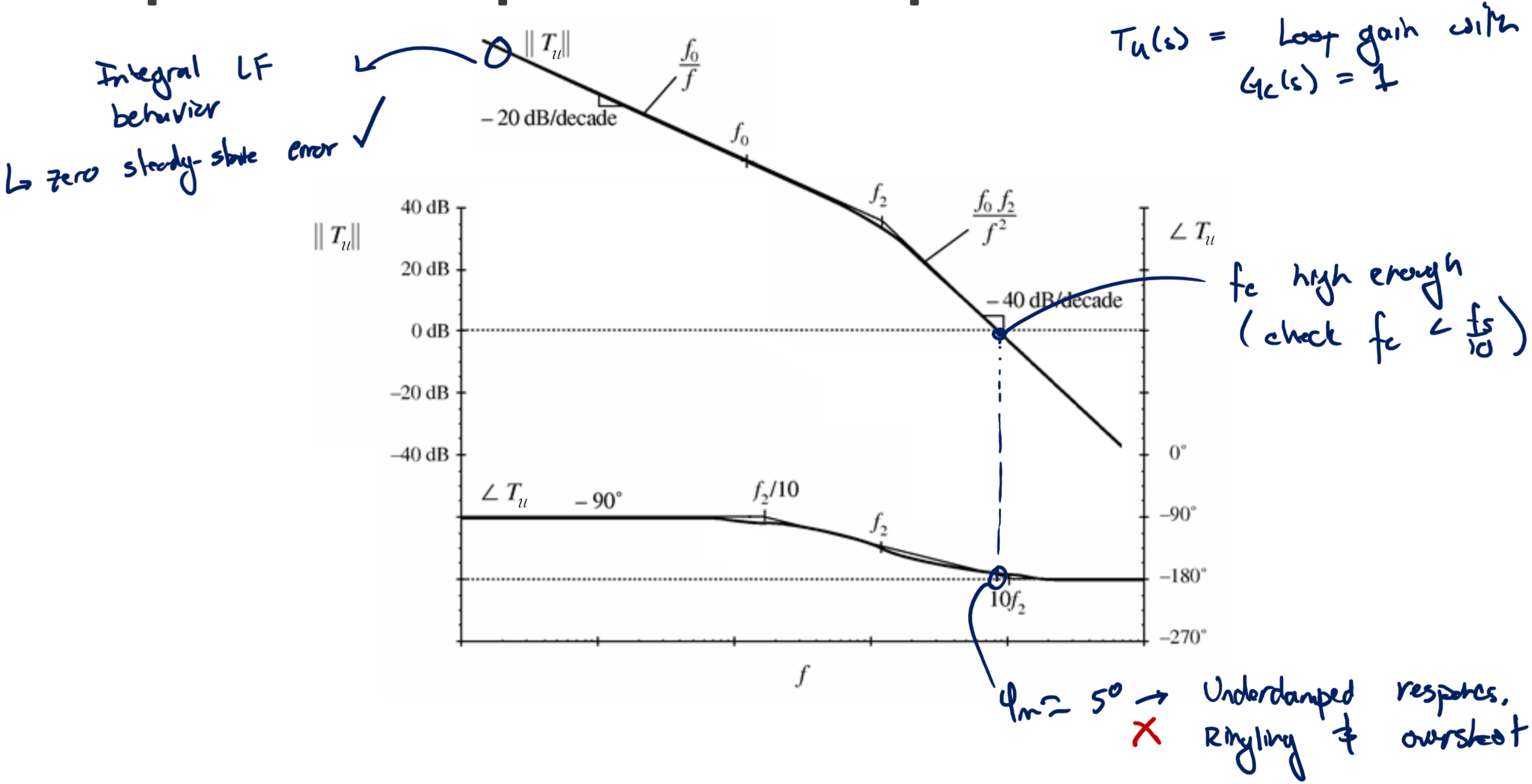
Design Approach

- Assume $G_c(s) = 1$, and plot the resulting uncompensated loop gain $T_u(s)$
- Examine uncompensated loop gain to determine the needs of the compensator
  - Is low-frequency loop gain amplitude $||T(0)||$ large enough to result in **low steady-state error**?
    - ↳ $||T(0)|| \to \infty$ zero steady-state error
  - Is φ_m sufficient for stability and requirements **on ringing/overshoot**?
  - Is f_c high enough for a sufficiently **fast response**?
- Construct compensator to address shortcomings of $T_u(s)$
  - Use "toolbox" of compensators

Additional Considerations:

- $f_c \lesssim f_s/10$ → Due to effects neglected in average model
- $||T(f < f_c)||$ = "large" } Based on phase margin test limitations
- $||T(f > f_c)||$ = "small" }

