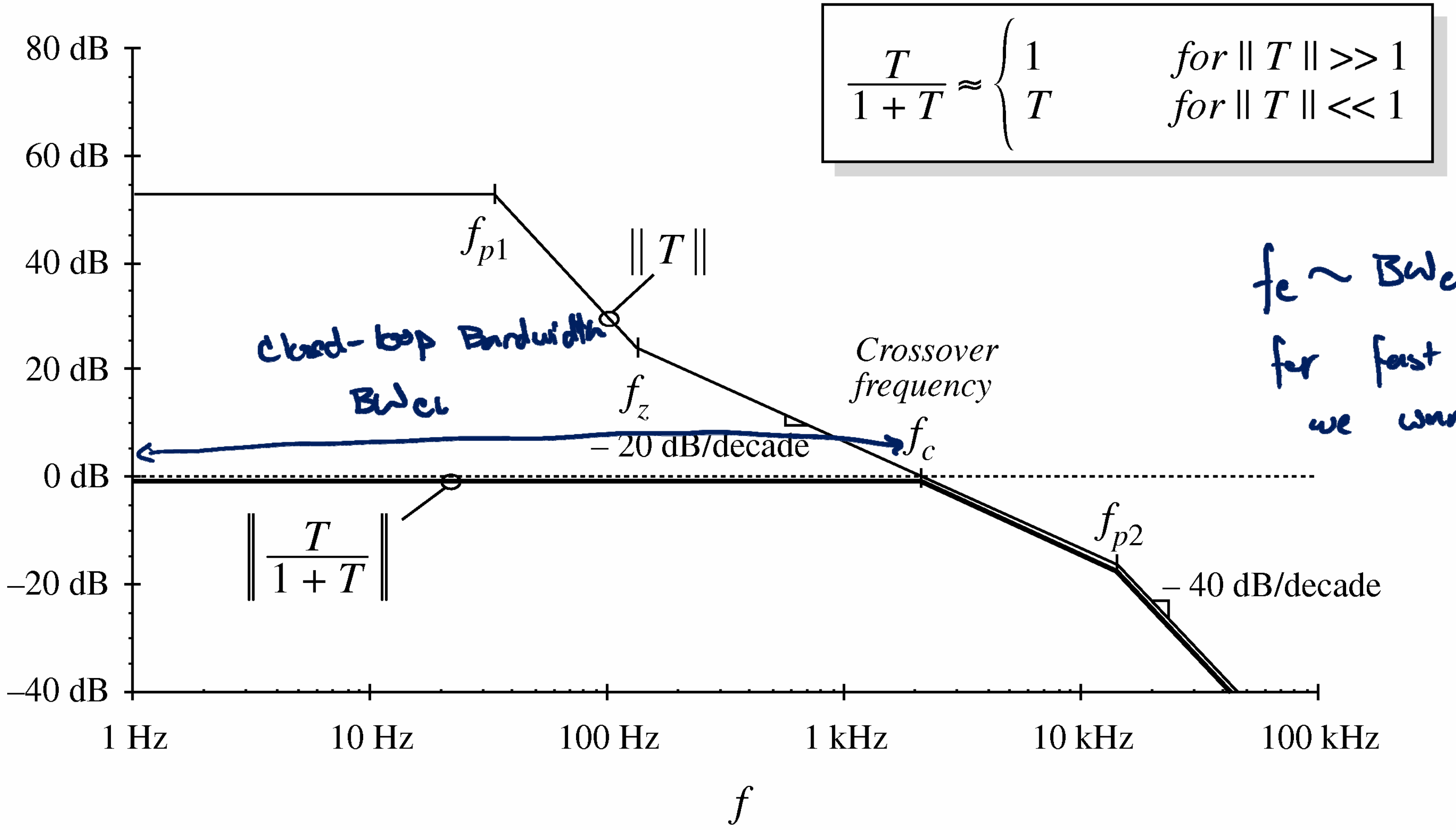
Tracking of Reference



$f_c \sim BW_{CL}$

for fast response we want "high" f_c

Determining Stability From T(s)

Assume original open-loop system was stable

Open-loop		Closed-loop
$G_{vd}(s)$	$\rightarrow$	$\frac{1}{H} \frac{T}{1+T}$
$G_{vg}(s)$		$G_{vg} \frac{1}{1+T}$

+ We can move the locations of poles & zeros
− We can destabilize the system if not compensated correctly

Stability Tests

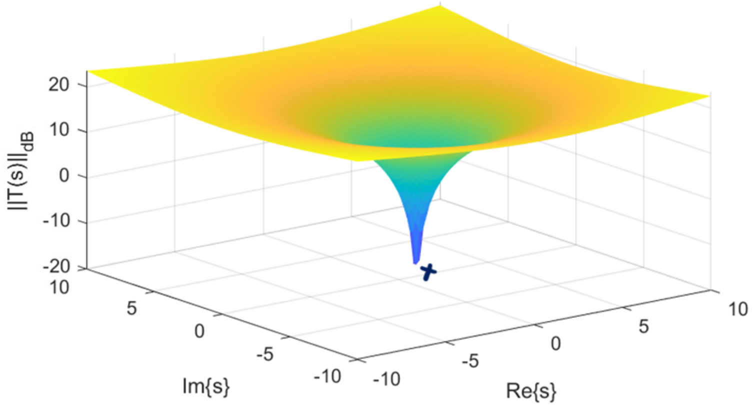
(1) Nyquist Stability Theorem : General result

