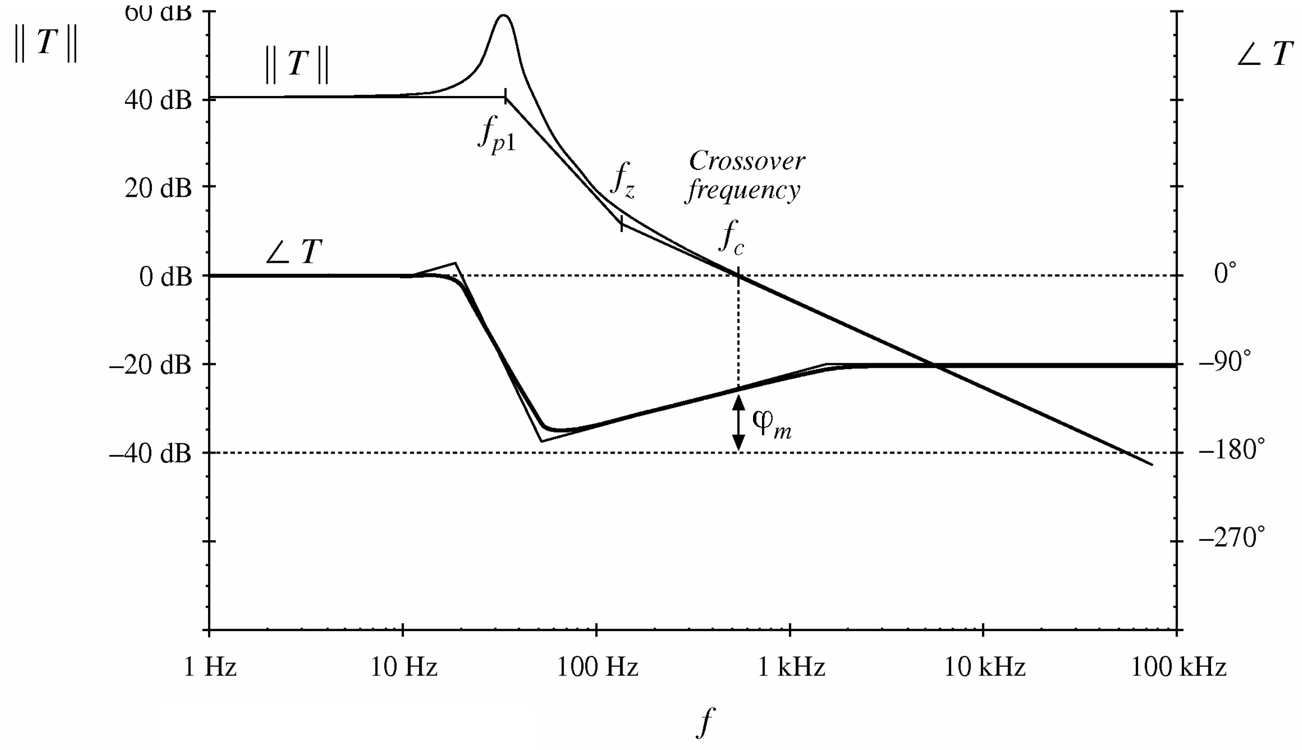