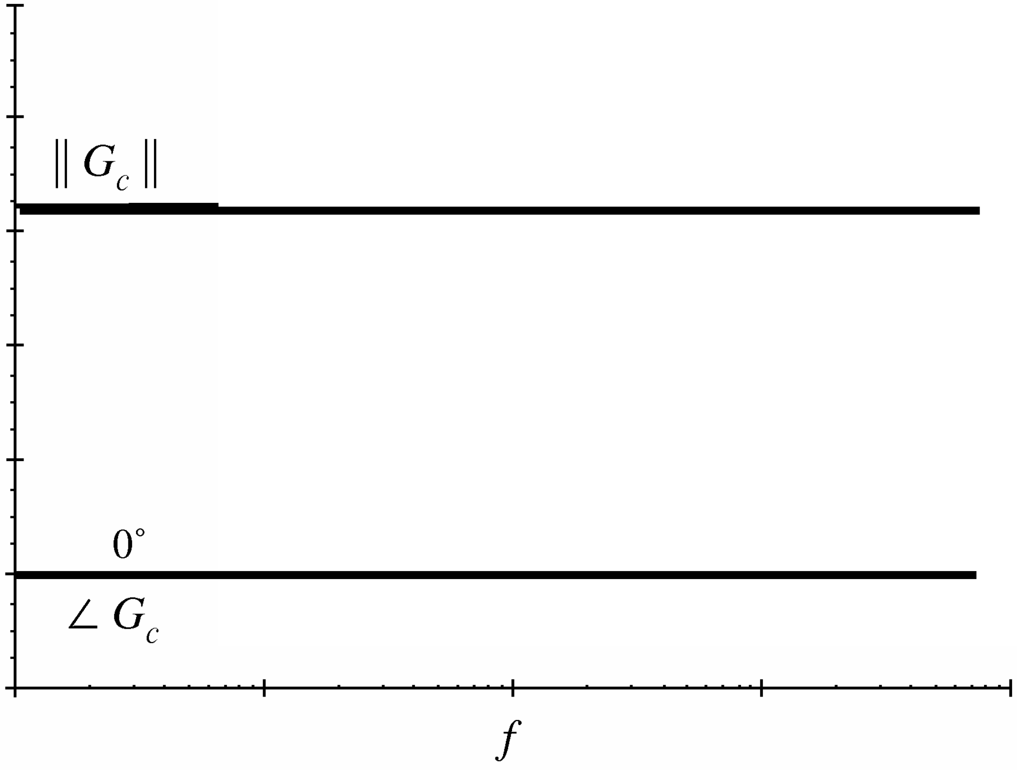
Example: Uncompensated Loop Gain



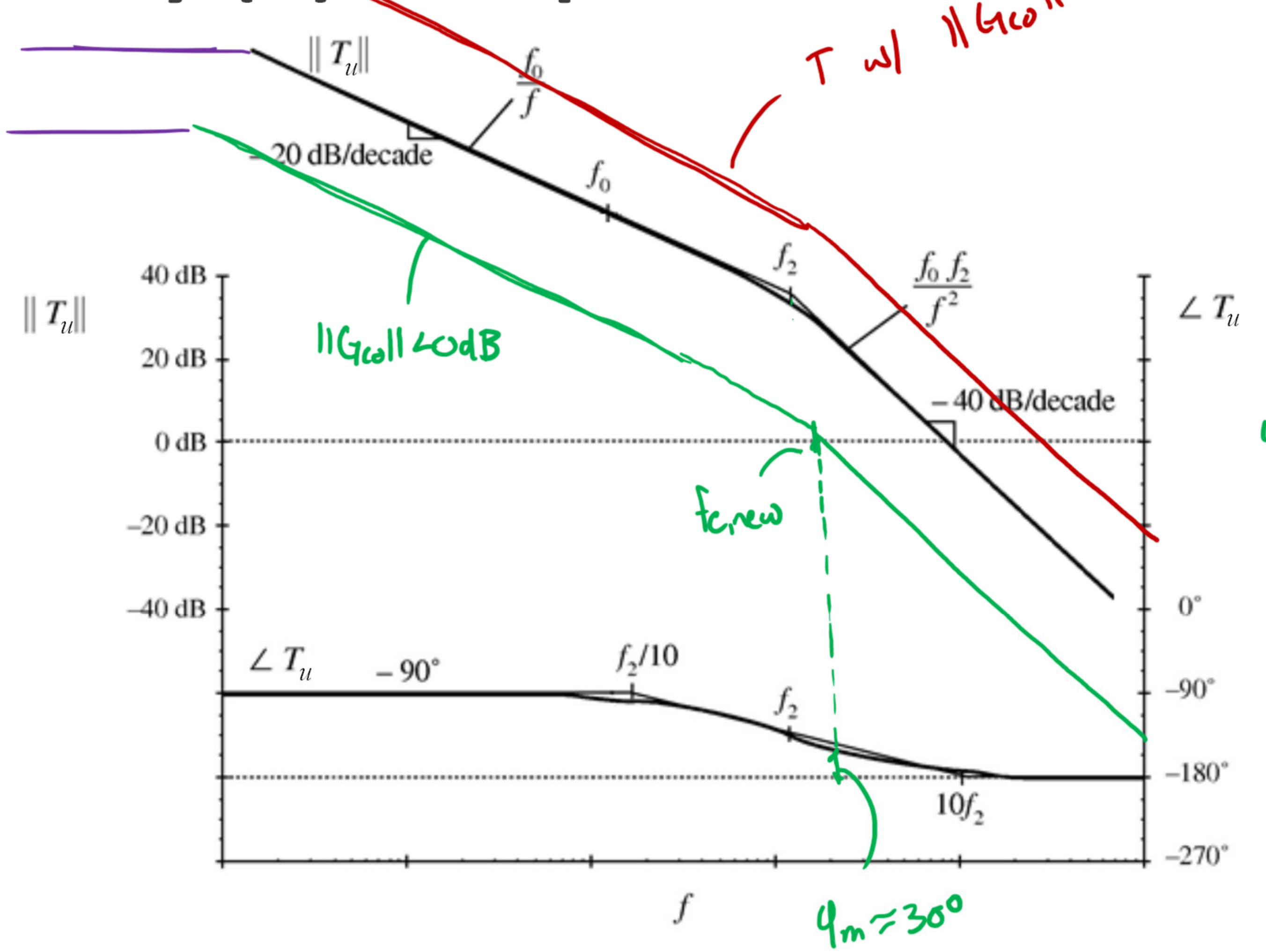
Proportional (P) Compensator

Real scalar ↓

