Block Diagram

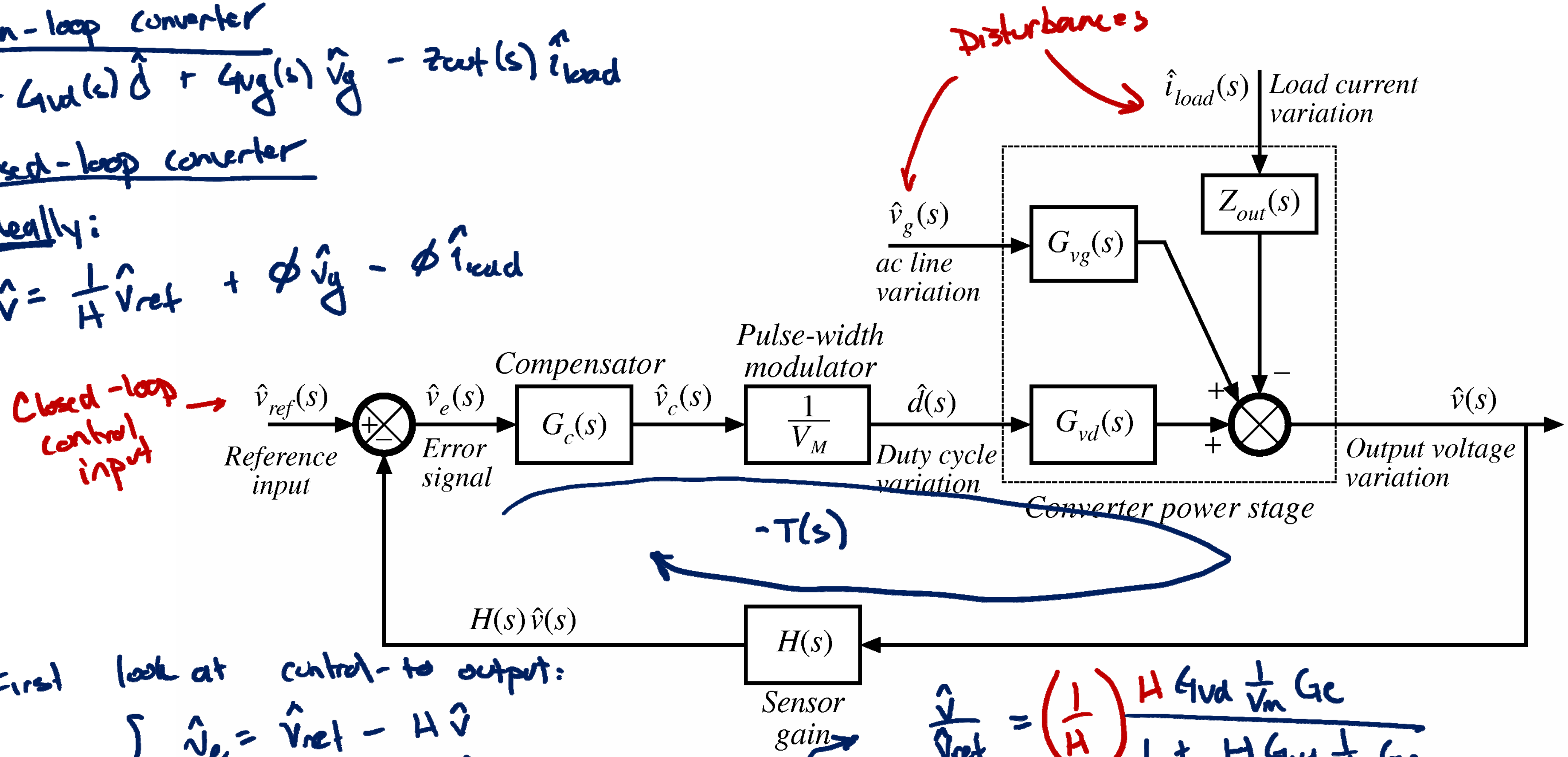
Open-loop converter

$$\hat{v} = G_{vd}(s)\,\hat{d} + G_{vg}(s)\,\hat{v}_g - Z_{out}(s)\,\hat{i}_{load}$$