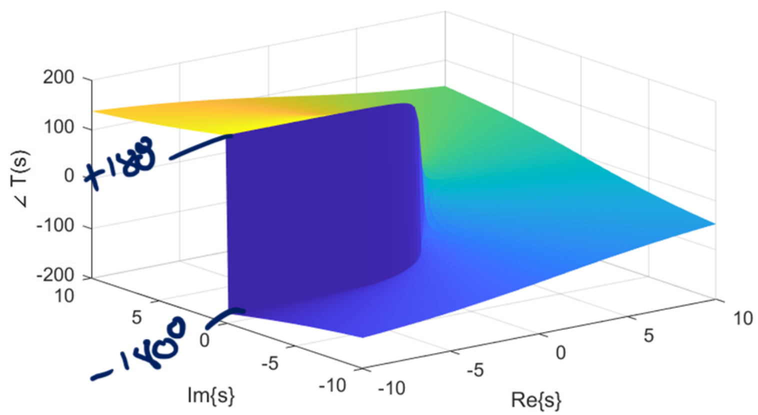
(2) Phase Margin Test : Limited case, but simplified & useful for certain systems

Closed-loop stability $\Longleftrightarrow$ closed-loop system has no RHP poles

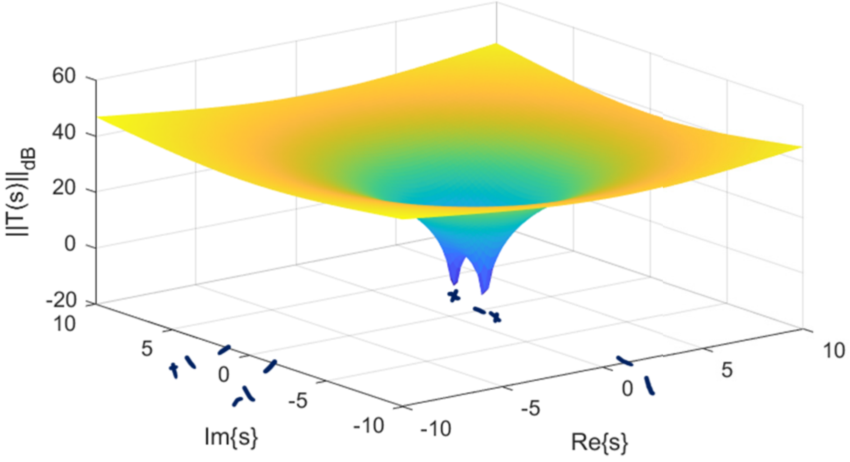
S-plane plots

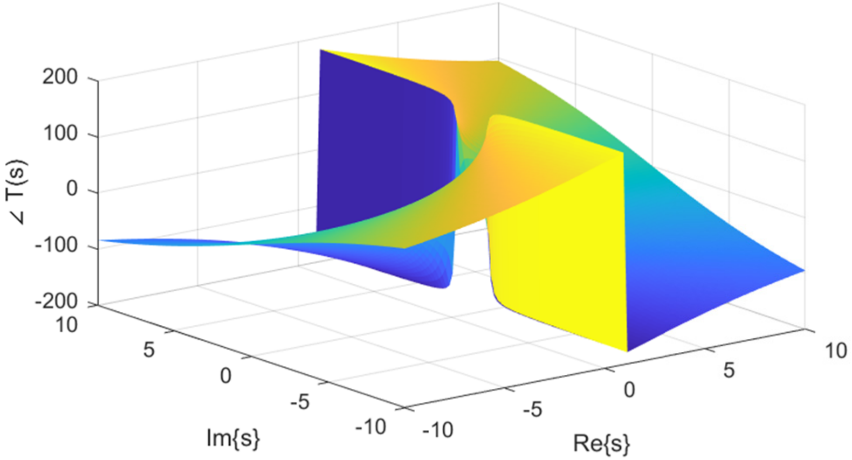
$$T(s) = (s - 1)$$





$$T(s) = (s - s_1)(s - s_1^*)$$





Cauchy's Principle