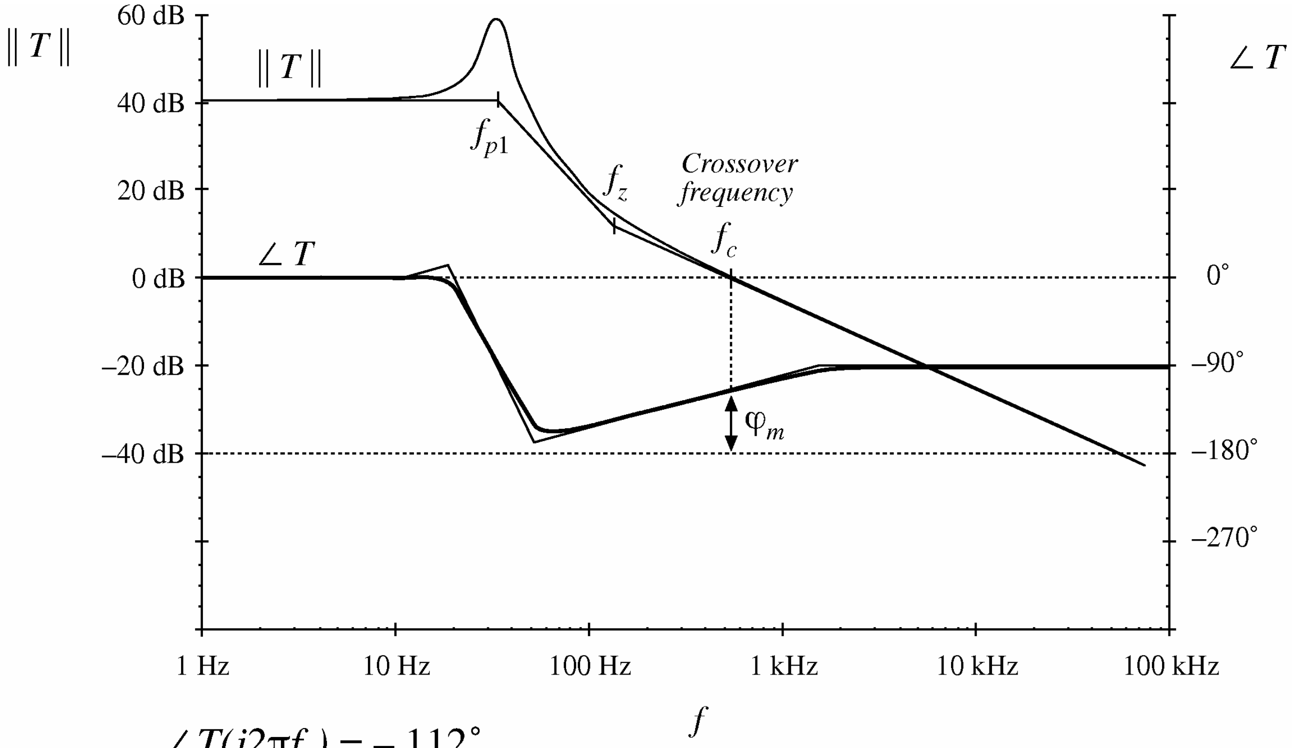
Alternative: The Phase Margin Test



