Alternative: The Phase Margin Test

Required:

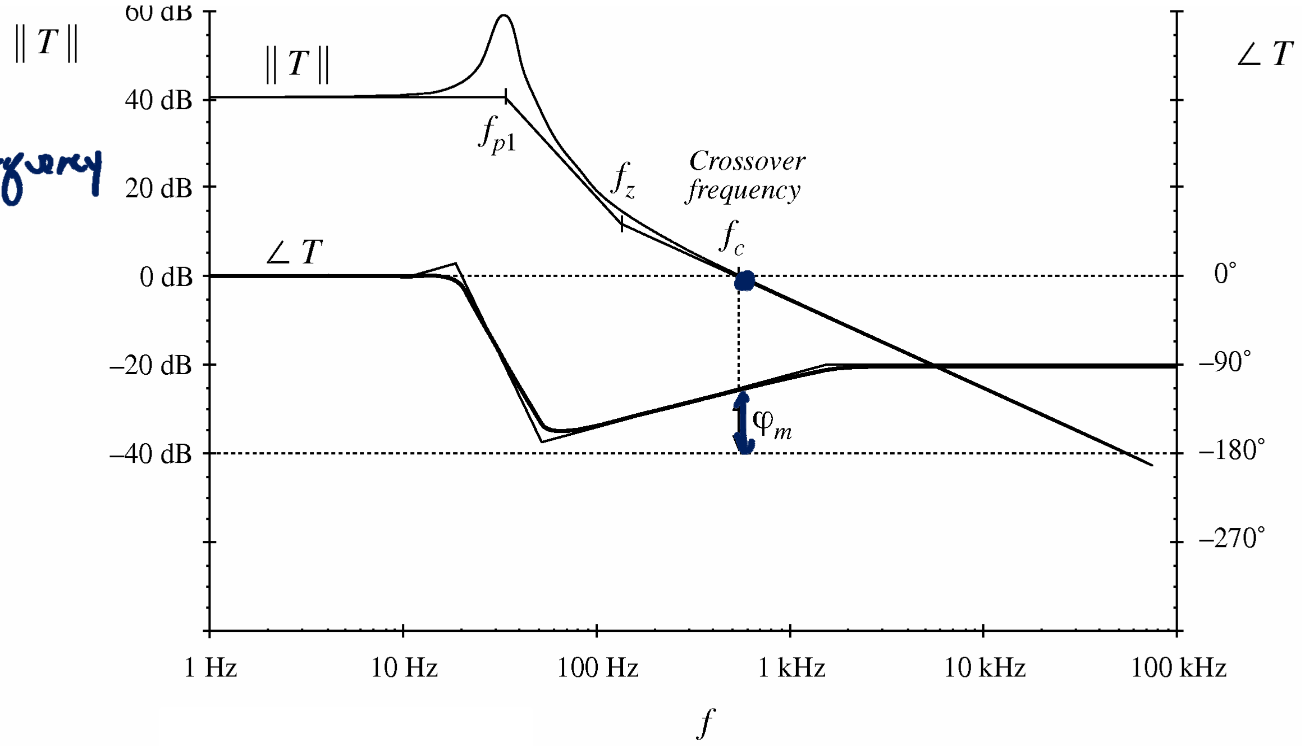
(1) $T(s)$ has no RHP poles

(2) $\|T(j\omega)\| = 0$ for exactly one ω_c

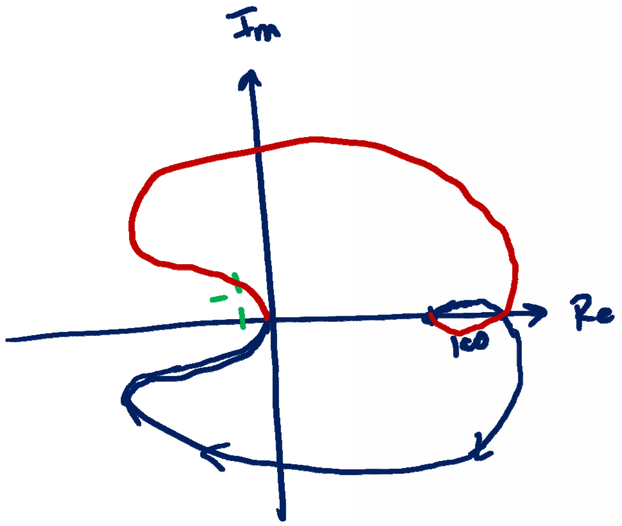
there is only one f_c = crossover frequency