Closed-loop converter

Ideally:

$$\hat{v} = \frac{1}{H}\hat{v}_{ref} + \phi\,\hat{v}_g - \phi\,\hat{i}_{load}$$



$T(s)$ = "Loop Gain"

→ Gain around the feedback loop neglecting negative sign

$$T(s) = H\,G_{vd}\,\frac{1}{V_m}\,G_c$$

First look at control-to-output:

$$\begin{cases} \hat{v}_e = \hat{v}_{ref} - H\hat{v} \\ \hat{v} = G_{vd}\,\frac{1}{V_m}\,G_c\,\hat{v}_e \end{cases}$$

$$\hat{v}\left(1 + H G_{vd}\frac{1}{V_m}G_c\right) = G_{vd}\frac{1}{V_m}G_c\,\hat{v}_{ref}$$

$$\frac{\hat{v}}{\hat{v}_{ref}} = \left(\frac{1}{H}\right)\frac{H\,G_{vd}\,\frac{1}{V_m}\,G_c}{1 + H\,G_{vd}\,\frac{1}{V_m}\,G_c}$$

$$\frac{\hat{v}}{\hat{v}_{ref}} = \frac{1}{H}\,\frac{T(s)}{1+T(s)}$$

Closed-Loop Transfer Functions

Disturbance-to-output (e.g. $\hat{v}_g \rightarrow \hat{v}$)

$$\hat{v} = G_{vg}\hat{v}_g - \hat{v}\, G_{vd} \frac{1}{V_m} G_c H$$

$$\hookrightarrow \frac{\hat{v}}{\hat{v}_g} = G_{vg} \frac{1}{1 + G_{vd}\frac{1}{V_m} G_c H} = G_{vg}\frac{1}{1+T(s)}$$

$$\frac{\hat{v}}{\hat{i}_{load}} = (-Z_{out})\frac{1}{1+T(s)}$$

Combining all three inputs by superposition

$$\hat{v} = \frac{1}{H}\left(\frac{T}{1+T}\right)\hat{v}_{ref} + G_{vg}\left(\frac{1}{1+T}\right)\hat{v}_g - Z_{out}\left(\frac{1}{1+T}\right)\hat{i}_{load}$$

Ideal

≈ 1 control

$\approx \emptyset$ $\approx \emptyset$

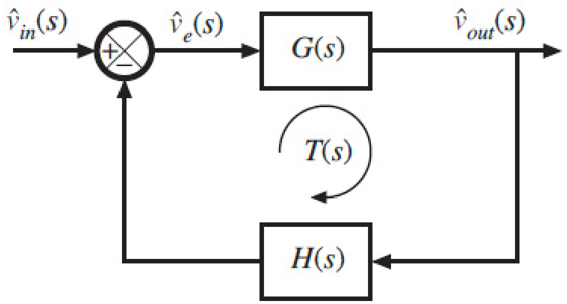
Disturbances

In both cases, if T is "very large" we approach ideal behavior

Open-Loop and Closed-Loop

Open loop

Closed loop



open-loop behaviors

$G_{vd}(s)$

$G_{vg}(s)$

$Z_{out}(s)$

→

G_{vd} $\frac{T}{1+T}$

G_{vg} $\frac{1}{1+T}$

Z_{out} $\frac{1}{1+T}$

Neither open- nor closed:

