

Chapter 2: Goals

- Develop techniques for easily determining output voltage of an arbitrary converter circuit
- Derive the principles of *inductor volt-second balance* and *capacitor charge (amp-second) balance*
- Introduce the key *small ripple approximation*
- Develop simple methods for selecting filter element values
- Illustrate via examples

Buck Converter Review

Pulse-width Modulation (PWM)

$T_s \rightarrow$ switching period

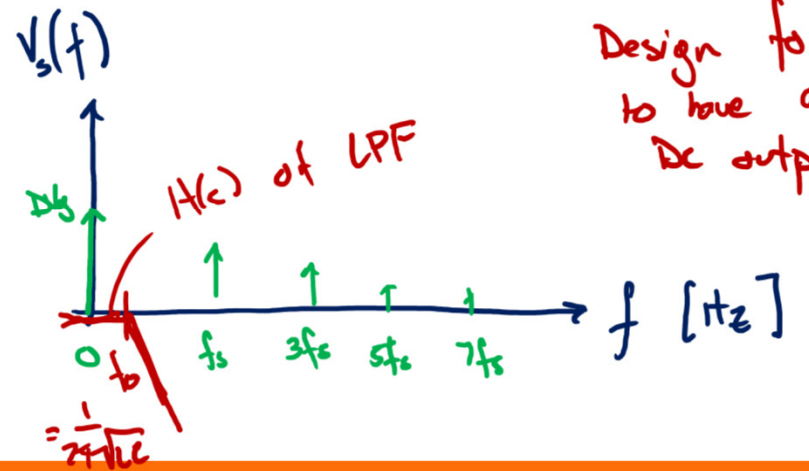
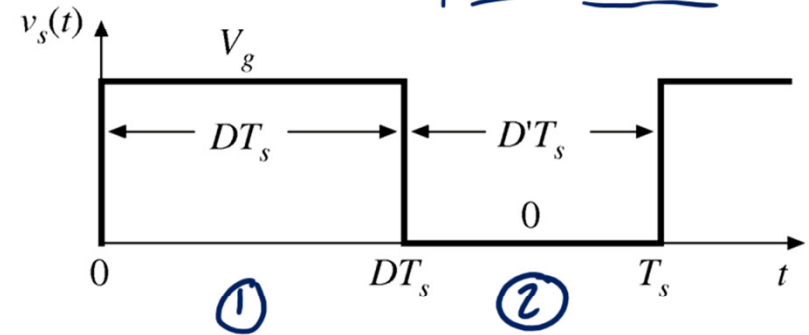
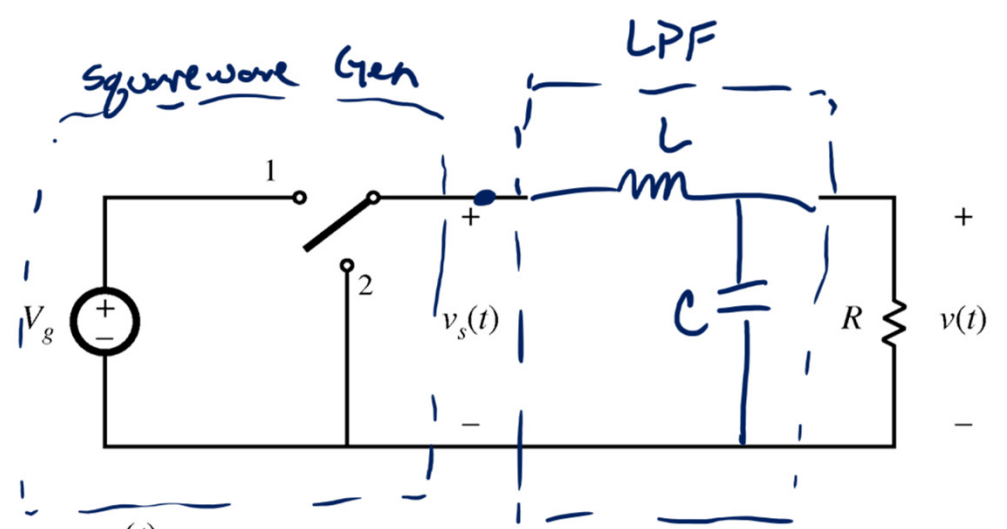
$f_s = \frac{1}{T_s} \rightarrow$ switching frequency