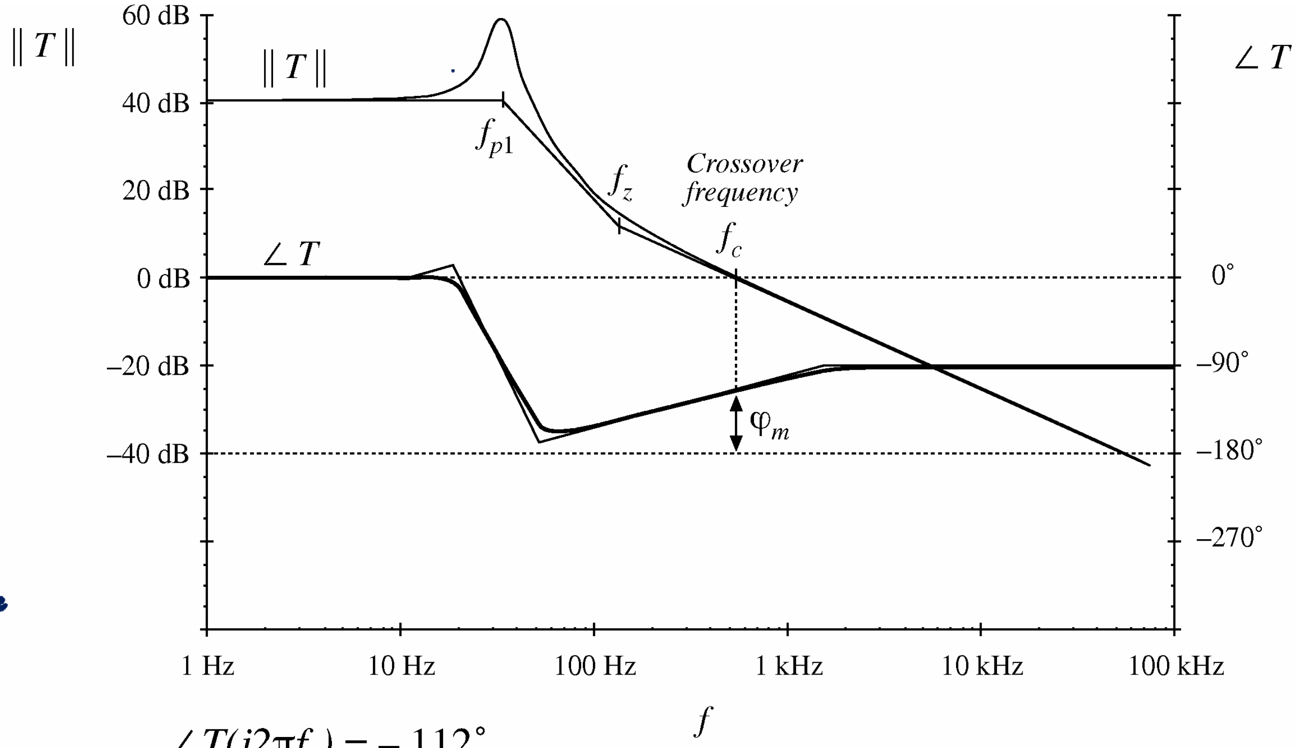
System is stable if

$$\varphi_m = 180^\circ + \measuredangle T(j\omega_c) > 0^\circ$$



Example: Stable System

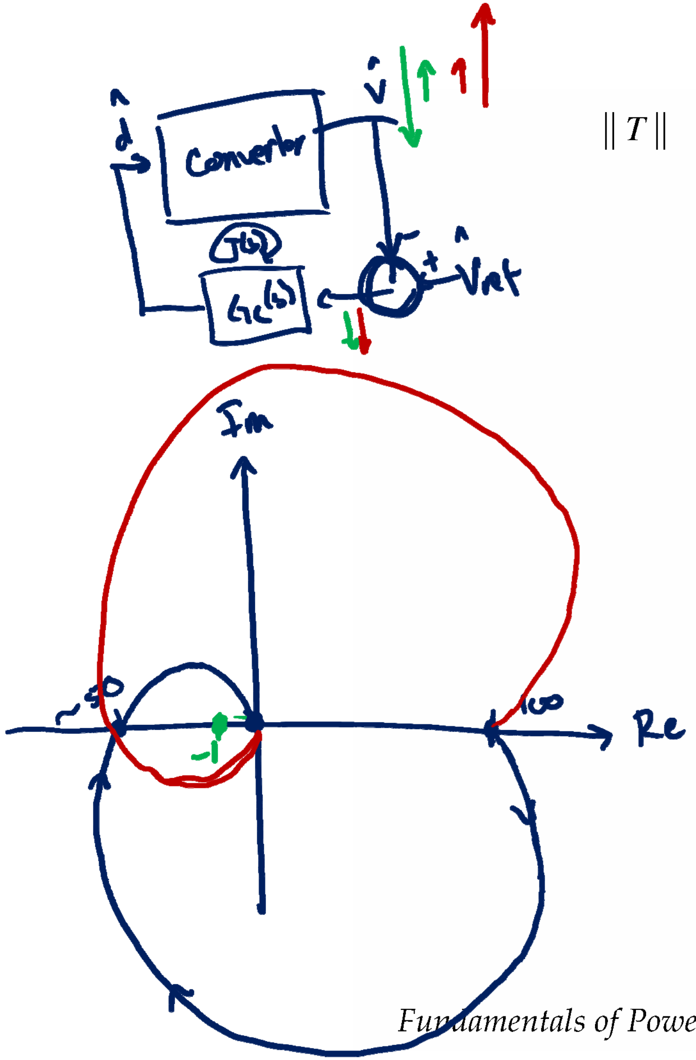


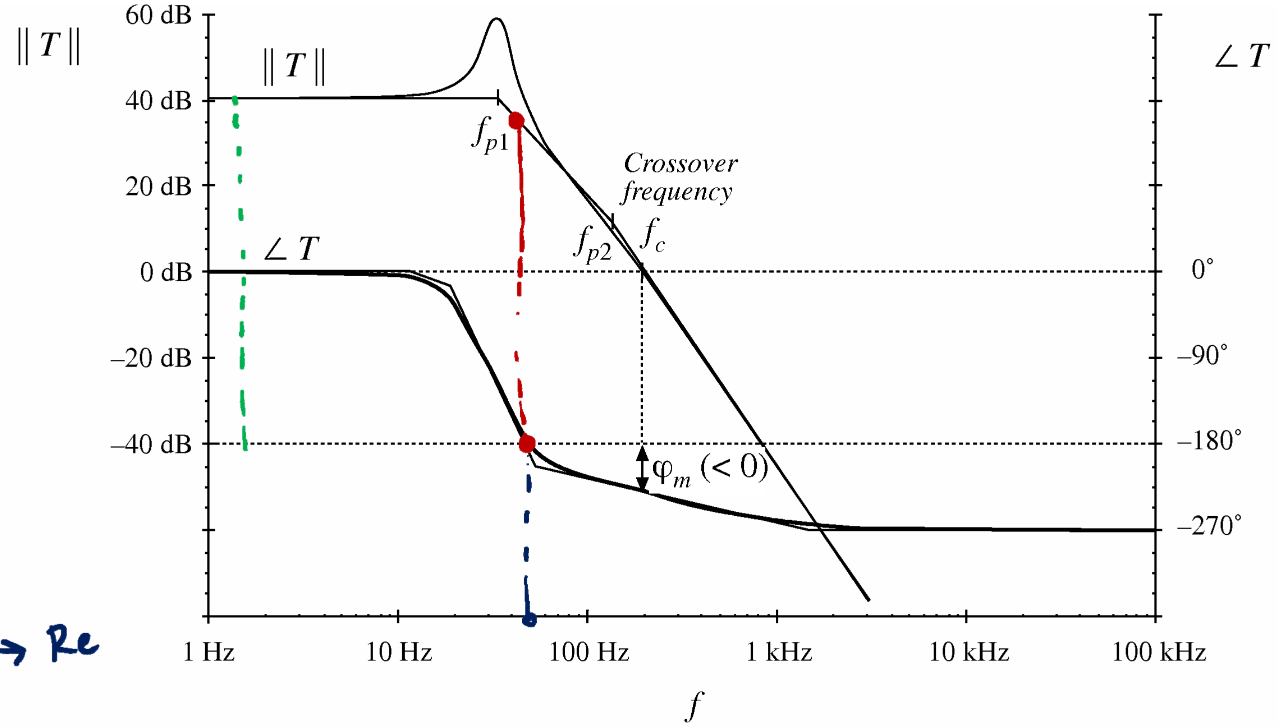


$\angle T(j2\pi f_c) = -112^\circ$

$\varphi_m = 180^\circ - 112^\circ = +68^\circ$