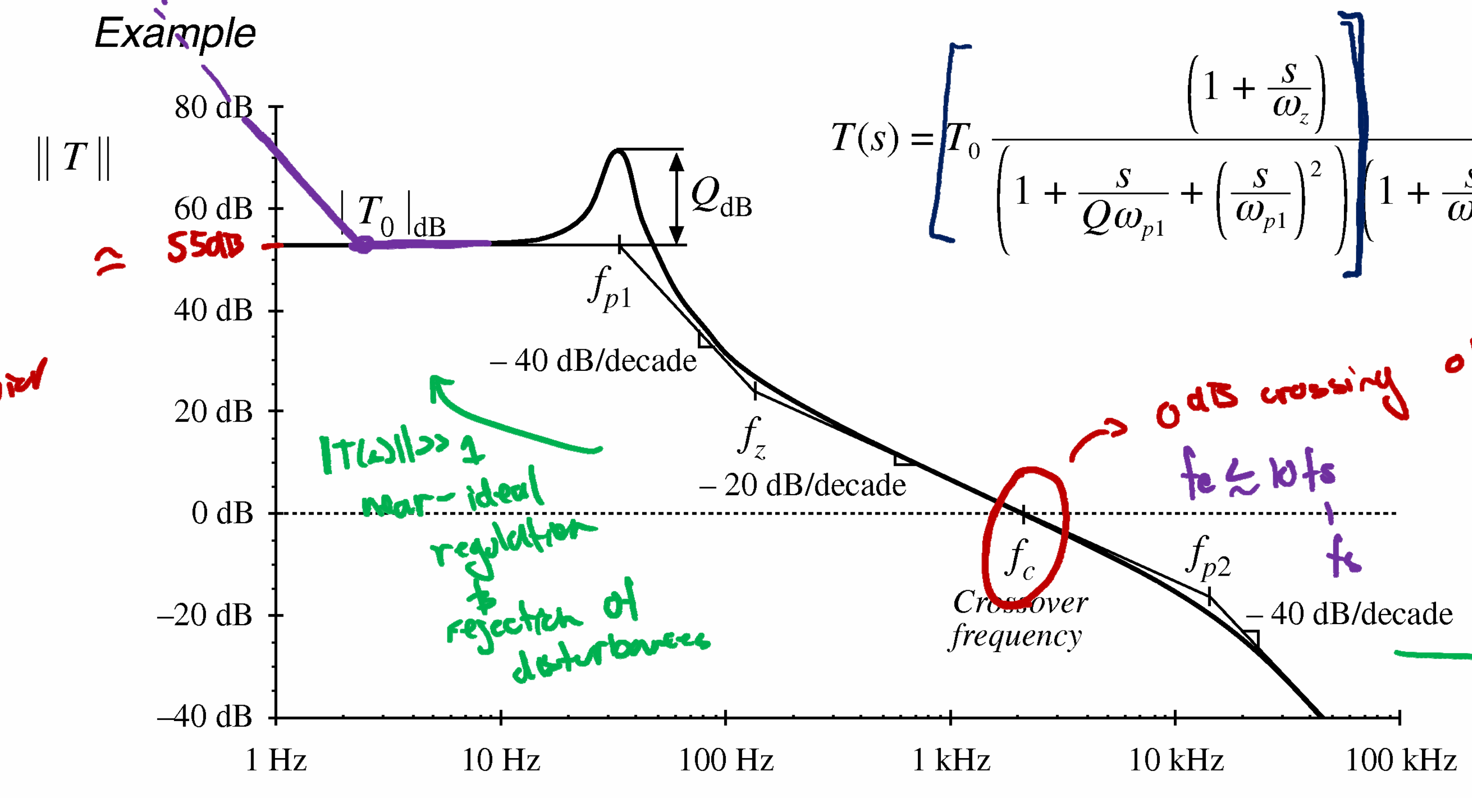
T(s) is "Loop Gain"