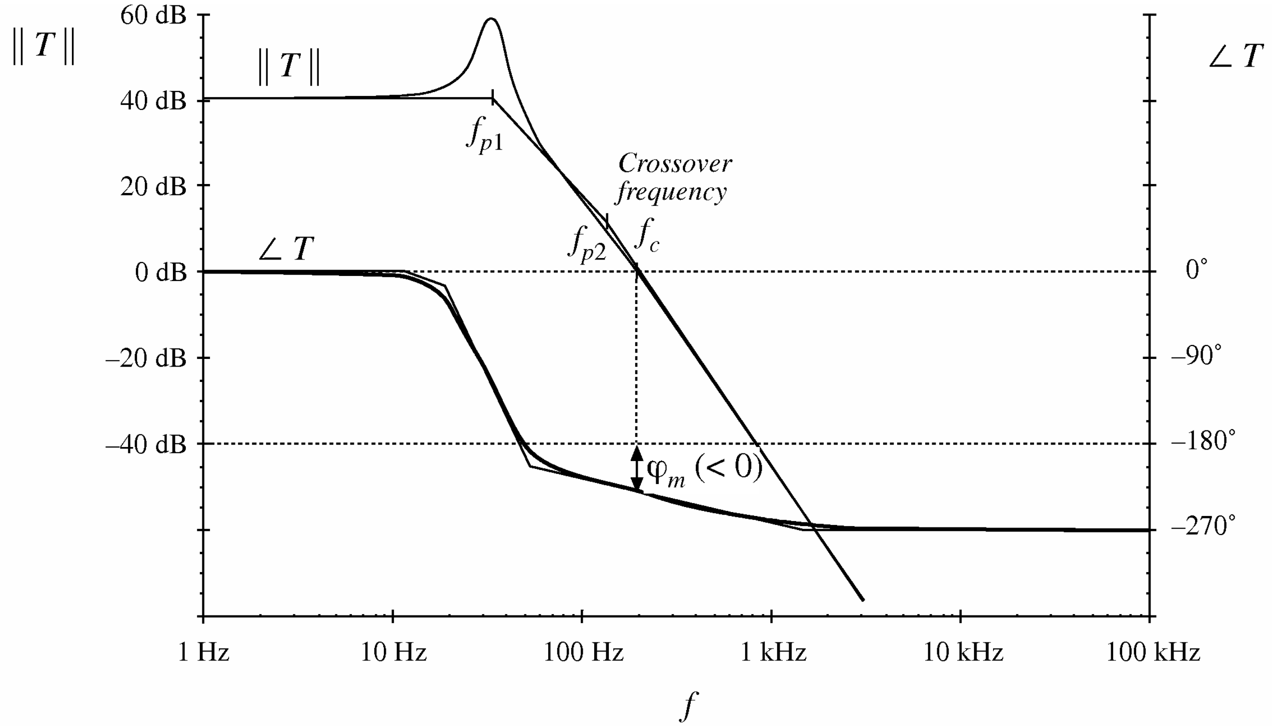
Example: Stable System



$$\angle T(j2\pi f_c) = -112^\circ$$

$$\varphi_m = 180^\circ - 112^\circ = +68^\circ$$

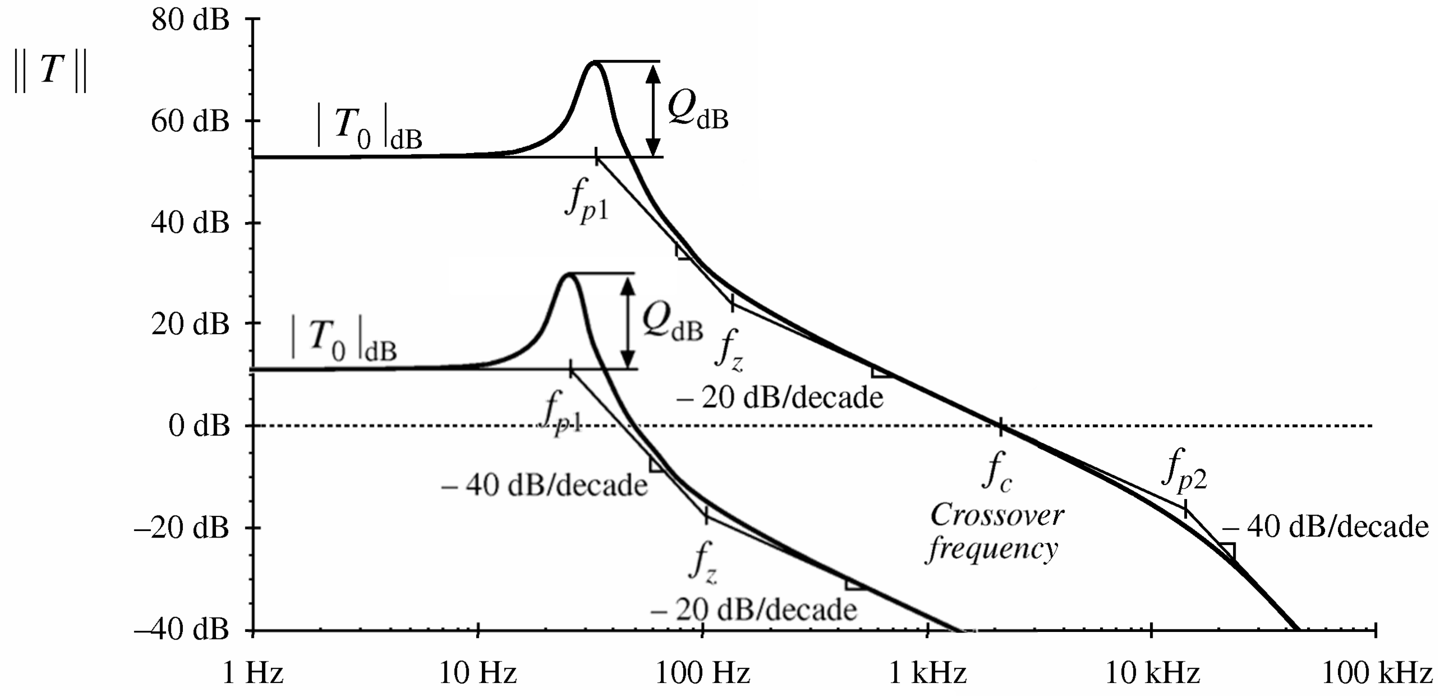
Example: Unstable System



$$\angle T(j2\pi f_c) = -230^\circ$$

$$\varphi_m = 180^\circ - 230^\circ = -50^\circ$$