$$G_c(s) = G_{c0}$$



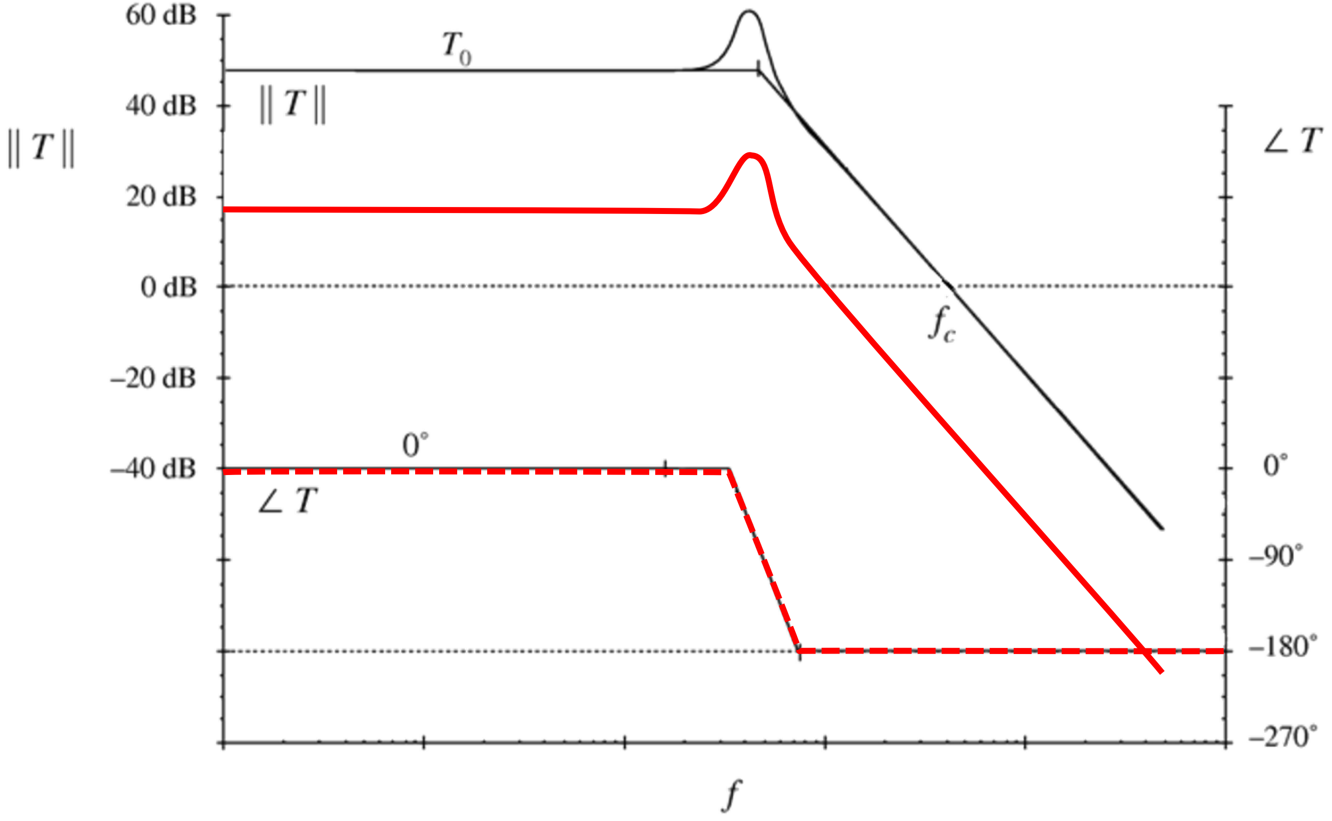
Stabilization by (P) Compensator



w/ (P) compensator
- shifting curve up & down

for systems w/ integral behavior