For some transfer function $T(s)$ & some closed contour Γ in the complex plane

If we traverse Γ (clockwise) & evaluate $\angle T(\Gamma)$, then

$$\boxed{N = Z - P}$$

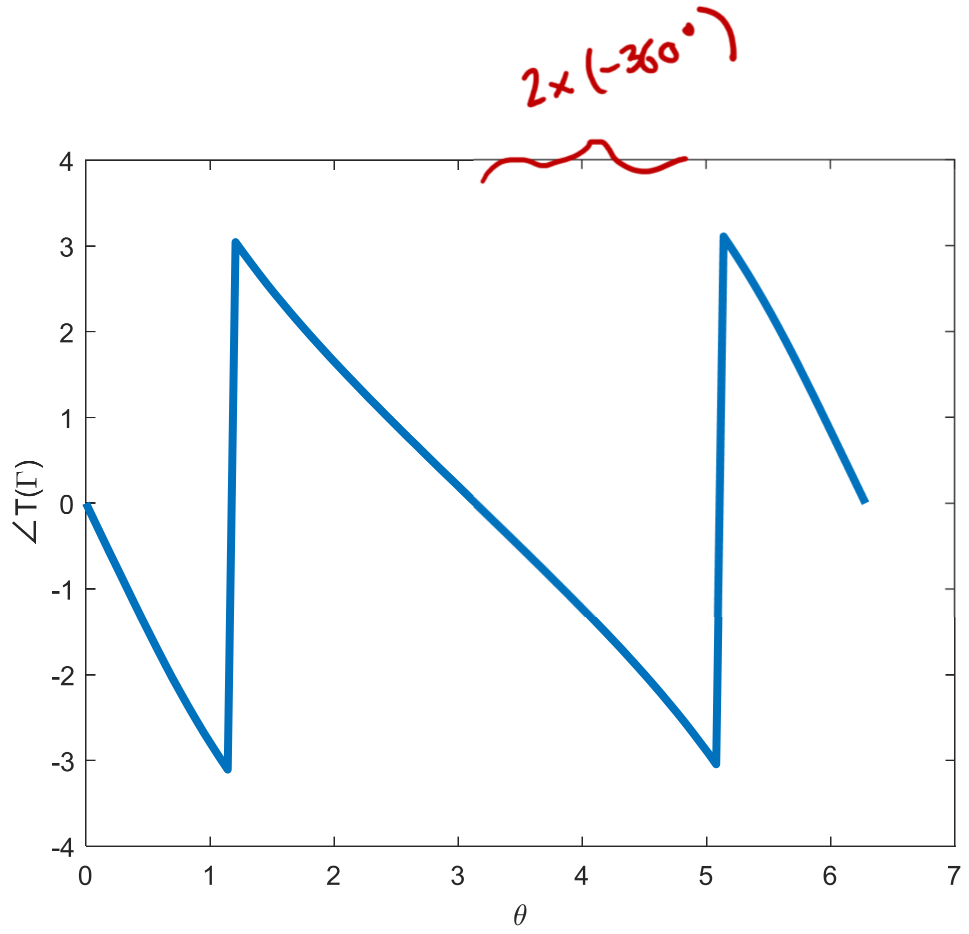
# of poles enclosed by Γ

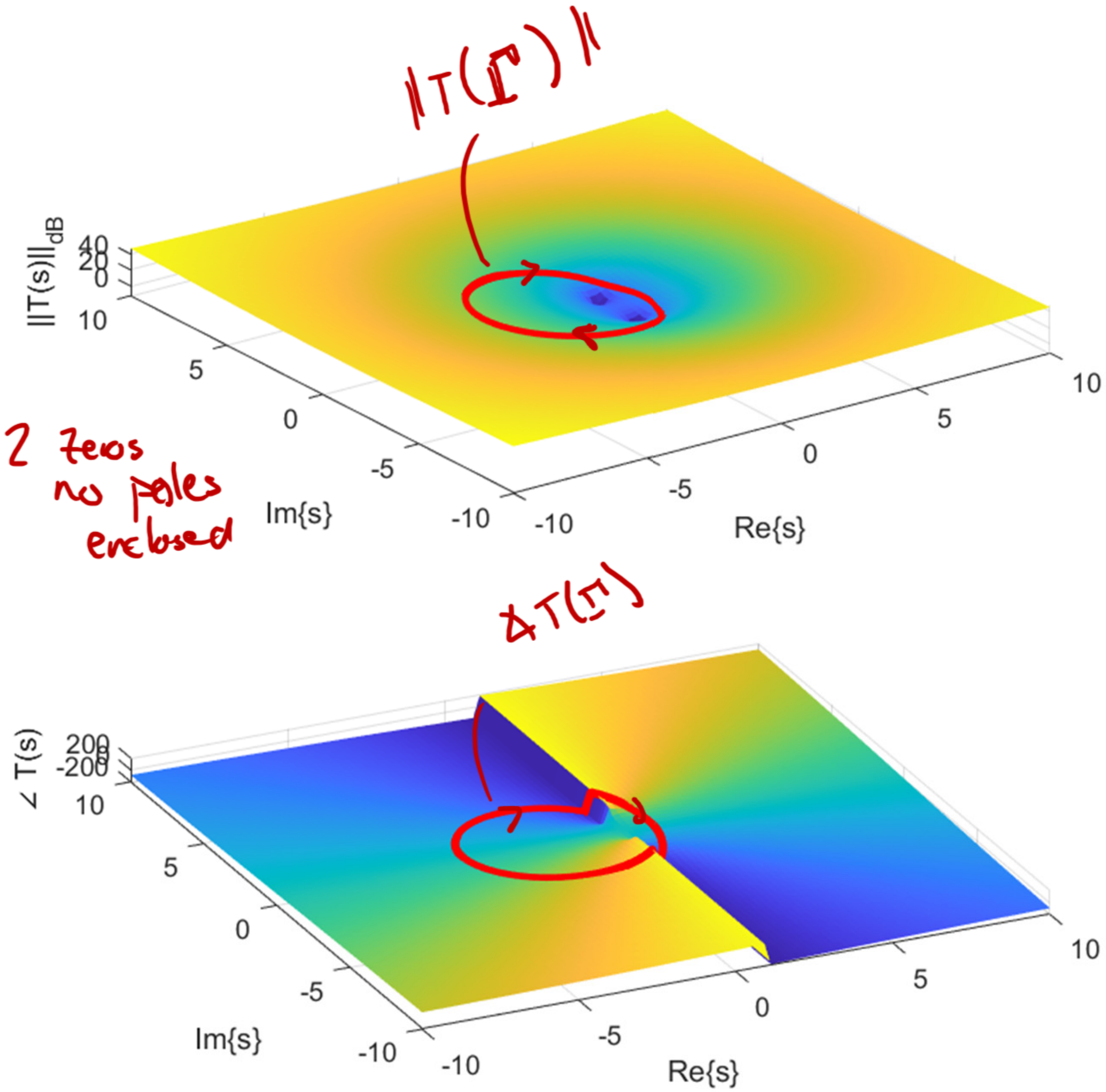
# of zeros enclosed by Γ

# of times phase goes through a 360° change

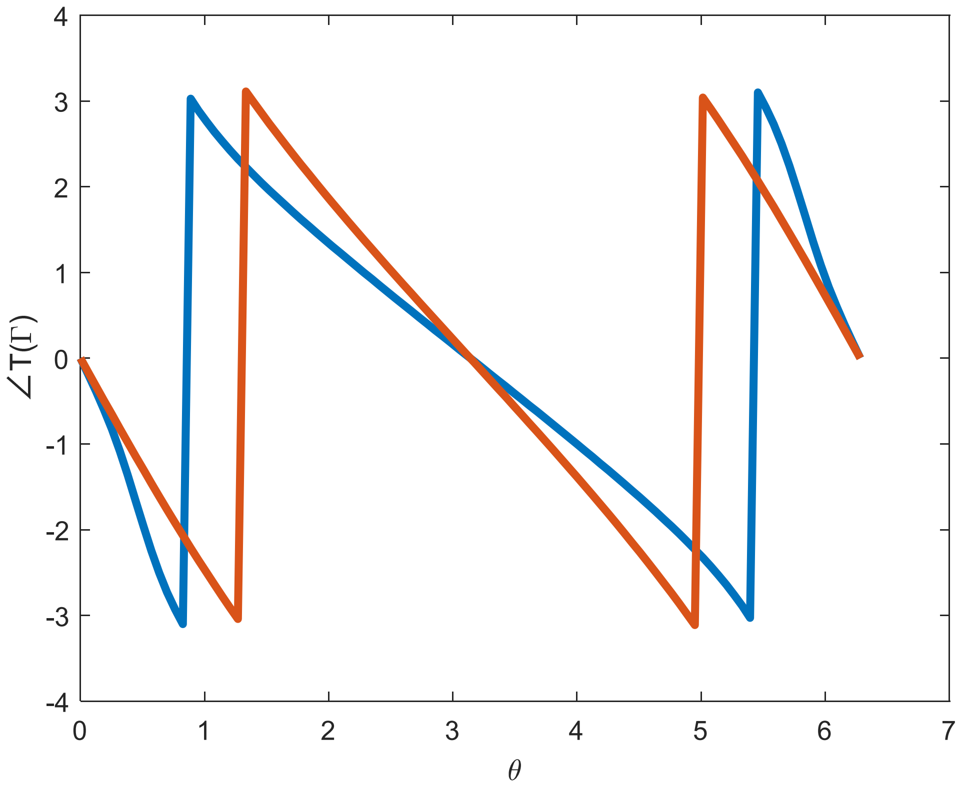
(−) if clockwise