T(s) vs T/1+T

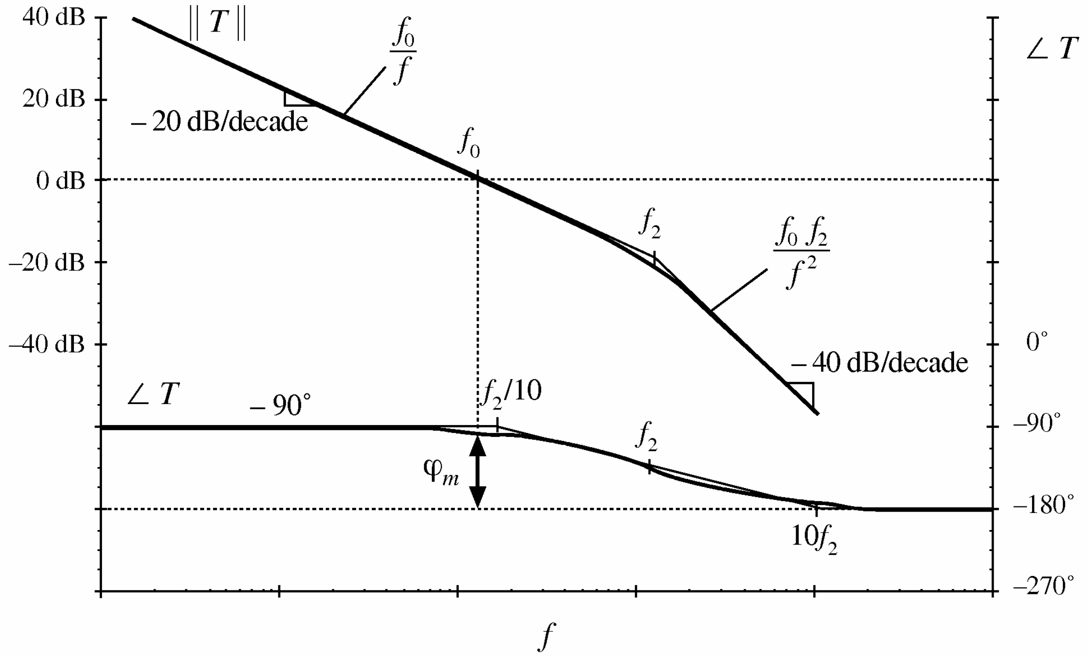


80 dB
60 dB
40 dB
20 dB
0 dB
–20 dB
–40 dB
|| T ||
| T0 |dB
QdB
fp1
fz
– 20 dB/decade
– 40 dB/decade
fc
Crossover frequency
fp2
– 40 dB/decade
| T0 |dB
QdB
fp1
fz
– 20 dB/decade
1 Hz
10 Hz
100 Hz
1 kHz
10 kHz
100 kHz