can ↑ φm at expense of fc

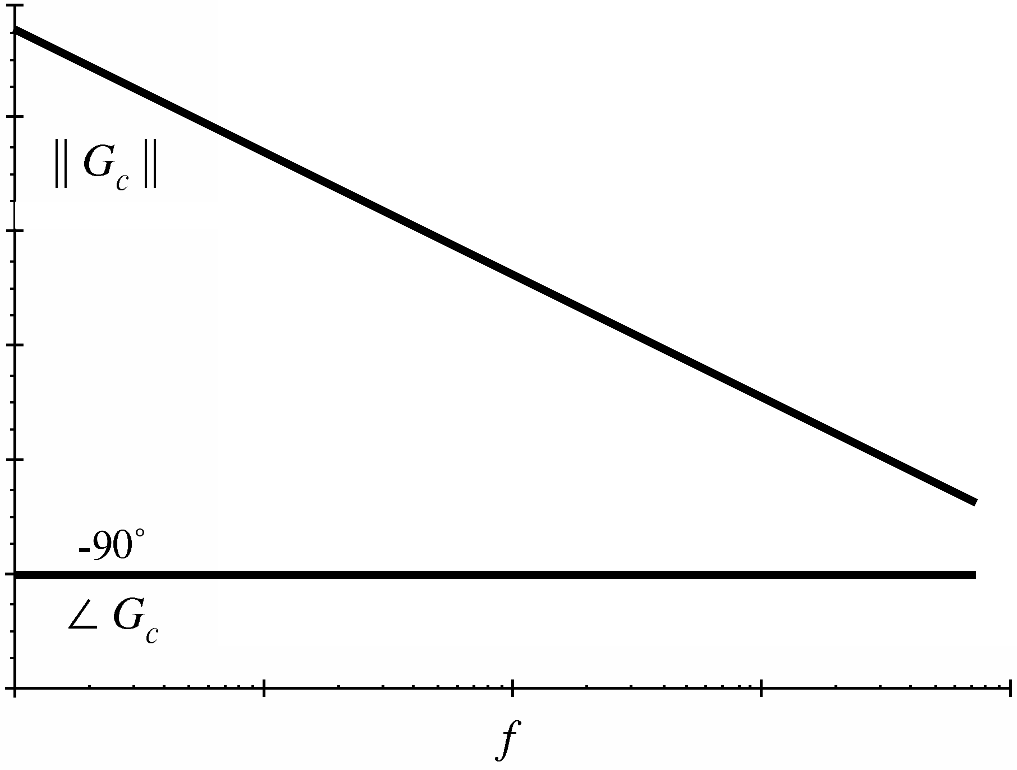
Another Example



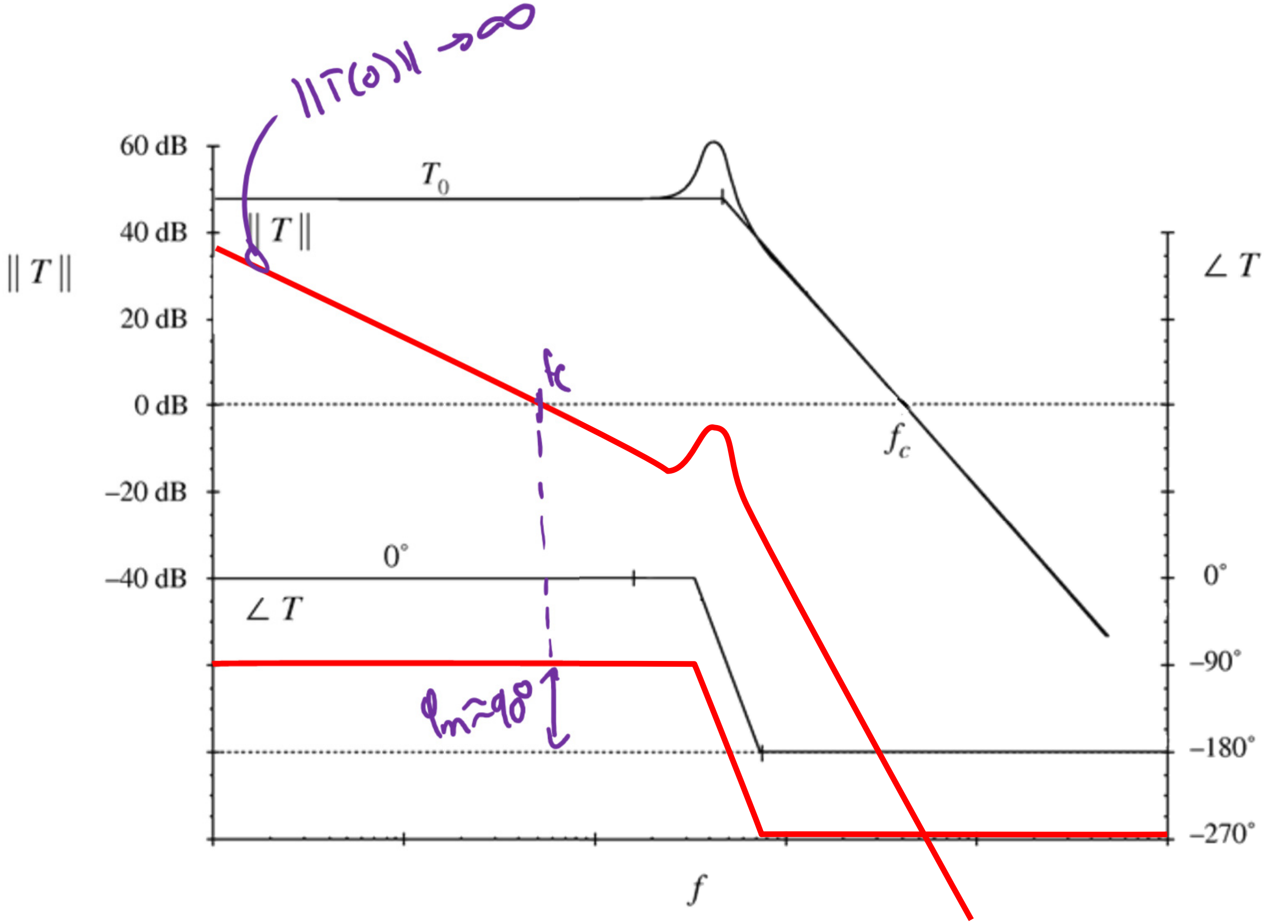
Proportional compensator can't fix φ_m here