Construction of the Loop Gain

start with $G_c = 1$

Example

$$T(s) = T_0 \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_{p1}} + \left(\frac{s}{\omega_{p1}}\right)^2\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$



at low frequency

$\frac{T}{1+T} = \frac{500}{500+1} \approx 99.8\%$ ← 500

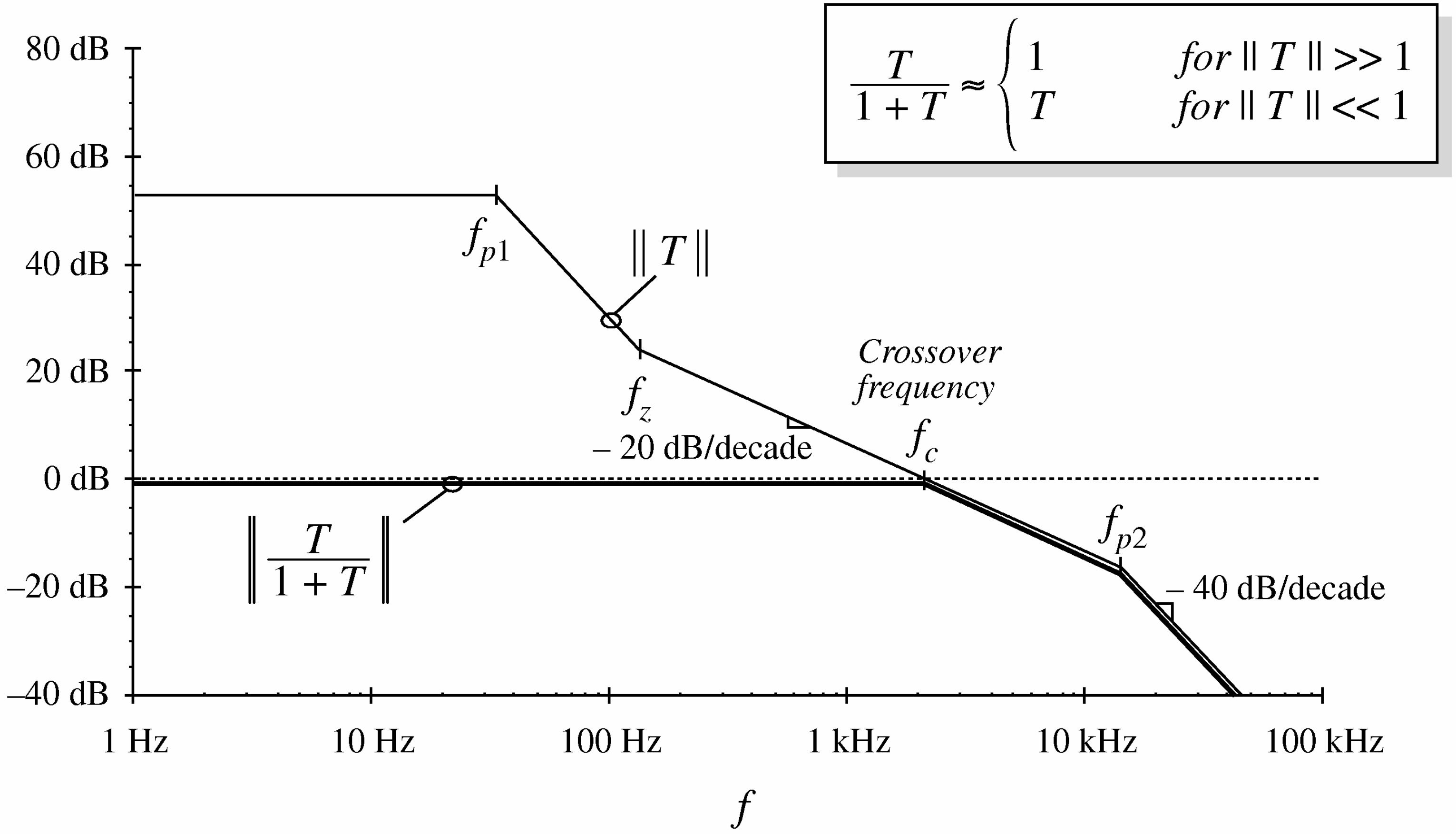
$\frac{1}{1+T} = \frac{1}{501} \approx 0.2\%$