= # of encirclements of origin when $T(\Gamma)$ plotted in complex plane.

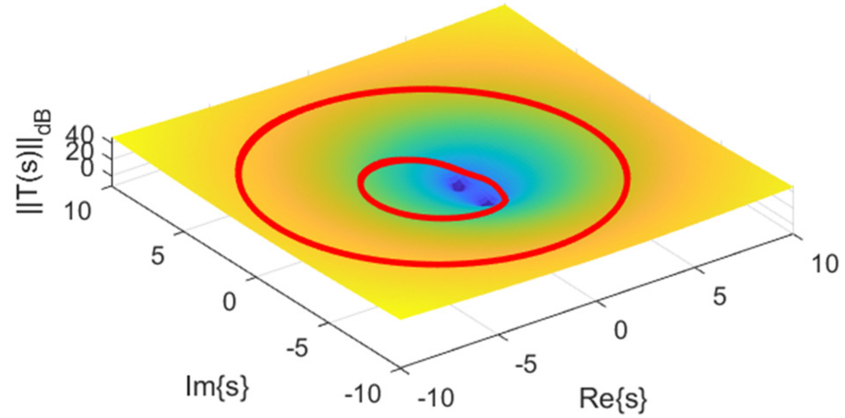

2x(-360°)
∠T(Γ)
θ


||T(Γ)||
||T(s)||dB
Im{s}
Re{s}
2 zeros
no poles
enclosed
∡T(Γ)
∠ T(s)
Im{s}
Re{s}

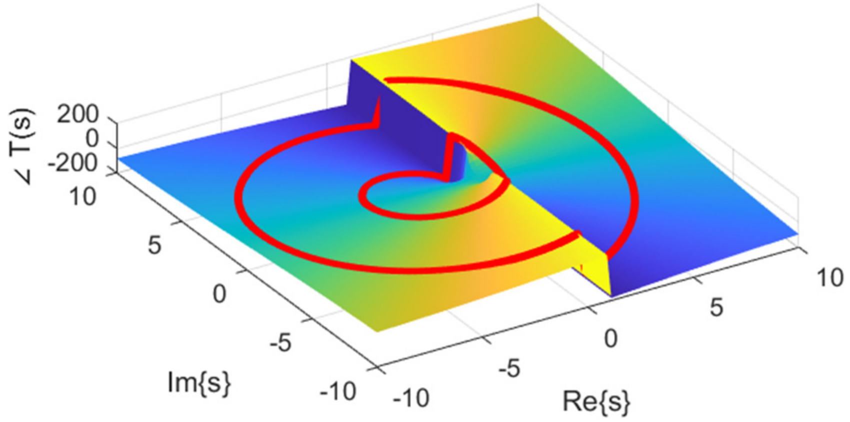
$$T(s) = (s - s_1)(s - s_1^*)$$



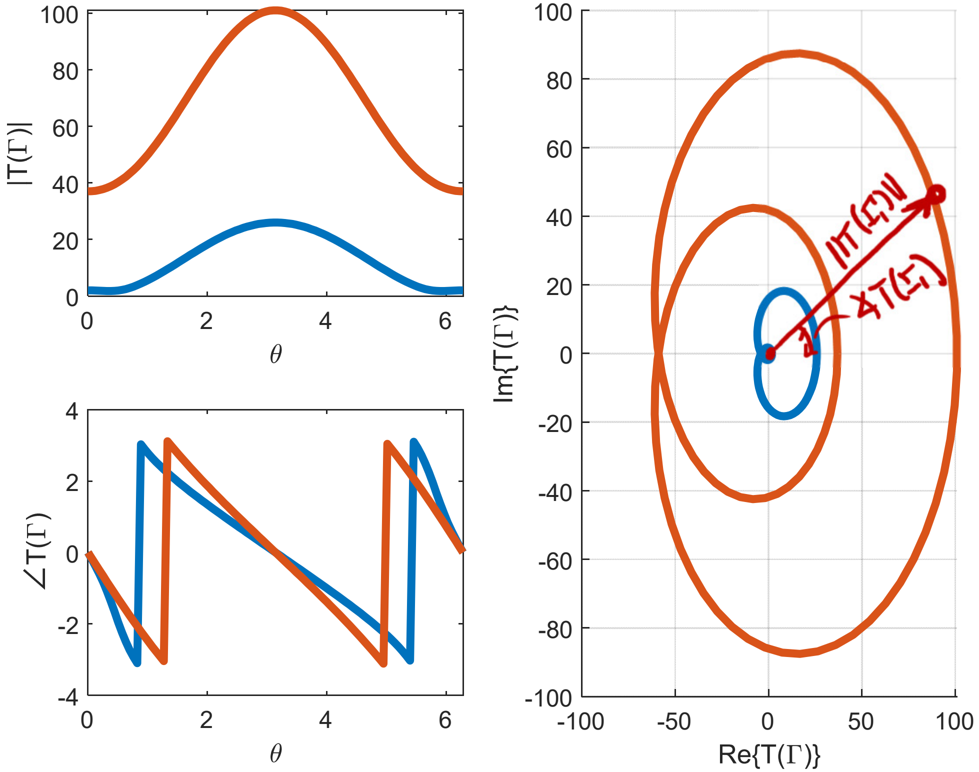
4
3
2
1
0
-1
-2
-3
-4
$\angle T(\Gamma)$
0
1
2
3
4
5
6
7
θ