Example: Unstable System

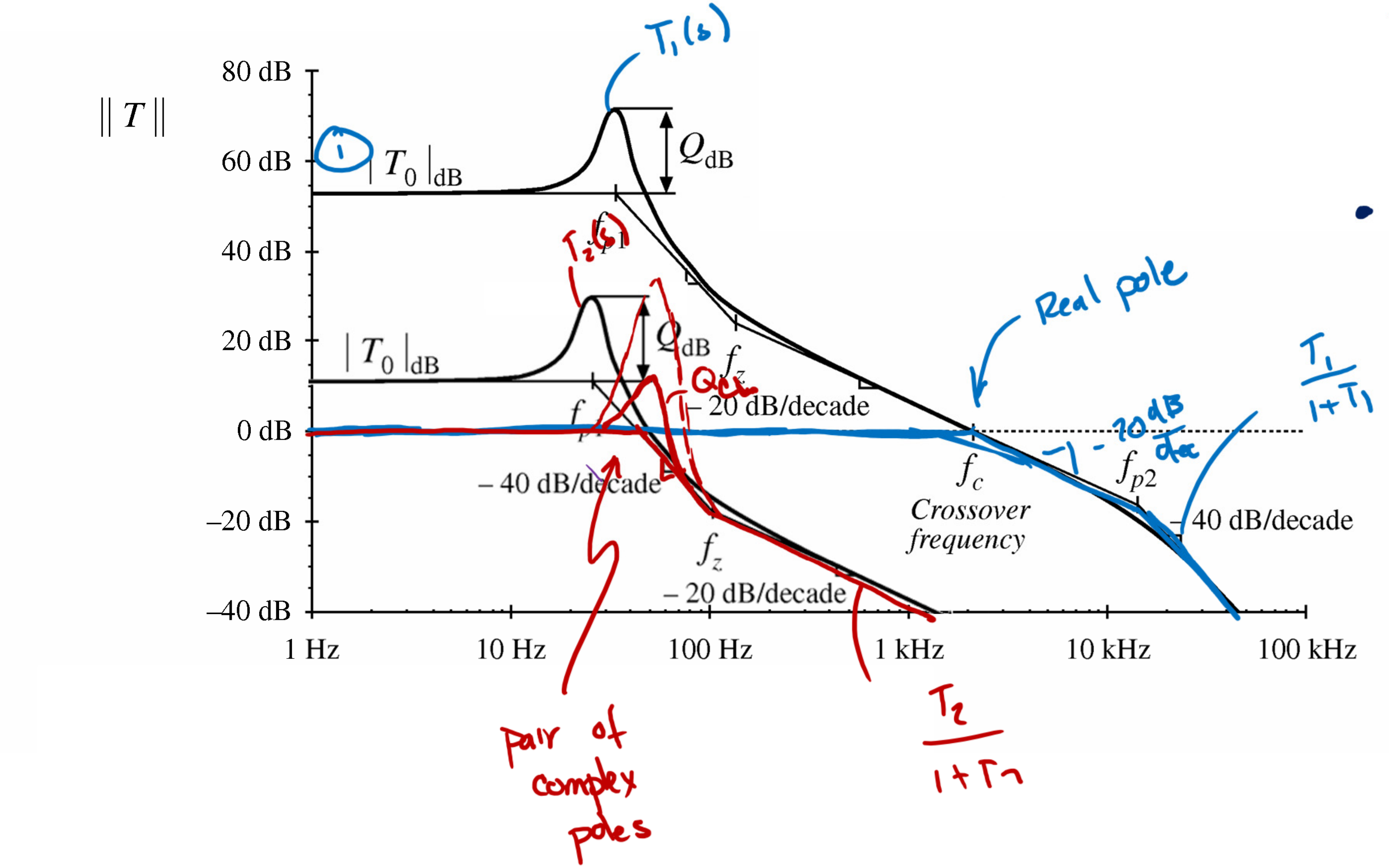




$$\angle T(j2\pi f_c) = -\,230^\circ$$

$$\varphi_m = 180^\circ - 230^\circ = -\,50^\circ$$

T(s) vs T/1+T



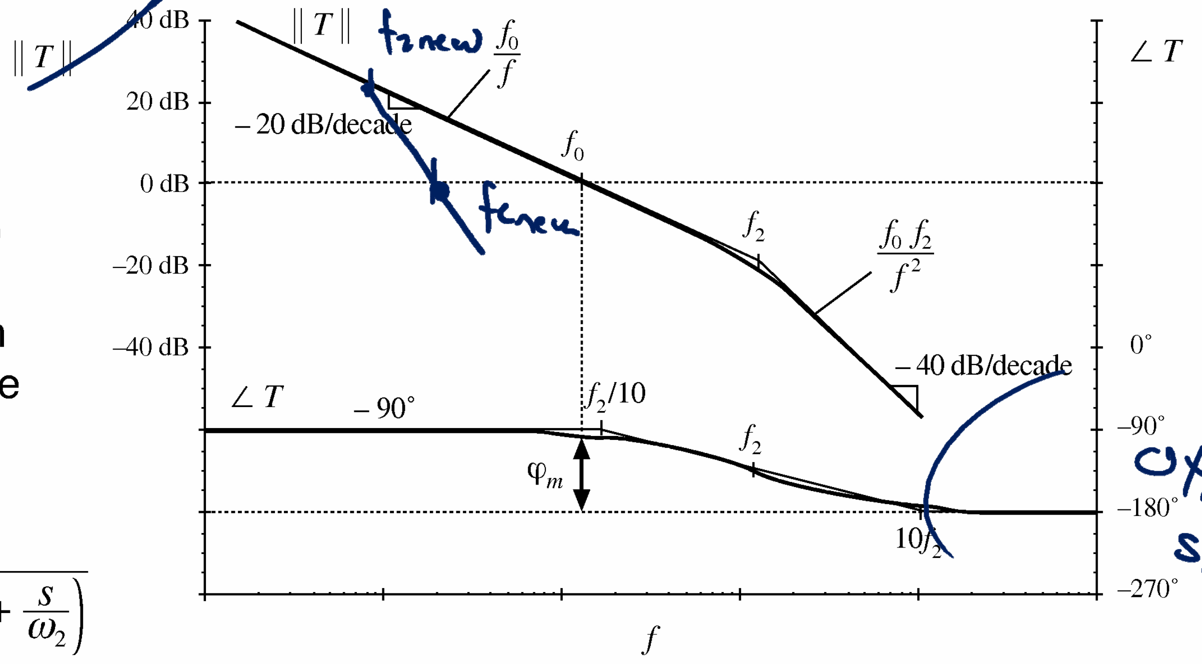
- $f_c \sim BW_{CL}$
  - Design high f_c for fast response
- Q_{CL} present when f_c crosses @ -40 dB/dec slope
  - will result in ringing

A Generic Second-Order System

other stuff

Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$



other stuff

Closed-Loop Response

If

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Then

$$\frac{T(s)}{1+T(s)} = \frac{1}{1+\frac{1}{T(s)}} = \frac{1}{1+\frac{s}{\omega_0}+\frac{s^2}{\omega_0\omega_2}}$$

or,

$$\frac{T(s)}{1+T(s)} = \frac{1}{1+\frac{s}{Q\omega_c}+\left(\frac{s}{\omega_c}\right)^2}$$

where

$$\omega_c = \sqrt{\omega_0\omega_2} = 2\pi f_c \qquad Q_{cl} = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