$0 \leq D \leq 1 \rightarrow$ Duty Cycle

$D' = 1 - D$

$M = \frac{V}{V_g} =$ "Conversion Ratio"

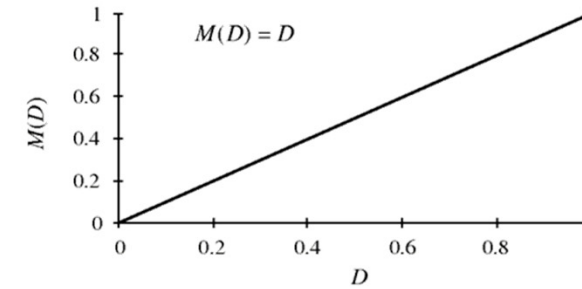
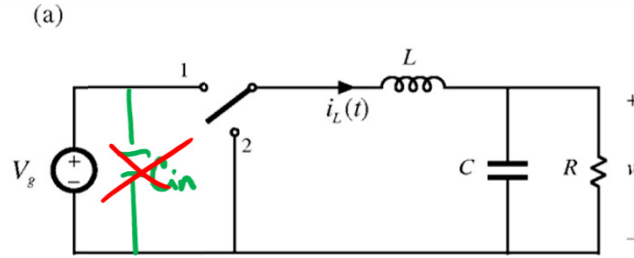
$$\langle v_s \rangle_{T_s} = \frac{1}{T_s} \int_0^{T_s} v_s(t) dt = \frac{1}{T_s} D V_g T_s = D V_g$$



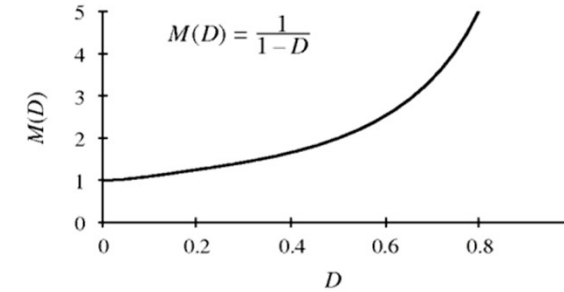
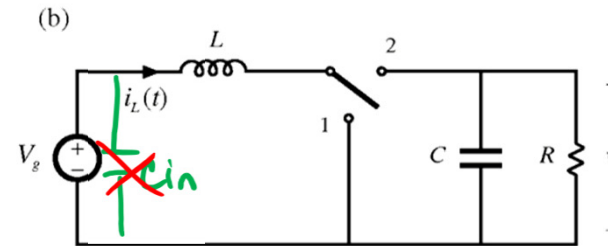
Design for $\ll f_s$
to have approximately
DC output

Three Basic DC-DC PWM Converters

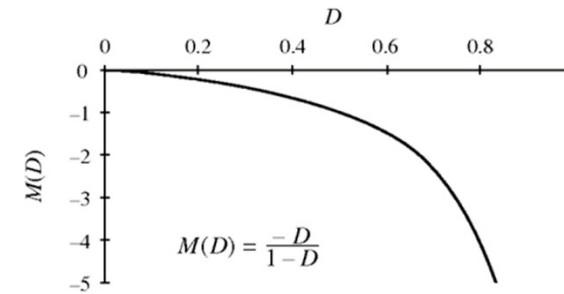
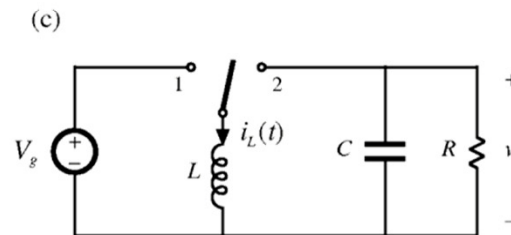
Buck



