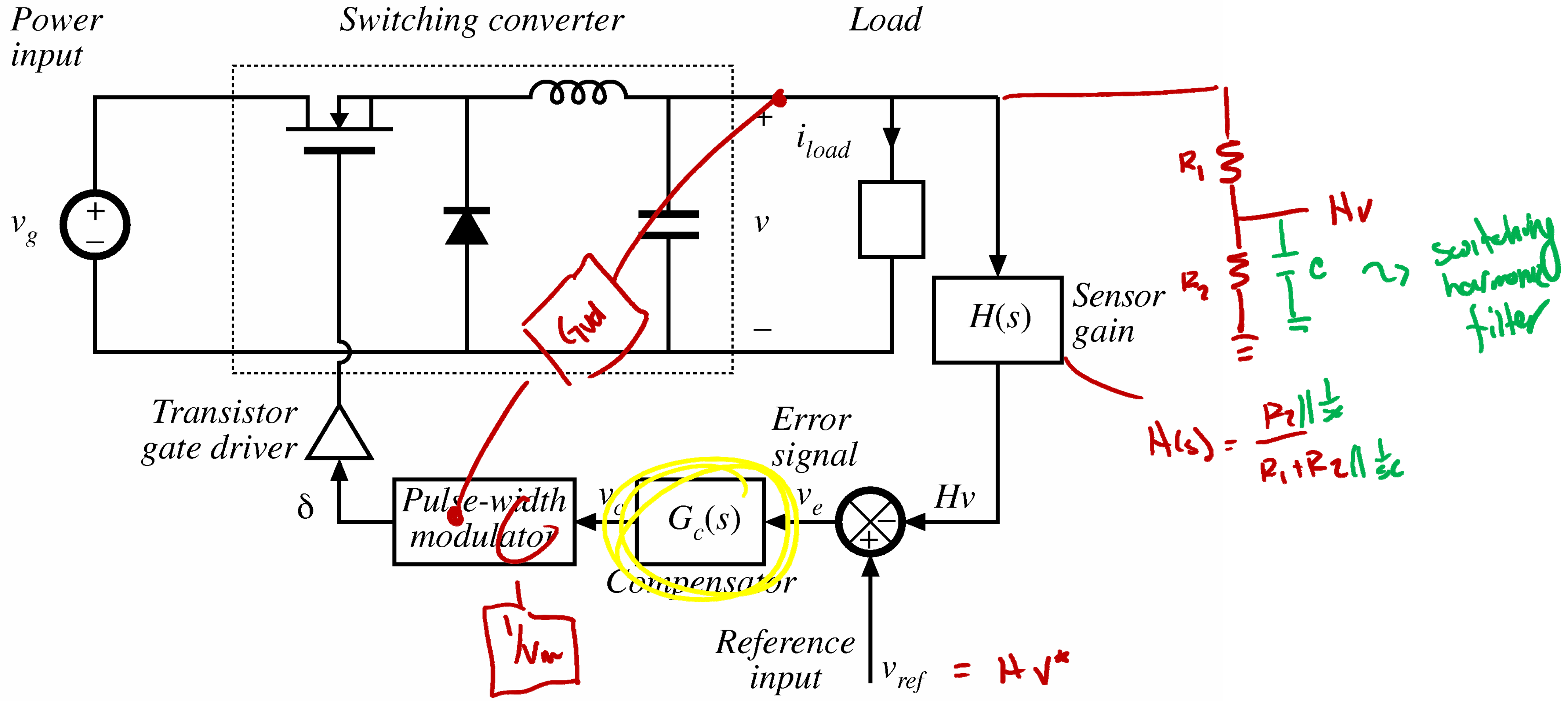
$$G_{vg} = \left(\frac{-D}{D'}\right) \frac{R \| \frac{1}{sC}}{R \| \frac{1}{sC} + \frac{sL}{D'^2}}$$



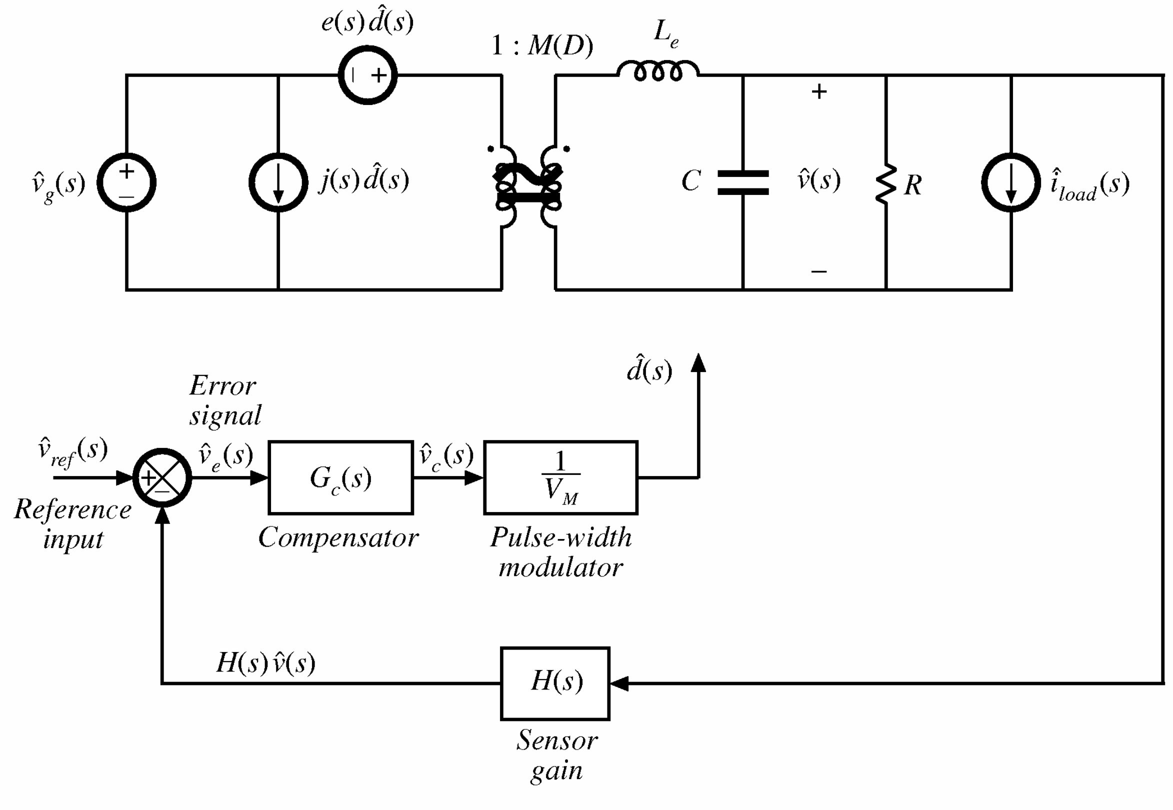
Chapter 9

CONTROLLER DESIGN

Closed-Loop Regulation



Small-Signal Closed-Loop Model



We want to design $G_c(s)$ to achieve

1. $\hat{v}_e \to \phi$ in steady state
2. Fast transient response
3. Minimal overshoot / ringing
4. Stable