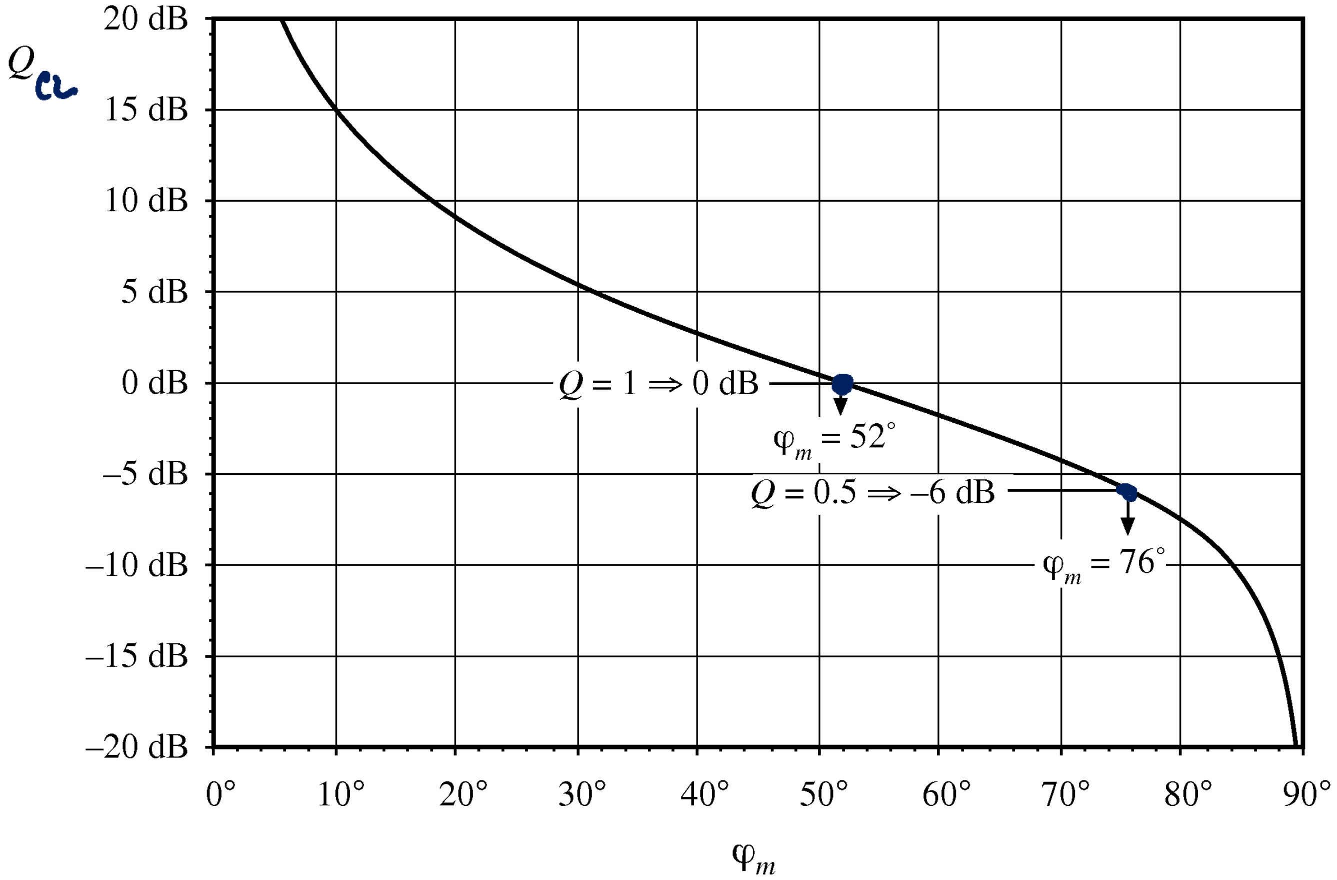
Q_{CL} vs. ϕ_m

Solve for exact crossover frequency, evaluate phase margin, express as function of φ_m. Result is:

$$Q = \frac{\sqrt{\cos \varphi_m}}{\sin \varphi_m}$$

$$\varphi_m = \tan^{-1} \sqrt{\frac{1 + \sqrt{1 + 4Q^4}}{2Q^4}}$$

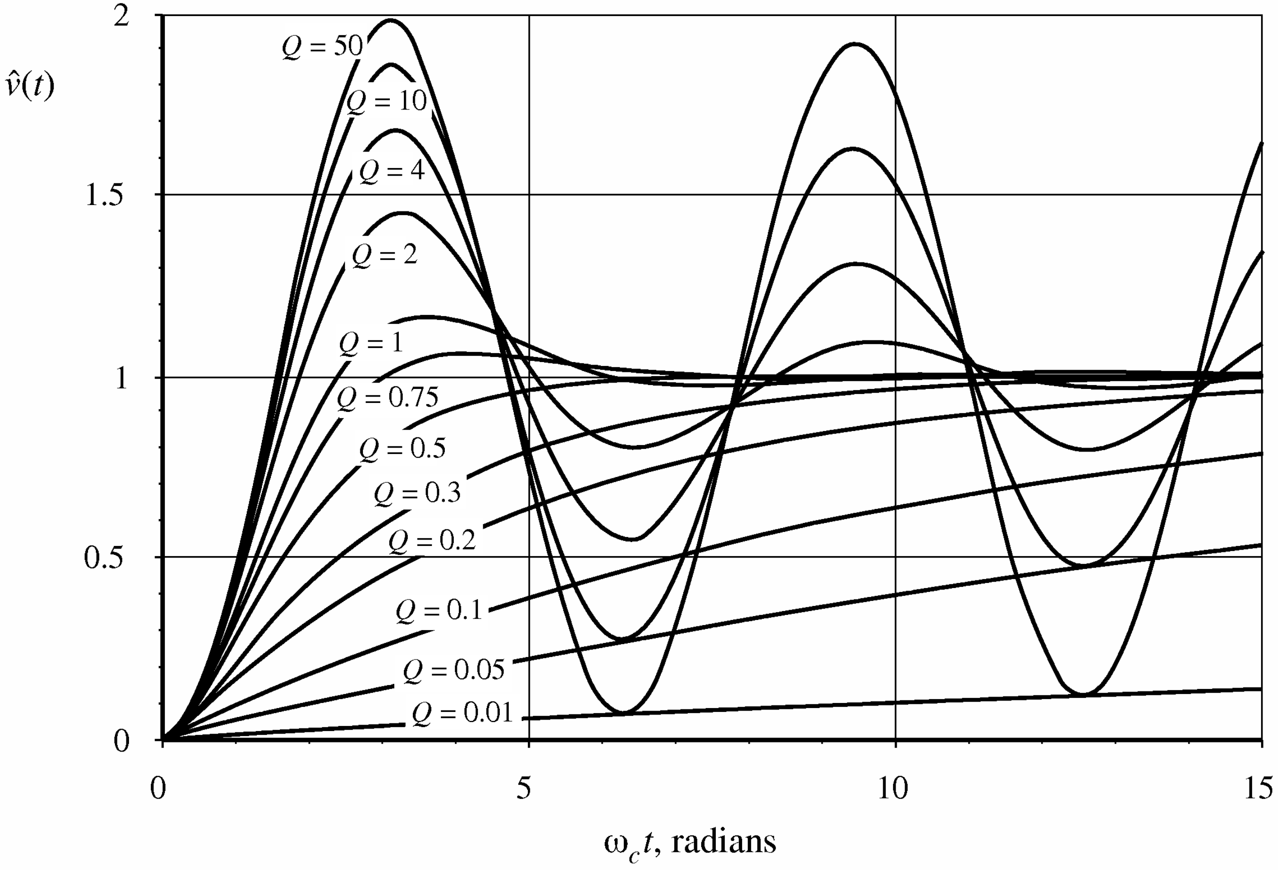
Q_{CL} vs. ϕ_m



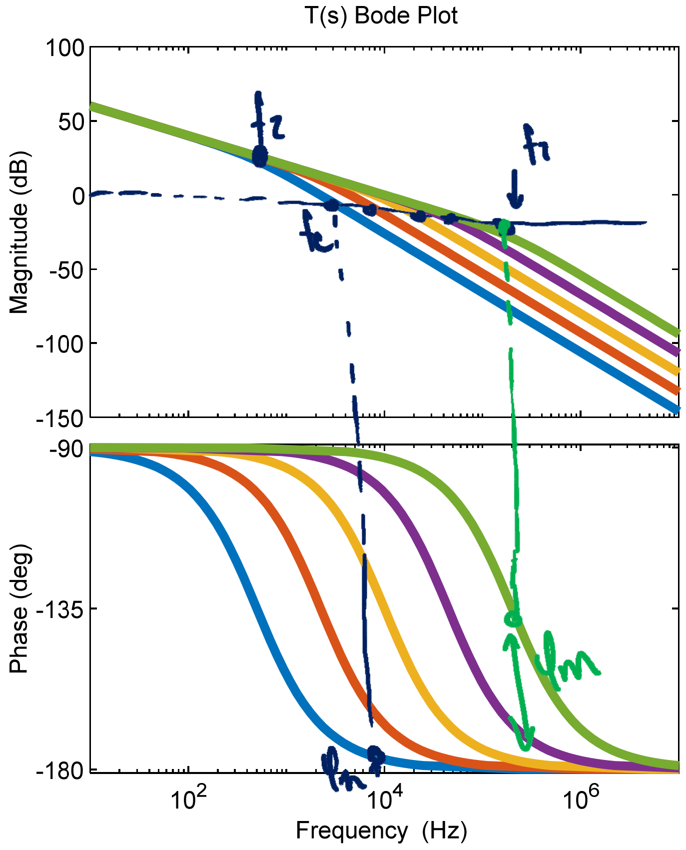
20 dB
15 dB
10 dB
5 dB
0 dB
–5 dB
–10 dB
–15 dB
–20 dB
Q_{CL}
0° 10° 20° 30° 40° 50° 60° 70° 80° 90°
φ_m
$Q = 1 \Rightarrow 0$ dB
$\varphi_m = 52°$
$Q = 0.5 \Rightarrow -6$ dB
$\varphi_m = 76°$