Boost



Buck-boost



The Small Ripple Approximation

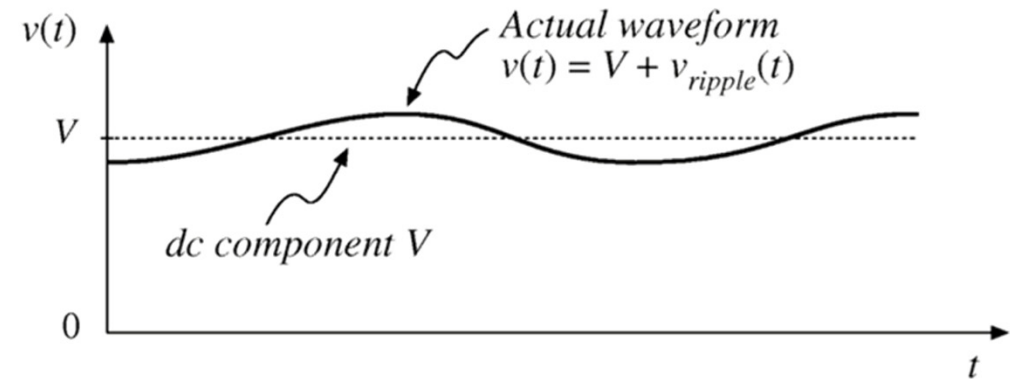
In a well-designed converter

$$v_{\text{ripple}}(t) \ll V$$

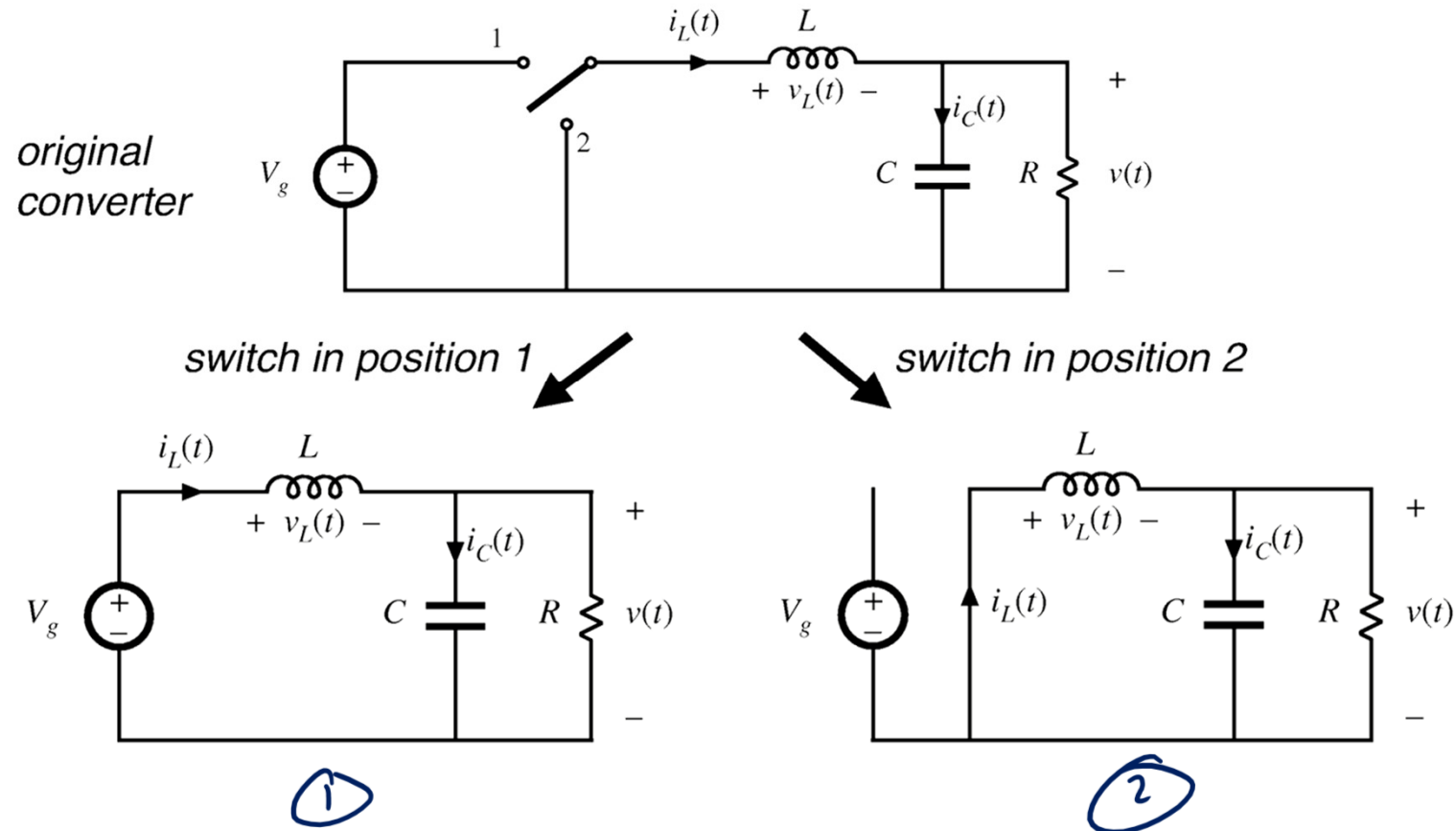
then we can approximate

$$v(t) \approx V$$

usually we can apply SRA to most/all L/C elements in PWM converters



Buck Switching Intervals: Inductor Current