Integral (I) Compensator

$$G_c(s) = \frac{K}{s}$$



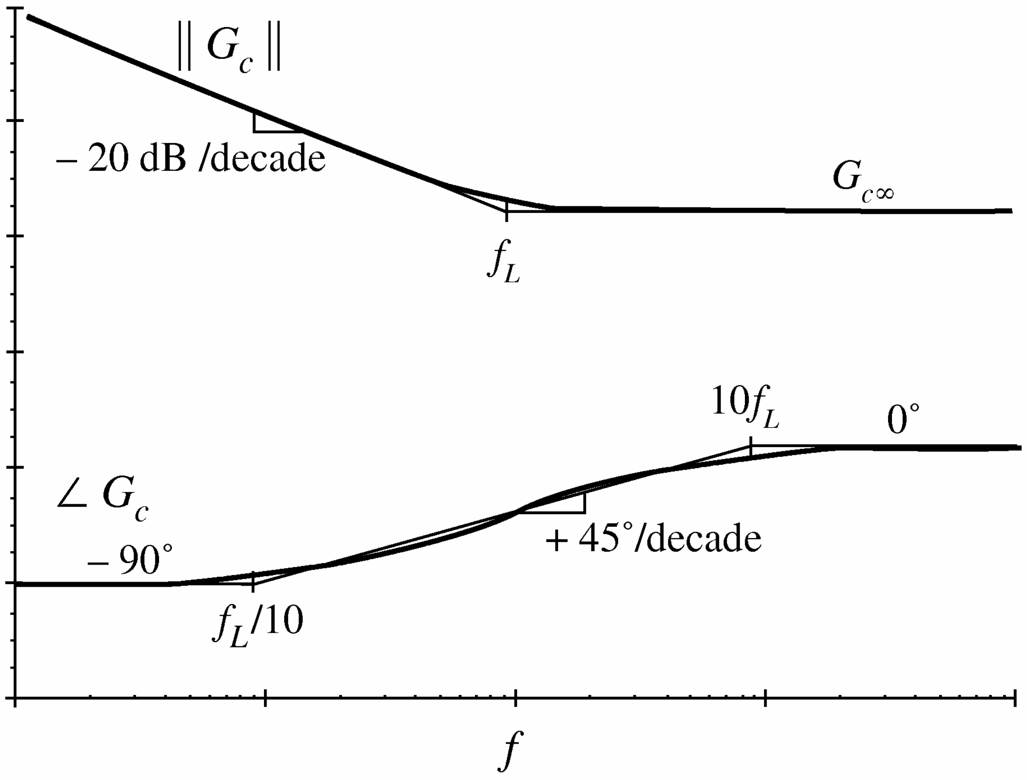
Stabilization by (I) Compensator



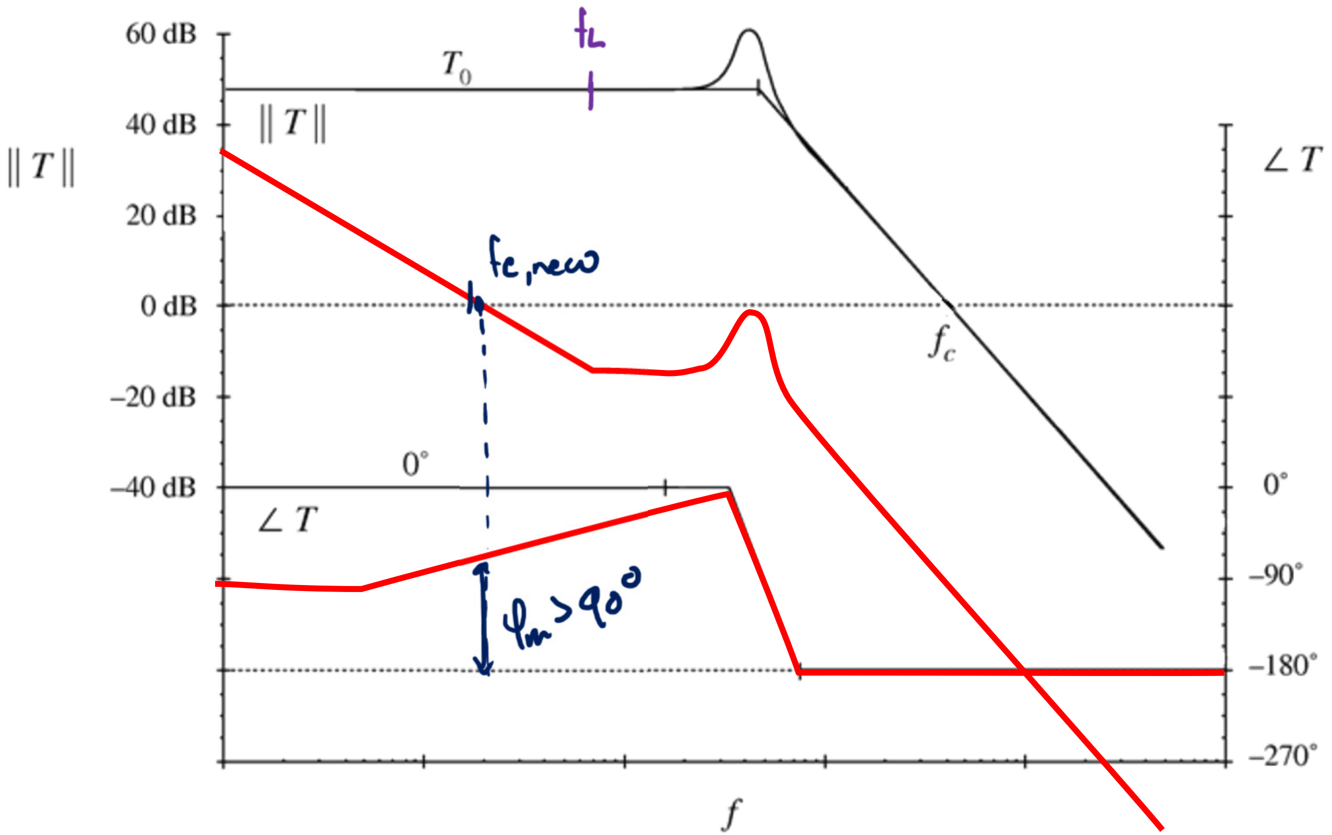
Lag (PI) Compensator

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Improves low-frequency loop gain and regulation