as $\omega \to 0$, DC behavior

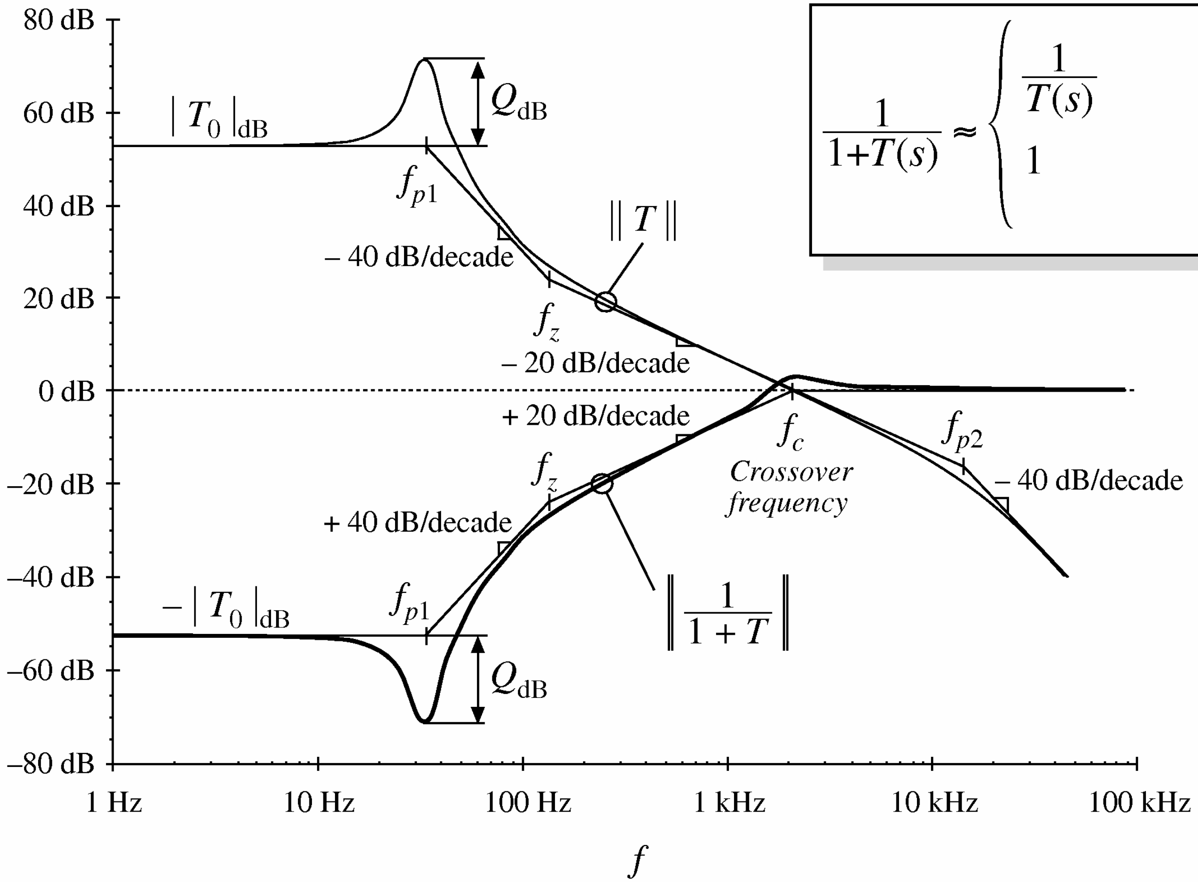
||T(s)|| << 1
no reference tracking & open-loop response to disturbances

At the crossover frequency f_c, $\| T \| = 1$

Tracking of Reference



Rejection of Disturbances



$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & \textit{for } \| T \| >> 1 \\ 1 & \textit{for } \| T \| << 1 \end{cases}$$