$\|T(s)\|_{dB}$
40
20
0
10
5
0
-5
-10
Im{s}
-10
-5
0
5
10
Re{s}

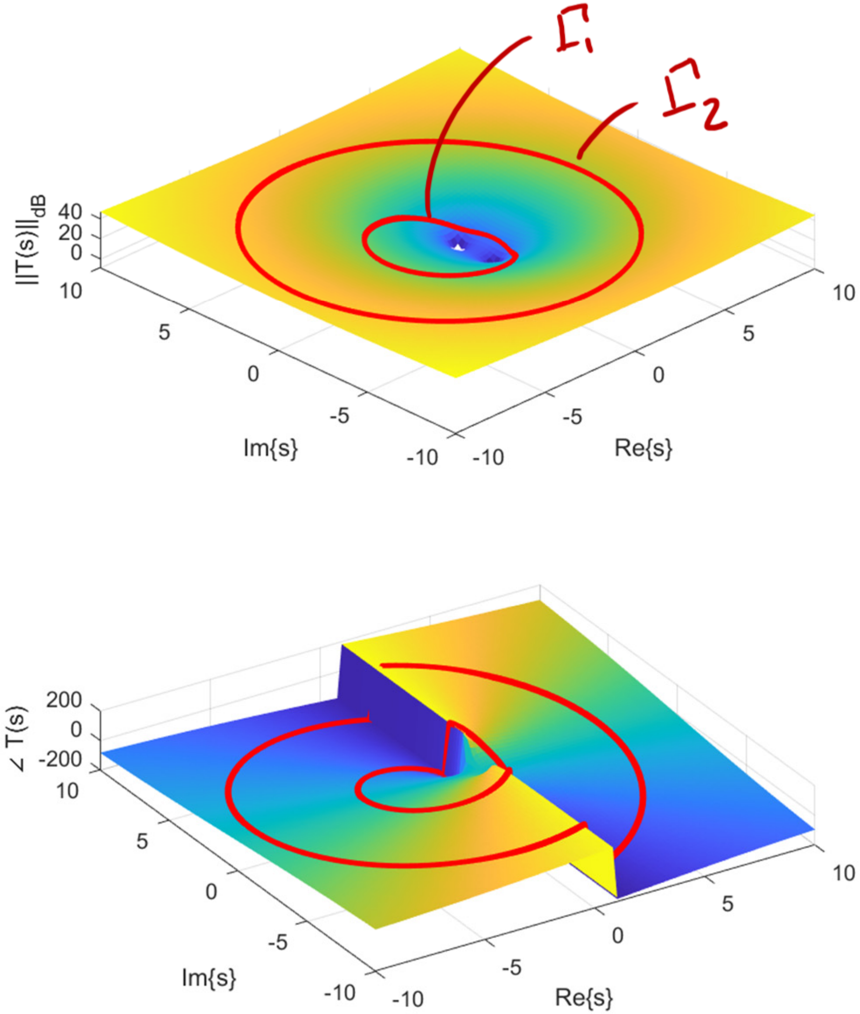


$\angle$ T(s)
200
0
-200
10
5
0
-5
-10
Im{s}
-10
-5
0
5
10
Re{s}



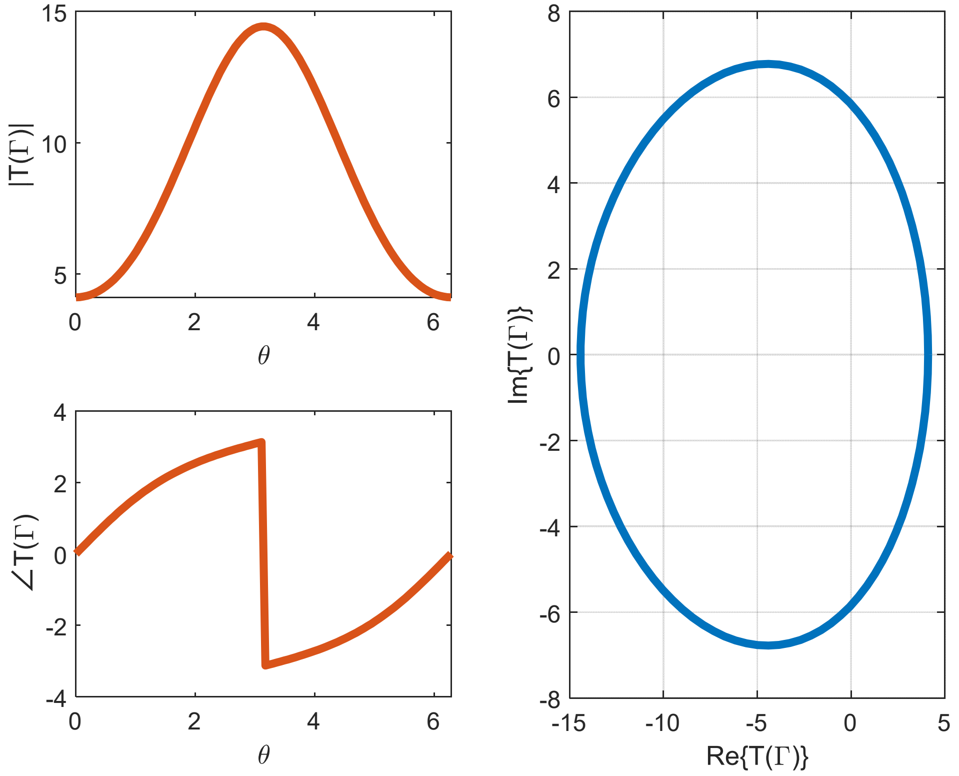
100
80
60
40
20
0
|T(Γ)|
0 2 4 6
θ
4
2
0
-2
-4
∠T(Γ)
0 2 4 6
θ
Im{T(Γ)}
Re{T(Γ)}
-100 -50 0 50 100
|T(Γ1)|
∠T(Γ1)