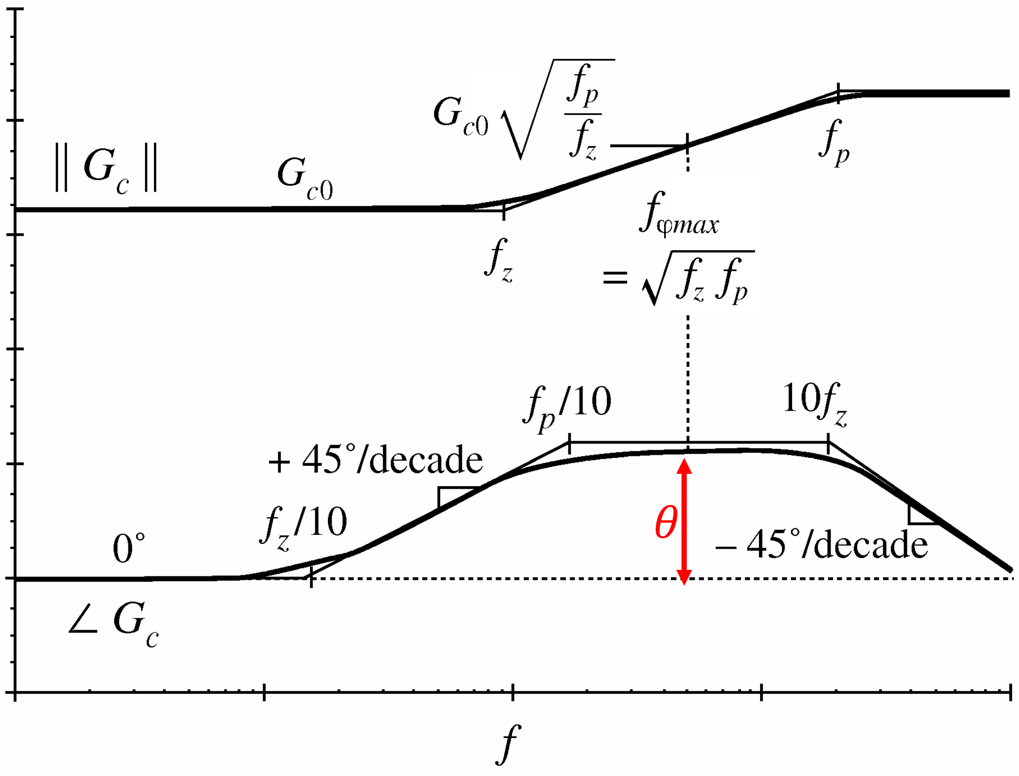
Stabilization by (PI) Compensator



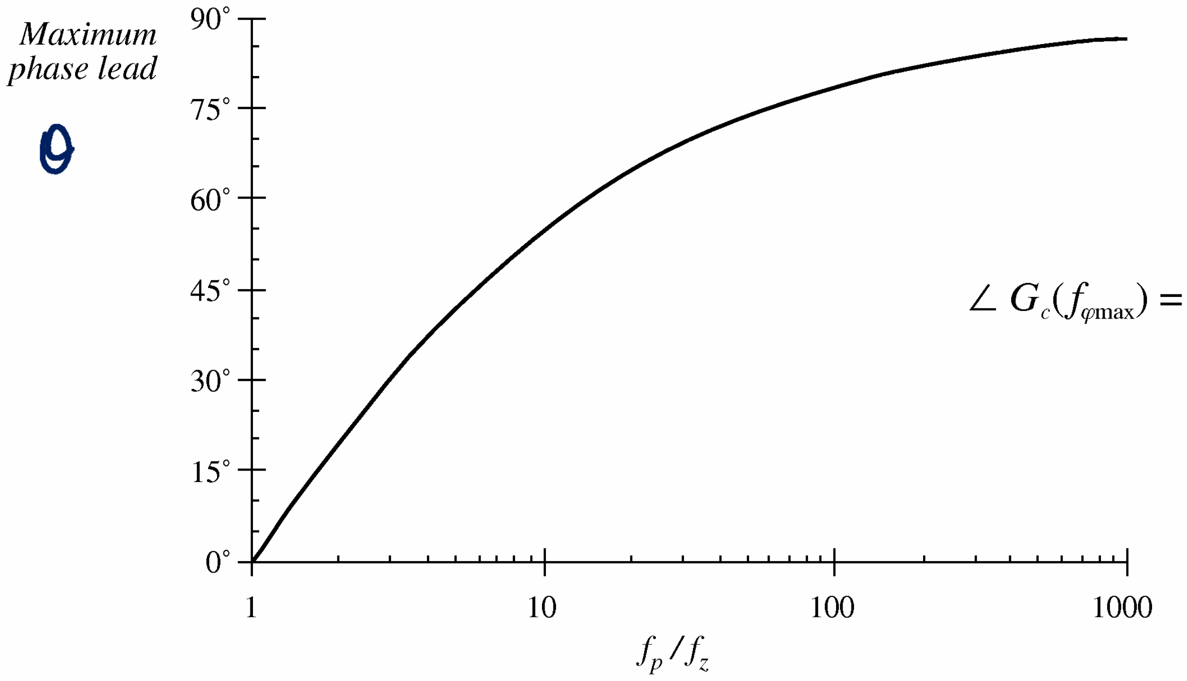
Lead (PD) Compensator

$$G_c(s) = G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

Improves phase margin



Maximum Phase Lead

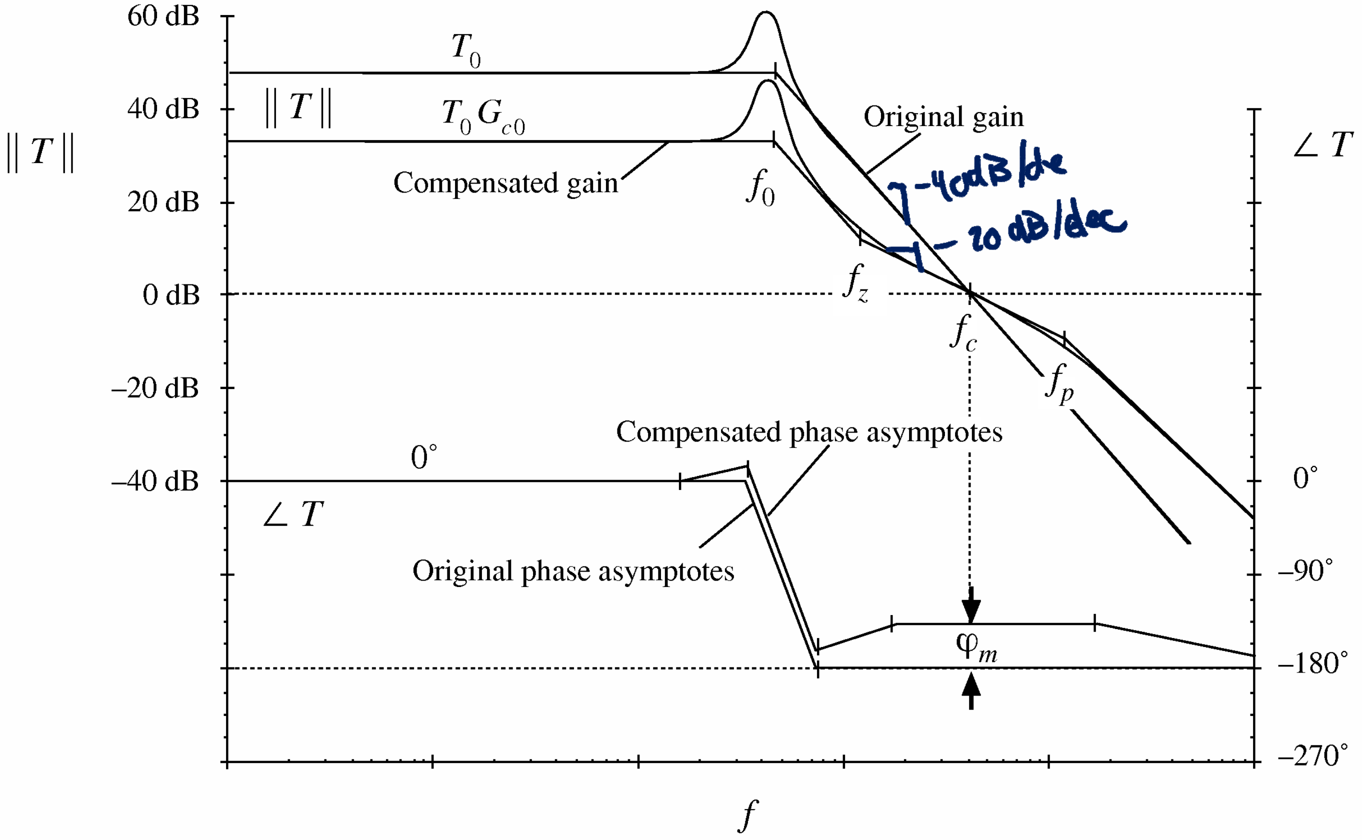


$$f_{\varphi\max} = \sqrt{f_z f_p}$$

$$\angle G_c(f_{\varphi\max}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2}\right)$$

$$\frac{f_p}{f_z} = \frac{1 + \sin(\theta)}{1 - \sin(\theta)}$$

Example Lead Compensator Design



$f_{\varphi max} = f_c$

$$f_z = f_{\varphi \max} \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}$$