A Generic Second-Order System

Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_2}\right)}$$



Closed-Loop Response

If

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Then

$$\frac{T(s)}{1+T(s)} = \frac{1}{1 + \frac{1}{T(s)}} = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0 \omega_2}}$$

or,

$$\frac{T(s)}{1+T(s)} = \frac{1}{1 + \frac{s}{Q\omega_c} + \left(\frac{s}{\omega_c}\right)^2}$$

where

$$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

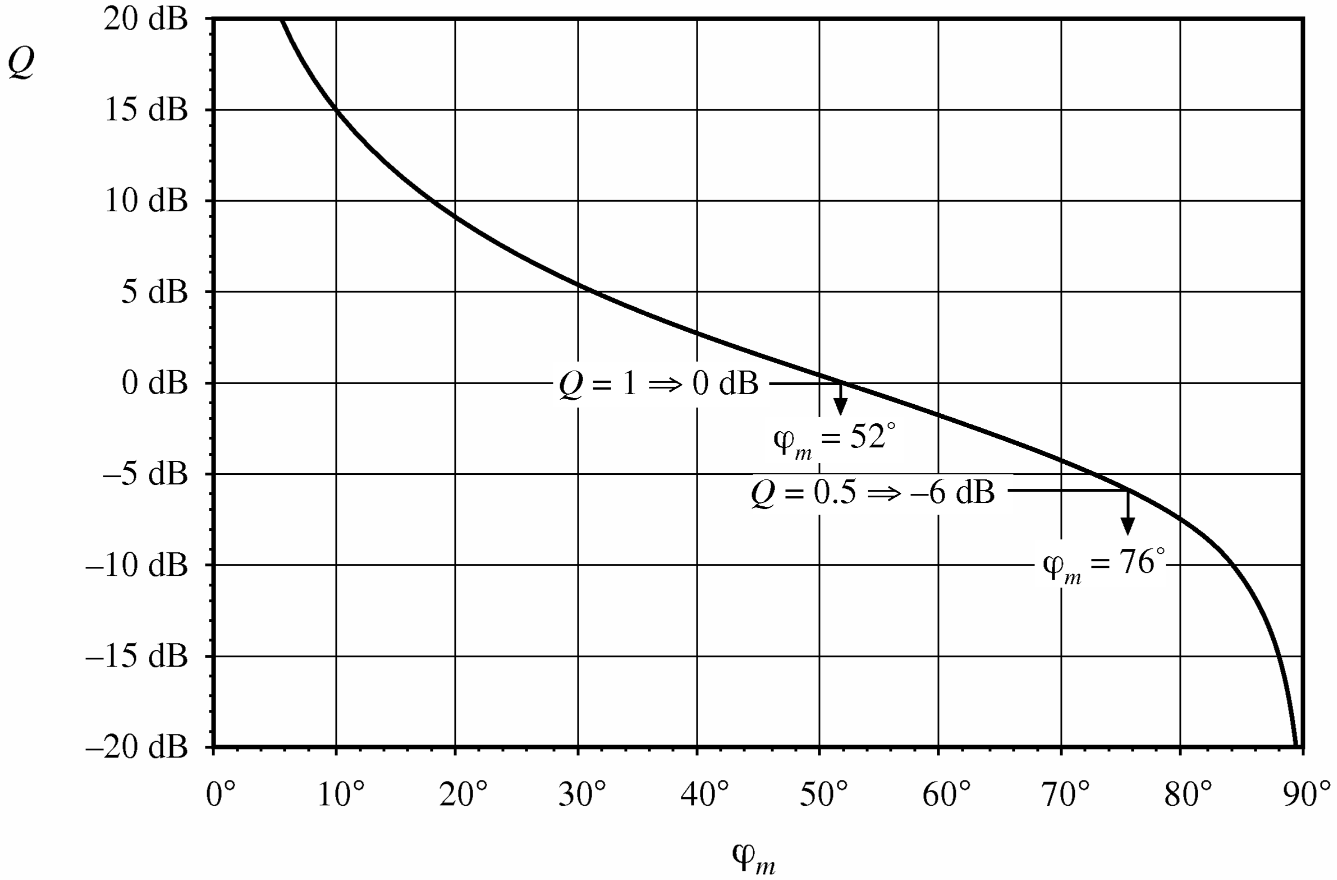
Q_{CL} vs. ϕ_m

Solve for exact crossover frequency, evaluate phase margin, express as function of φ_m. Result is:

$$Q = \frac{\sqrt{\cos\,\varphi_m}}{\sin\,\varphi_m}$$

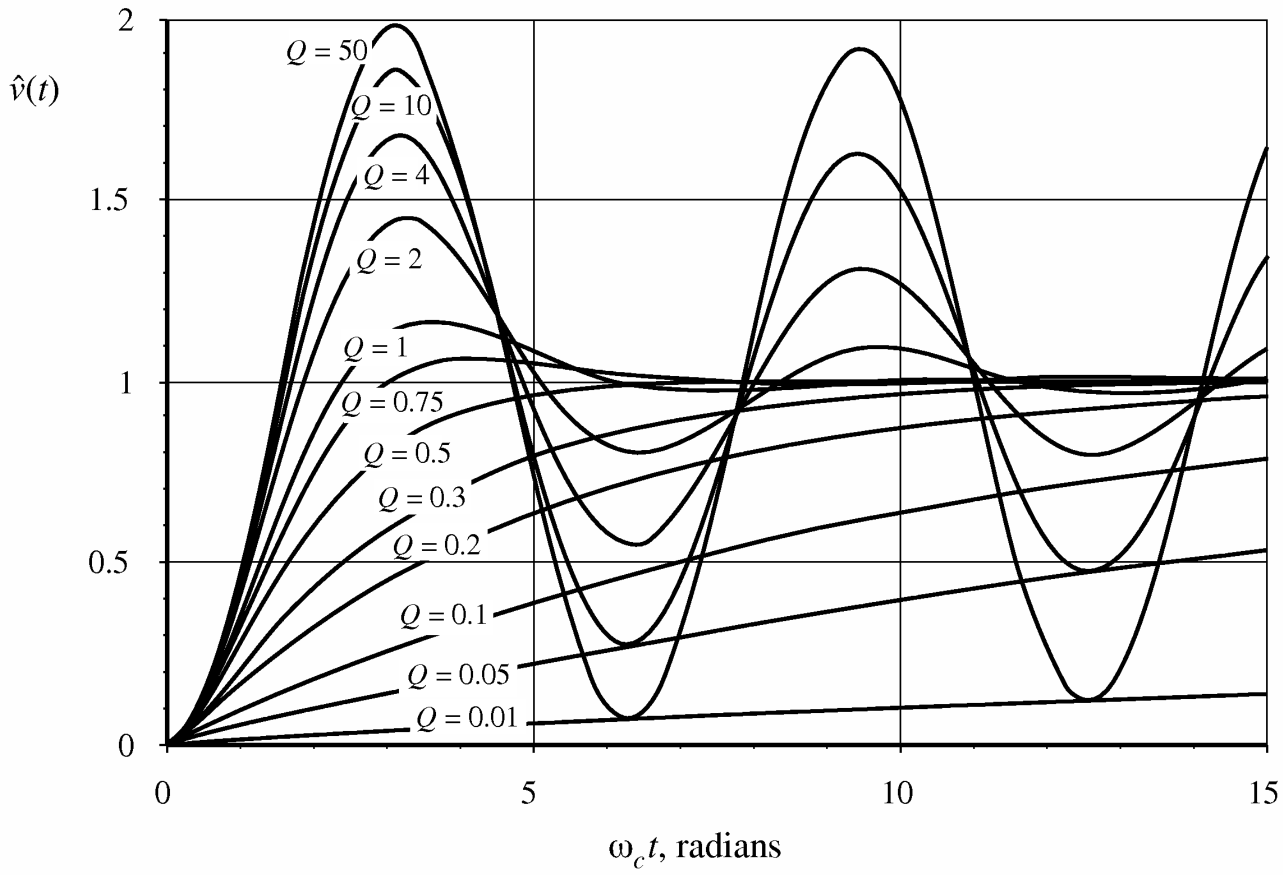
$$\varphi_m = \tan^{-1}\sqrt{\frac{1+\sqrt{1+4Q^4}}{2Q^4}}$$