Subinterval 1

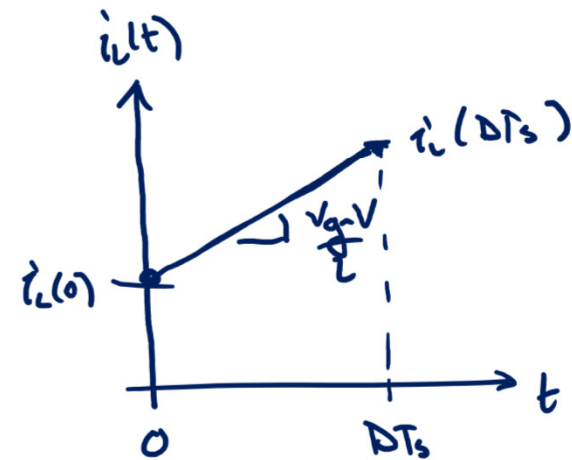
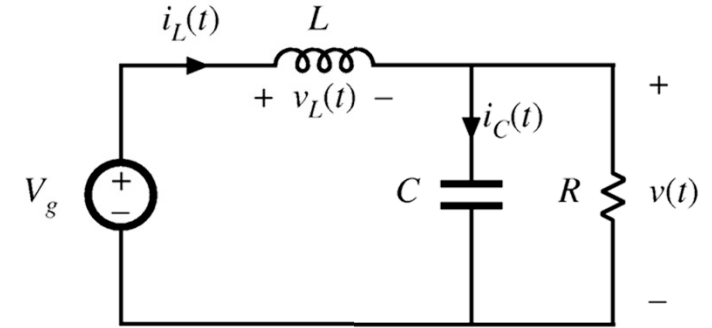
$$N_L(t) = L \frac{di_L}{dt}$$

$$V_g - N(t) = L \frac{di_L}{dt}$$

Apply SRA. on $v(t)$

$$V_g - V = L \frac{di_L}{dt}$$

$$\frac{V_g - V}{L} = \frac{di_L}{dt}$$



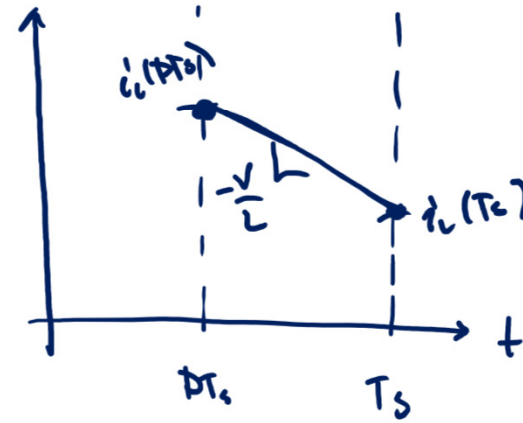
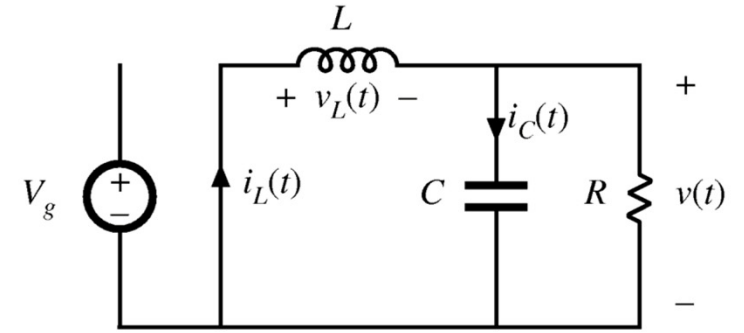
Subinterval 2