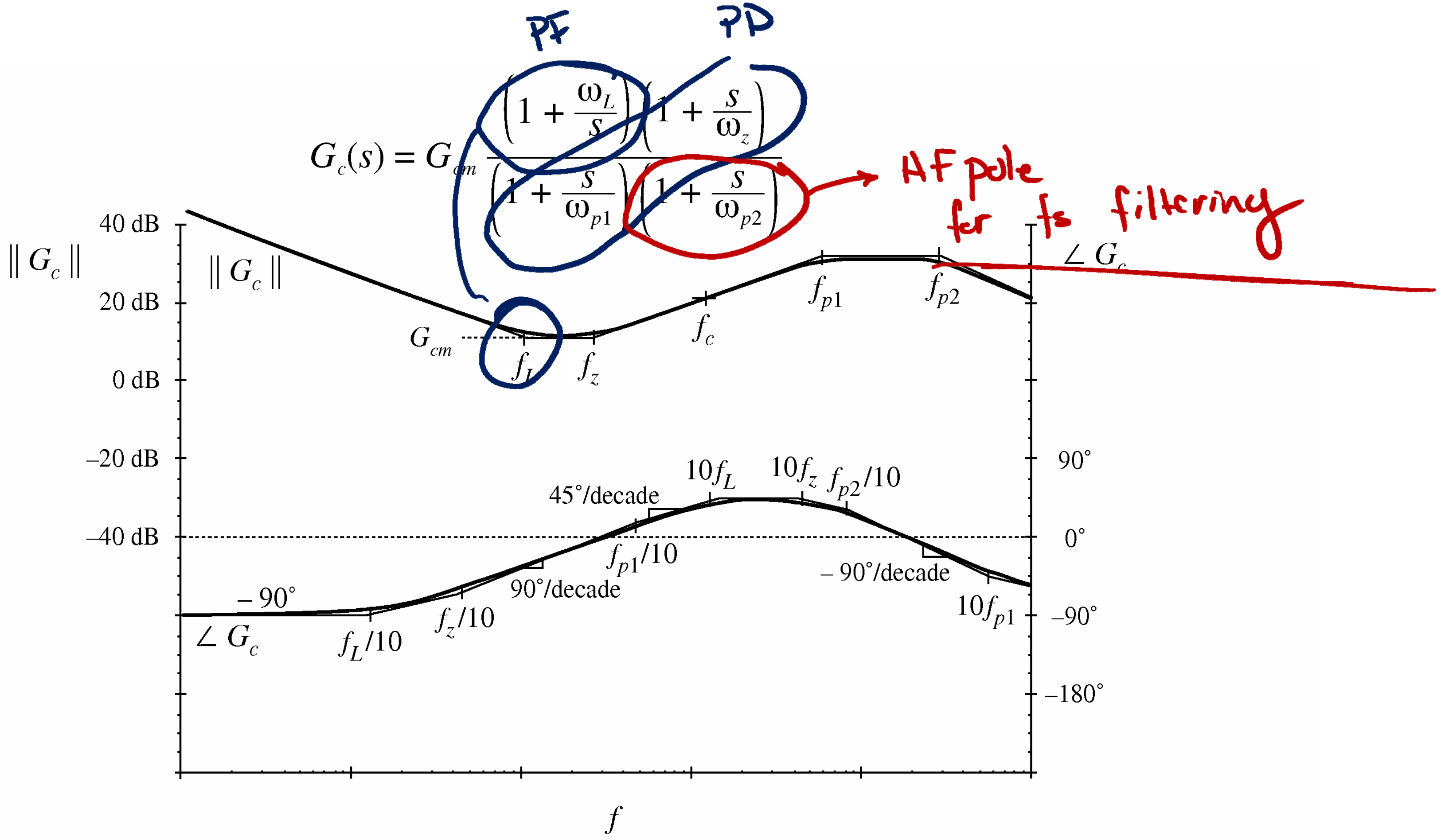
$$f_p = f_{\varphi \max} \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}}$$

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$

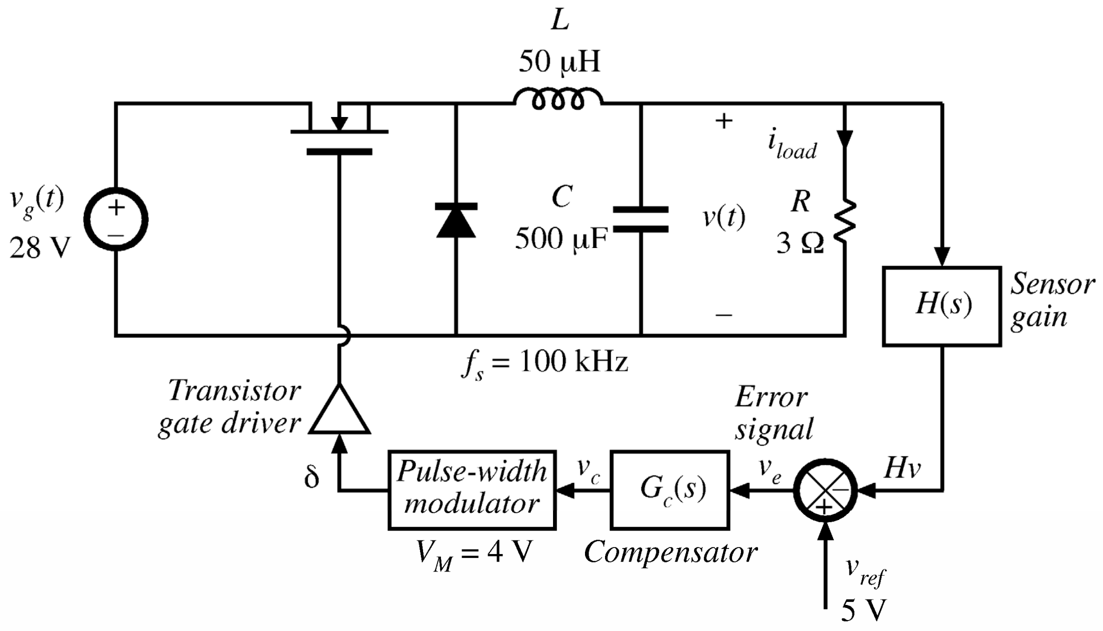
↳ to keep fc

Combined (PID) Compensator

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{\omega_L}{s}\right)\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