Q_{CL} vs. ϕ_m

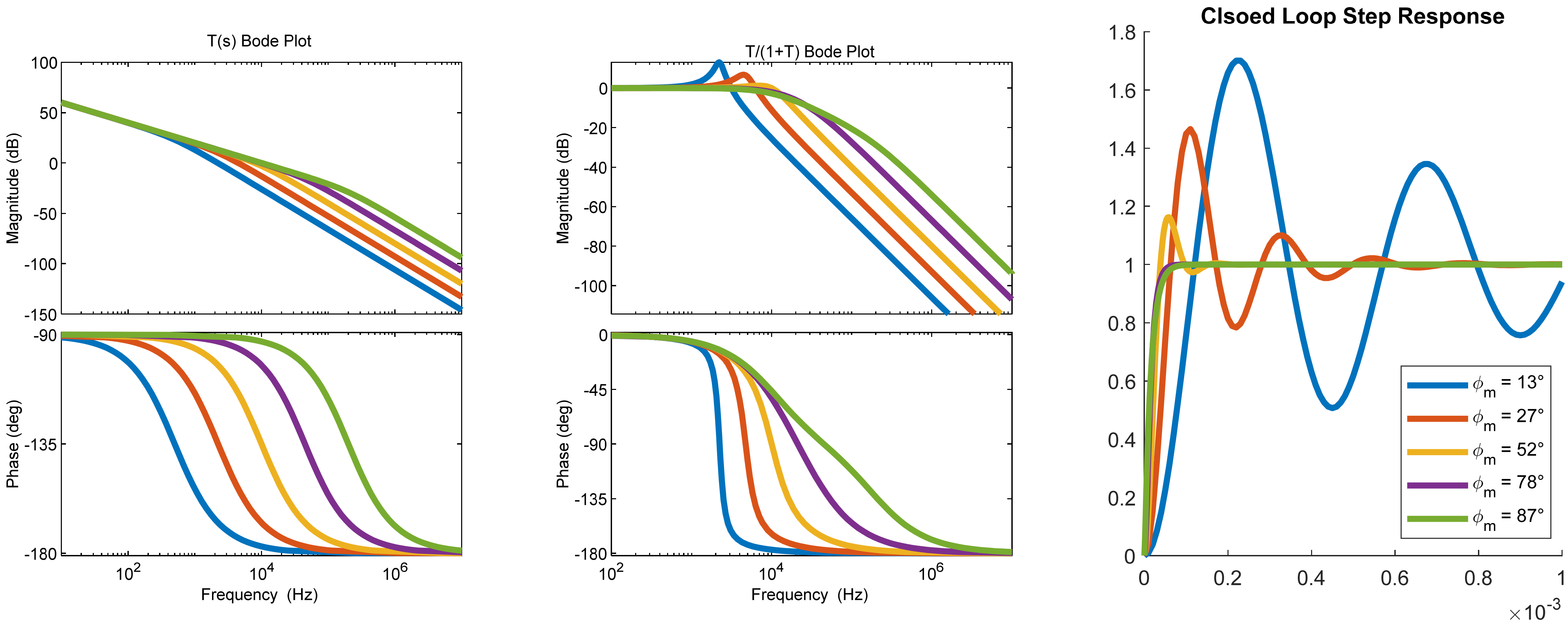


Q
20 dB
15 dB
10 dB
5 dB
0 dB
–5 dB
–10 dB
–15 dB
–20 dB
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
φm
Q = 1 ⇒ 0 dB
φm = 52°
Q = 0.5 ⇒ –6 dB
φm = 76°