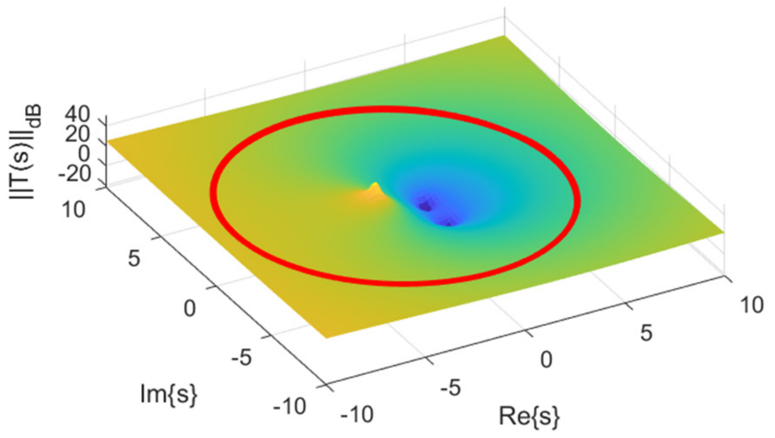
$$T(s) = (s - s_1)(s - s_1^*)$$

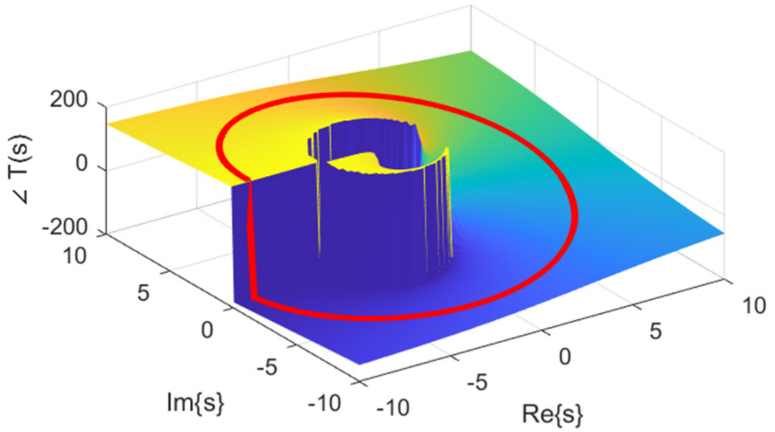


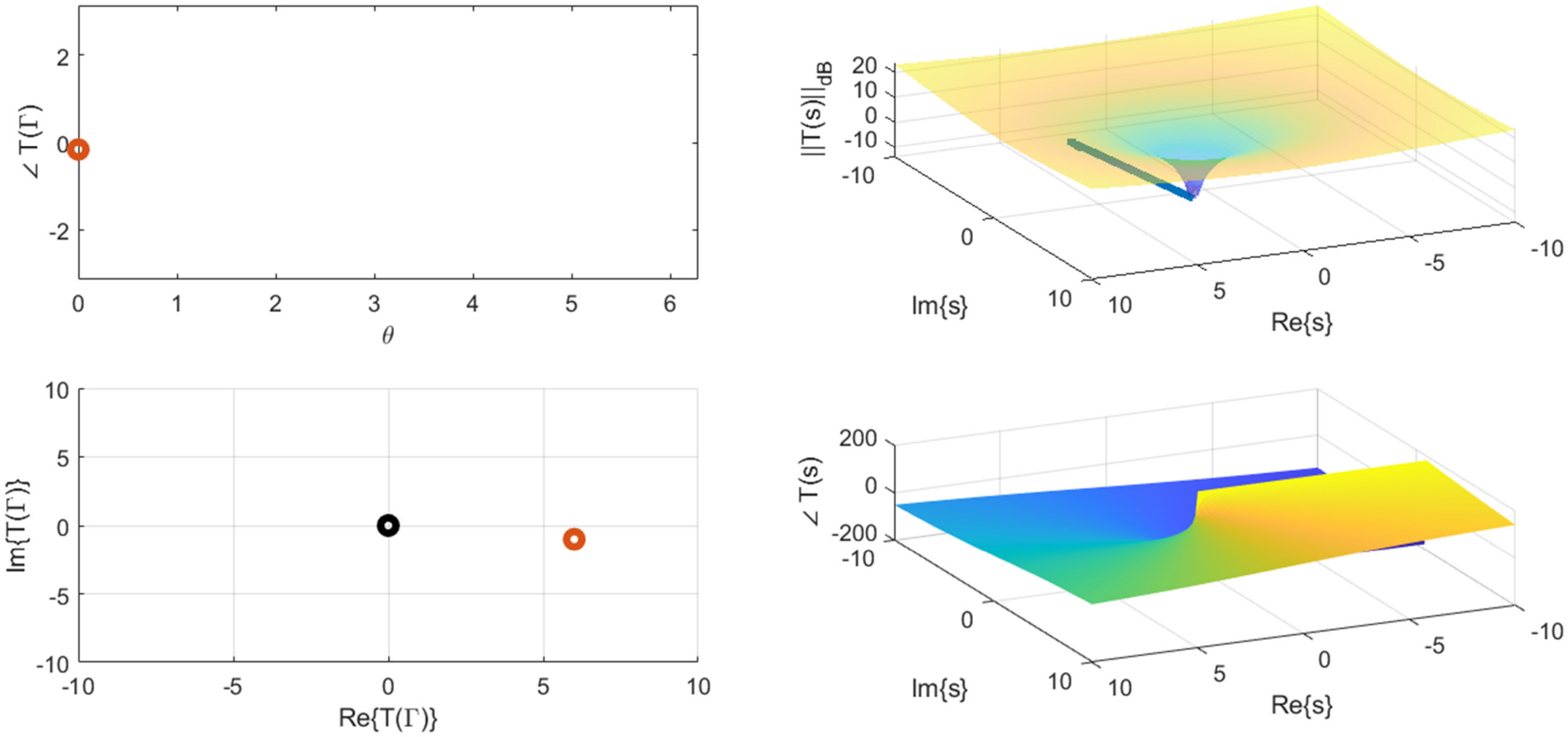
Γ1
Γ2
||T(s)||dB
40
20
0
10
5
0
-5
-10
Im{s}
Re{s}
∠ T(s)
200
0
-200