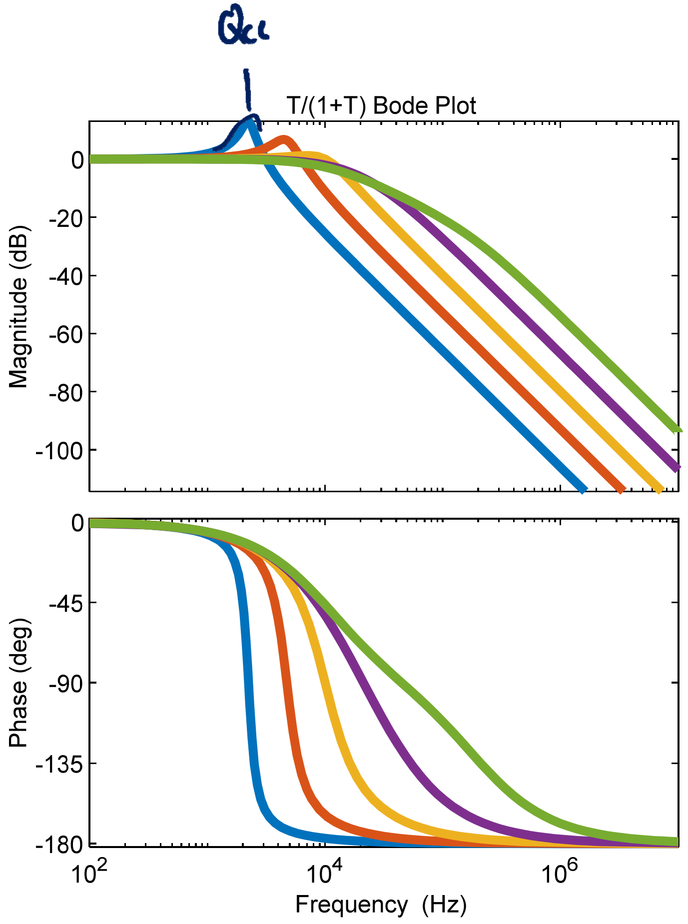
Closed-Loop Step Response vs. Q_{CL}

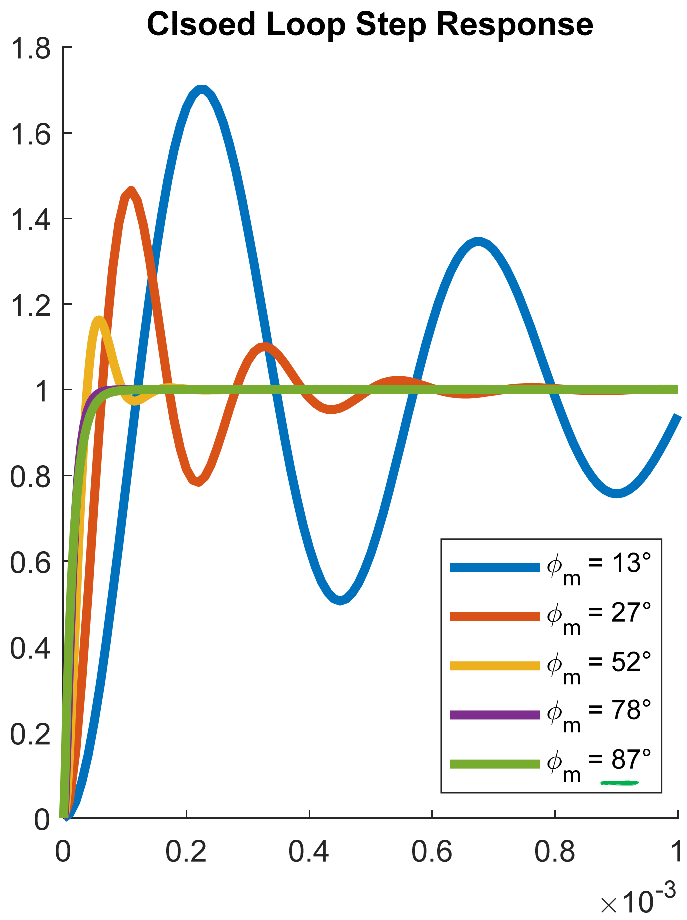
$\zeta = \frac{1}{2Q}$



Design of Phase Margin: Constant LF Gain Case







Design of Phase Margin: Constant f_c Case