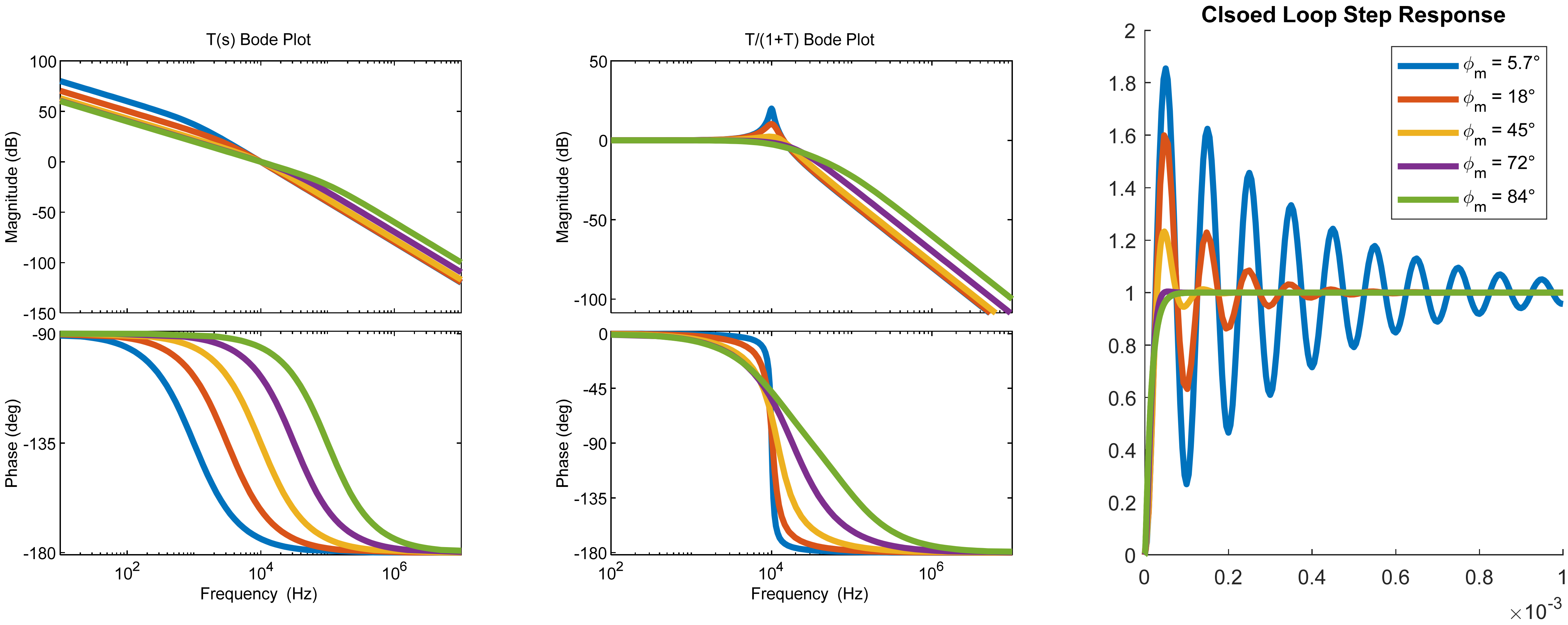
Closed-Loop Step Response vs. Q_{CL}



Design of Phase Margin: Constant LF Gain Case



Design of Phase Margin: Constant f_c Case



9.5 – Compensator Design