$$T(s) = \frac{(s - s_1)(s - s_1^*)}{s - p_1}$$

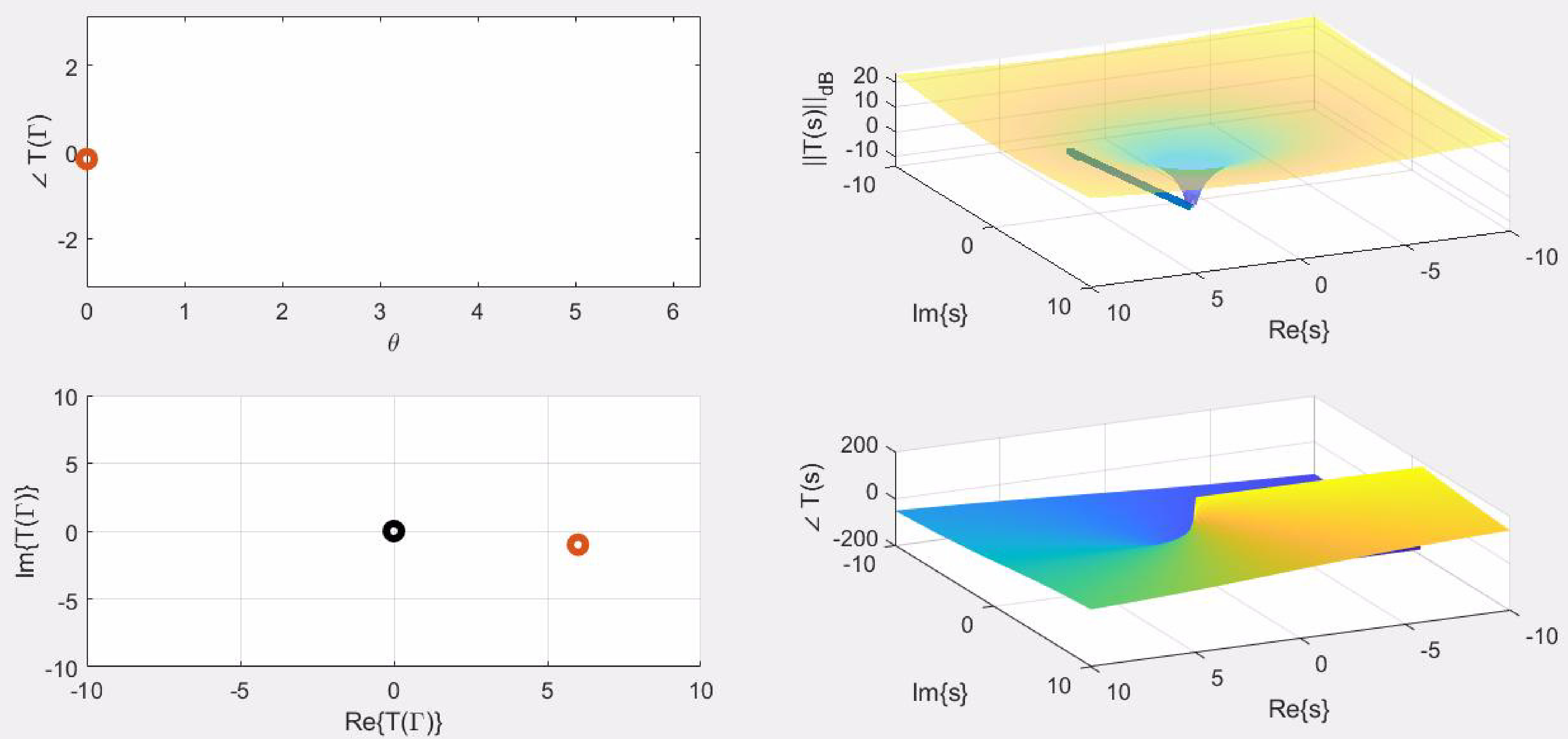




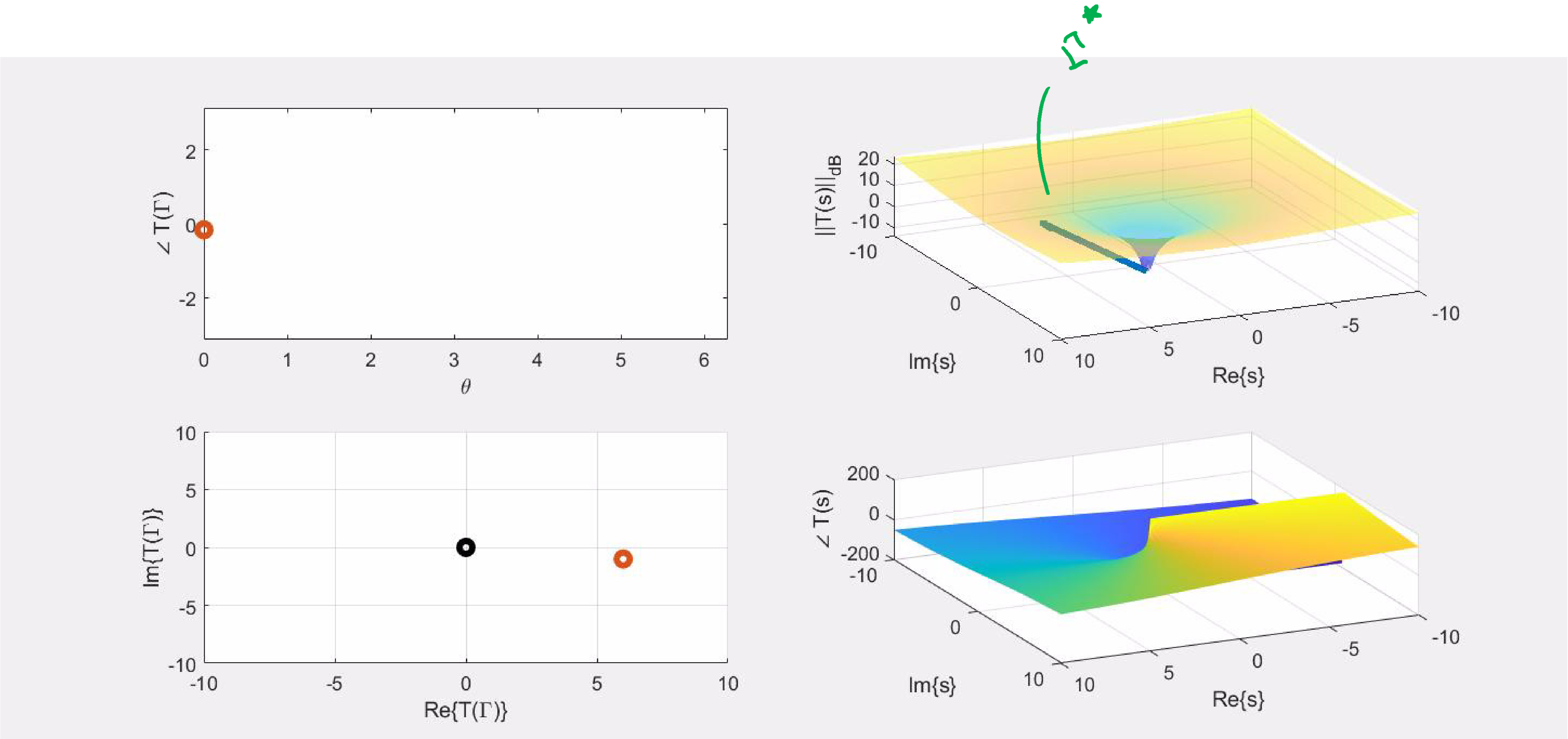




2
0
-2
$\angle T(\Gamma)$
0 1 2 3 4 5 6
θ
$||T(s)||_{dB}$
20
10
0
-10
-10
0
Im{s}
10
10
5
0
-5
-10
Re{s}
10
5
0
-5
-10
Im{T(Γ)}
-10 -5 0 5 10
Re{T(Γ)}
200
0
-200
$\angle T(s)$
-10
0
Im{s}
10
10
5
0
-5
-10
Re{s}



∠T(Γ)
θ
||T(s)||_dB
Im{s}
Re{s}
Im{T(Γ)}
Re{T(Γ)}
∠T(s)



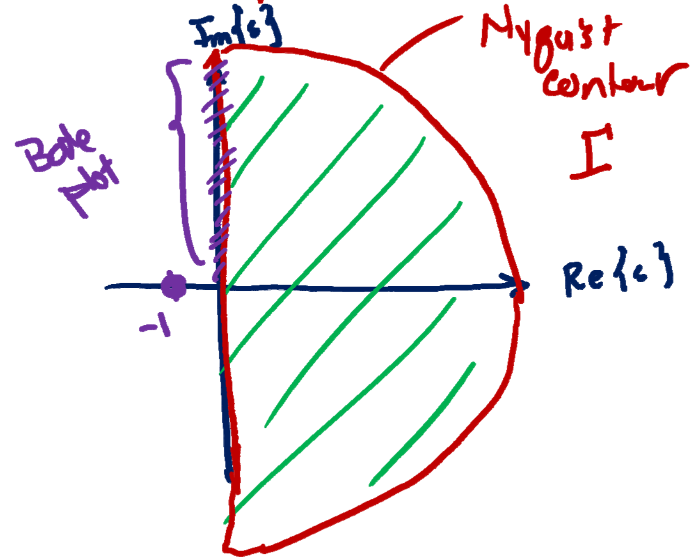
$\angle T(\Gamma)$
θ
$\mathrm{Im}\{T(\Gamma)\}$
$\mathrm{Re}\{T(\Gamma)\}$
$||T(s)||_{dB}$
Im{s}
Re{s}
$\angle T(s)$
Im{s}
Re{s}
Γ^*

Nyquist Contour & Stability Test

Use this to determine stability of $\frac{T(s)}{1+T(s)}$ & $\frac{1}{1+T(s)}$ → Do these have RHP poles?

we know $T(s)$ has no RHP poles if open-loop system is stable

RHP poles of $\frac{T(s)}{1+T(s)}$ or $\frac{1}{1+T(s)}$ are RHP zeros of $1+T(s)$



→ Plot $1+T(\Gamma)$ in the complex plane

# of times it encircles the origin = # of RHP zeros in $1+T(s)$ ⟺ RHP poles in closed loop transfer functions

# of origin encirclements of $1+T(\Gamma)$ ≡ # of encirclements of $(-1, 0)$ of $T(\Gamma)$