$$N_L = L \frac{di_L}{dt}$$
$$\emptyset - v(t) = L \frac{di_L}{dt}$$

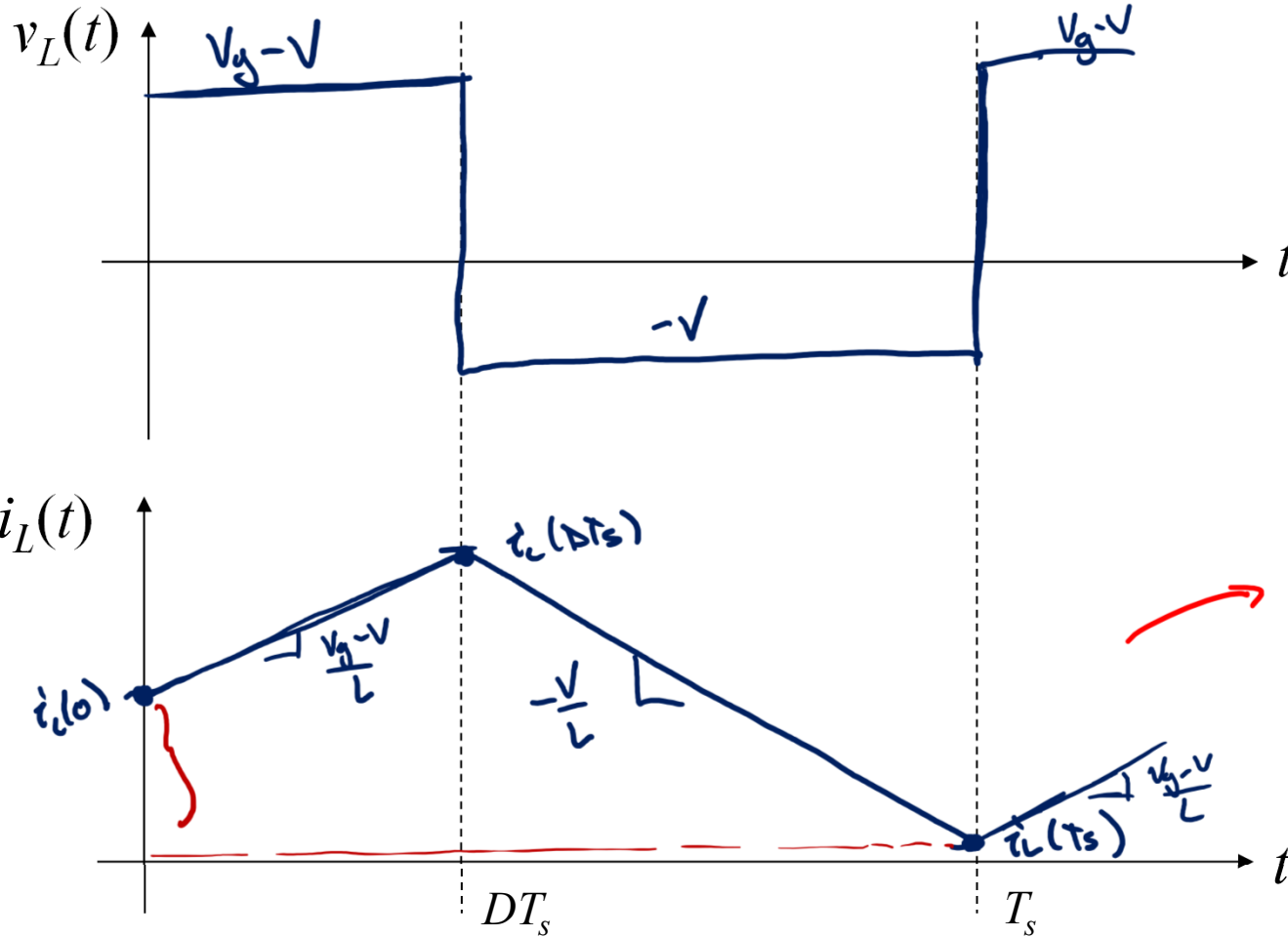
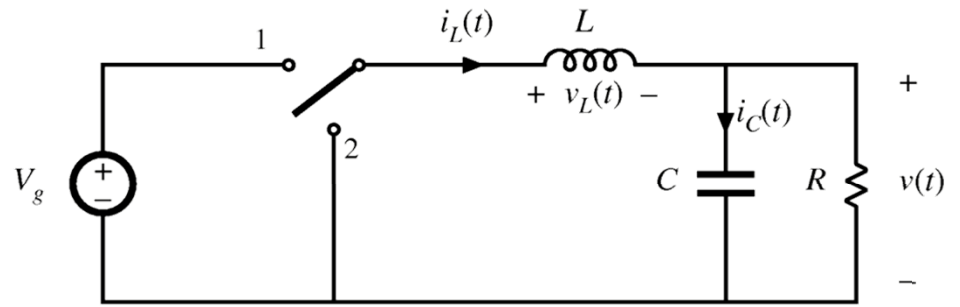
Apply SRA on $v(t)$

$$0 - V = L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{-V}{L}$$



Current Waveform

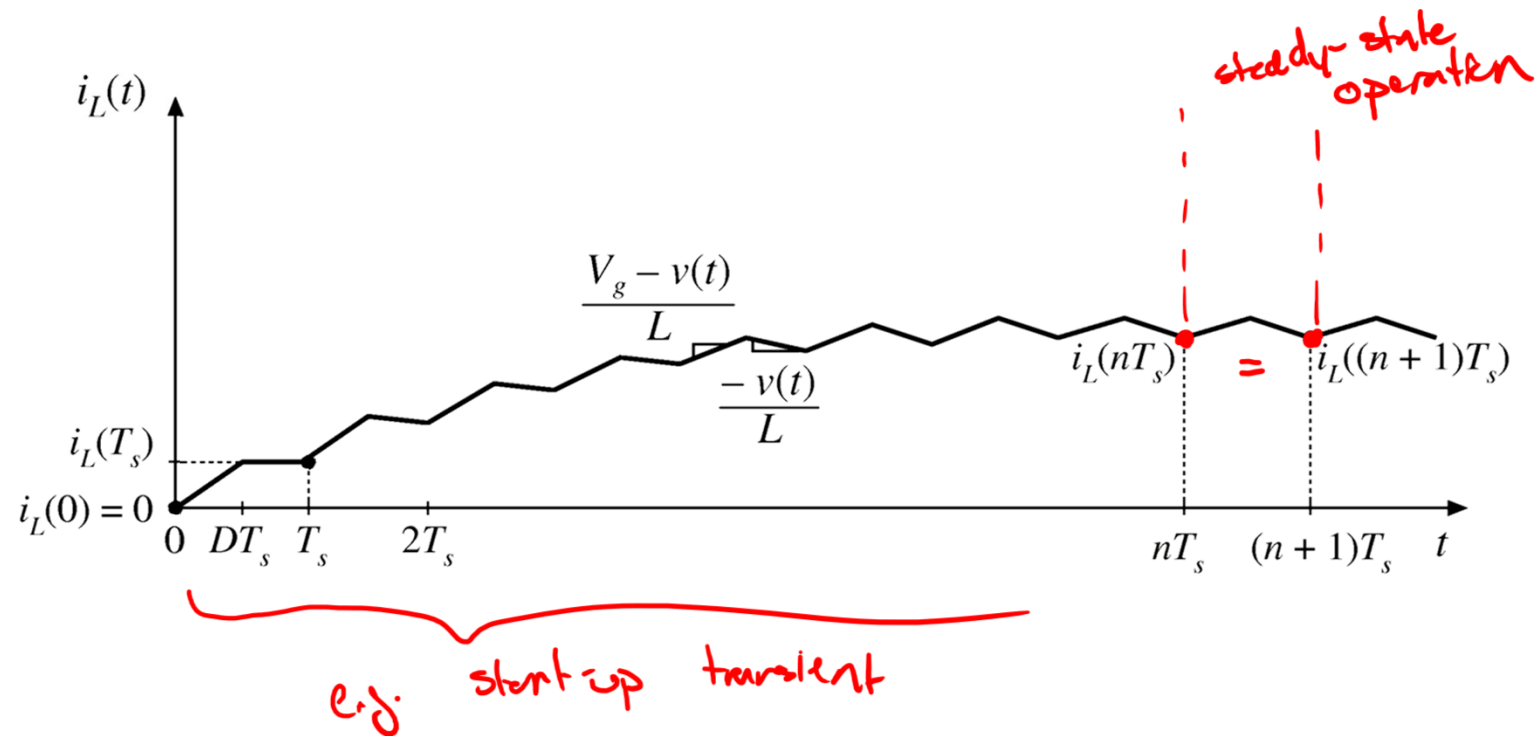


Inductor

$$v_L = L \frac{di_L}{dt}$$

$i_L(0) \neq i_L(T_s)$
 we're in a transient
 i.e. not in steady-state

Transient vs. Steady-State Operation



Volt-Second Balance

In steady-state

$$i_L(0) = i_L(T_s) = i_L(kT_s)$$

For this to be true:

$$\Delta i_{L1} + \Delta i_{L2} = 0$$

$$\frac{V_g - V}{L} DT_s = \Delta i_{L1} = -\Delta i_{L2} = -\left[\frac{-V}{L}(1-D)T_s\right]$$

~~$$\frac{V_g - V}{L} DT_s - \frac{V}{L}(1-D)T_s = 0$$~~

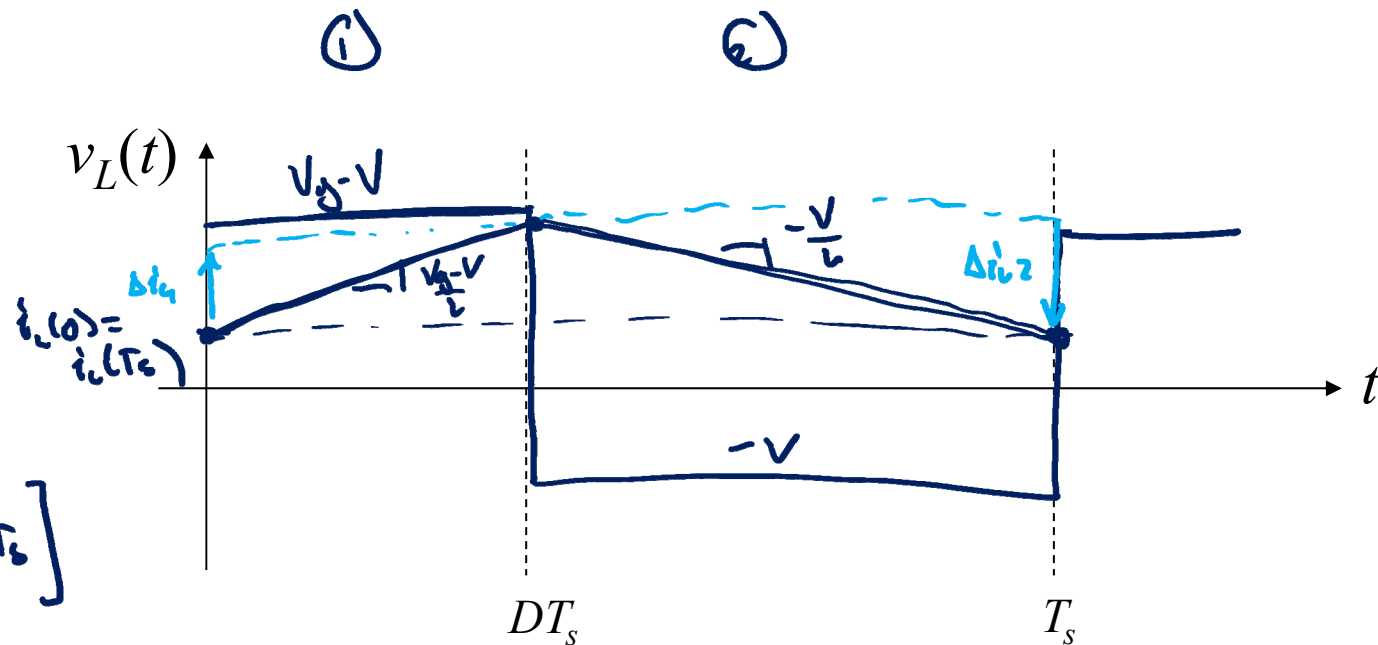
$$(V_g - V)D - V(1-D) = 0$$

$$V_g(D) - V(D + 1 - D) = 0$$

$$V_g(D) - V = 0$$

$$V = DV_g$$

$$\frac{V}{V_g} = M = D$$



Derivation of Volt-second Balance

$$\int_0^{T_s} v_L(t) dt = \int_0^{T_s} L \frac{di_L}{dt} dt$$

$$\frac{1}{L} \int_0^{T_s} v_L(t) dt = \int_0^{T_s} \frac{di_L}{dt} dt = \frac{i_L(T_s) - i_L(0)}{\text{in steady-state, } = \phi}$$

$$\frac{L}{T_s} \left(\frac{1}{L} \int_0^{T_s} v_L(t) dt \right) = \phi \frac{L}{T_s}$$

$$\frac{1}{T_s} \int_0^{T_s} v_L(t) dt = \phi \rightarrow$$

$$\boxed{\langle v_L \rangle_{T_s} = \phi}$$

in steady-state