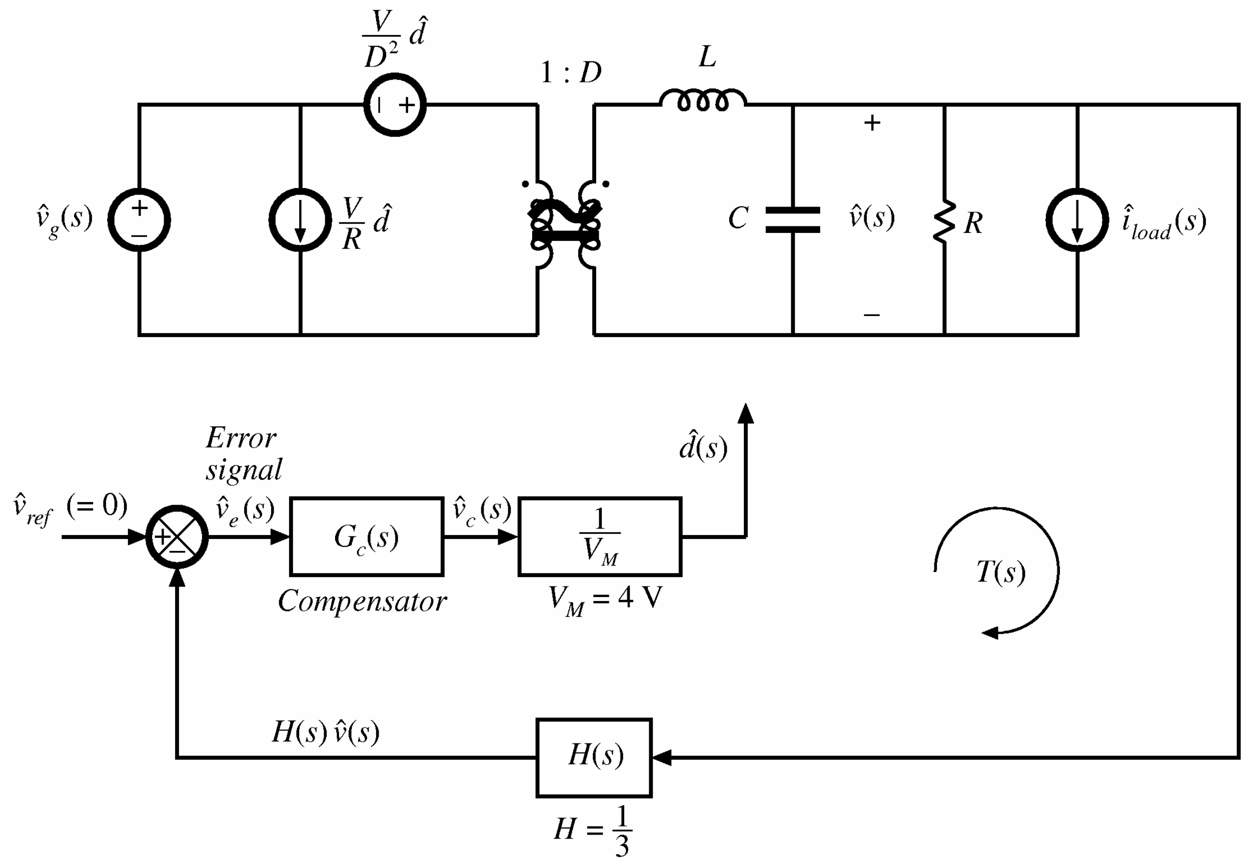


Example Design of Buck Compensator



Input voltage	$V_g = 28\text{V}$
Output	$V = 15\text{V},\ I_{load} = 5\text{A},\ R = 3\Omega$
Quiescent duty cycle	$D = 15/28 = 0.536$
Reference voltage	$V_{ref} = 5\text{V}$
Quiescent value of control voltage	$V_c = DV_M = 2.14\text{V}$
Gain $H(s)$	$H = V_{ref}/V = 5/15 = 1/3$

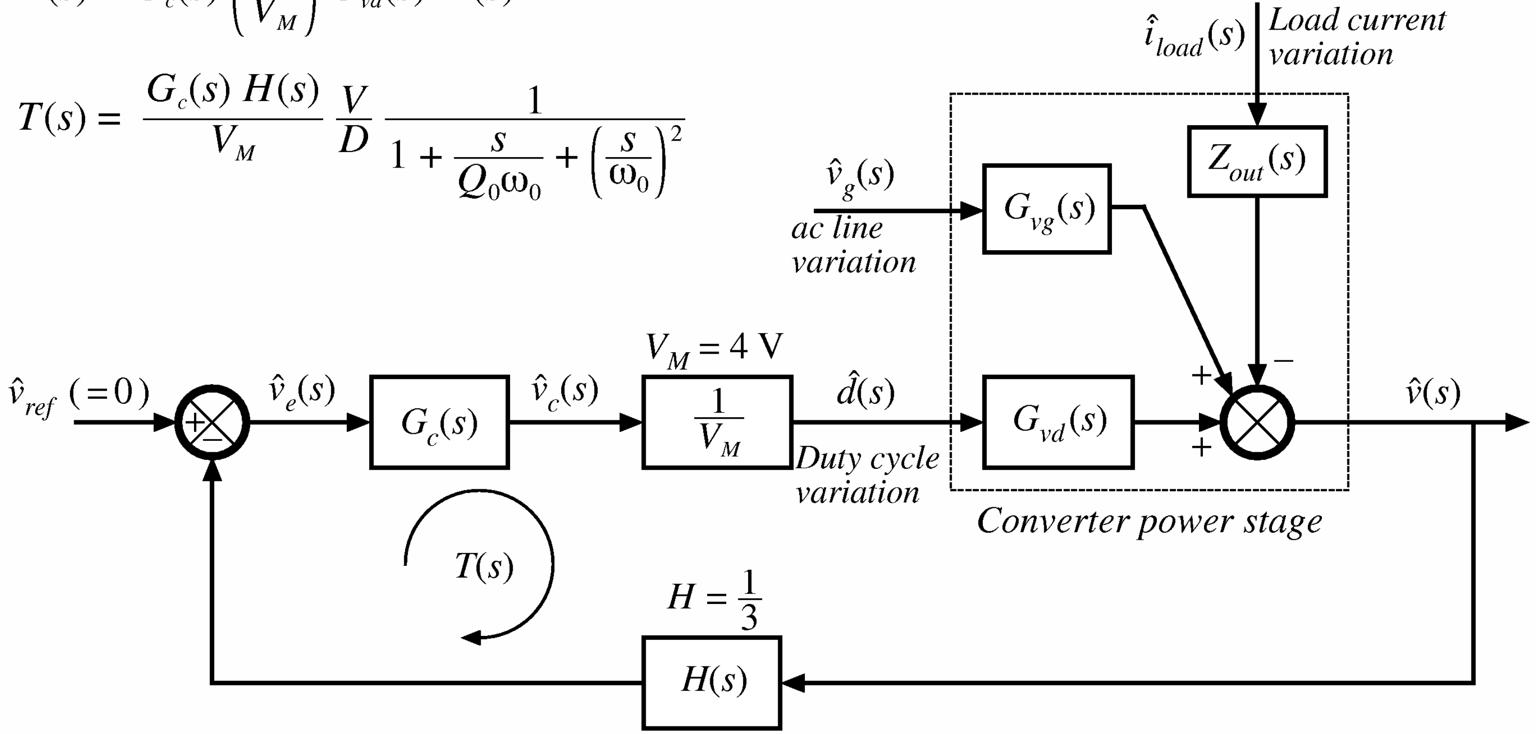
AC Power Stage Model



System Block Diagram

$$T(s) = G_c(s)\left(\frac{1}{V_M}\right) G_{vd}(s)\, H(s)$$

$$T(s) = \frac{G_c(s)\, H(s)}{V_M} \frac{V}{D} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

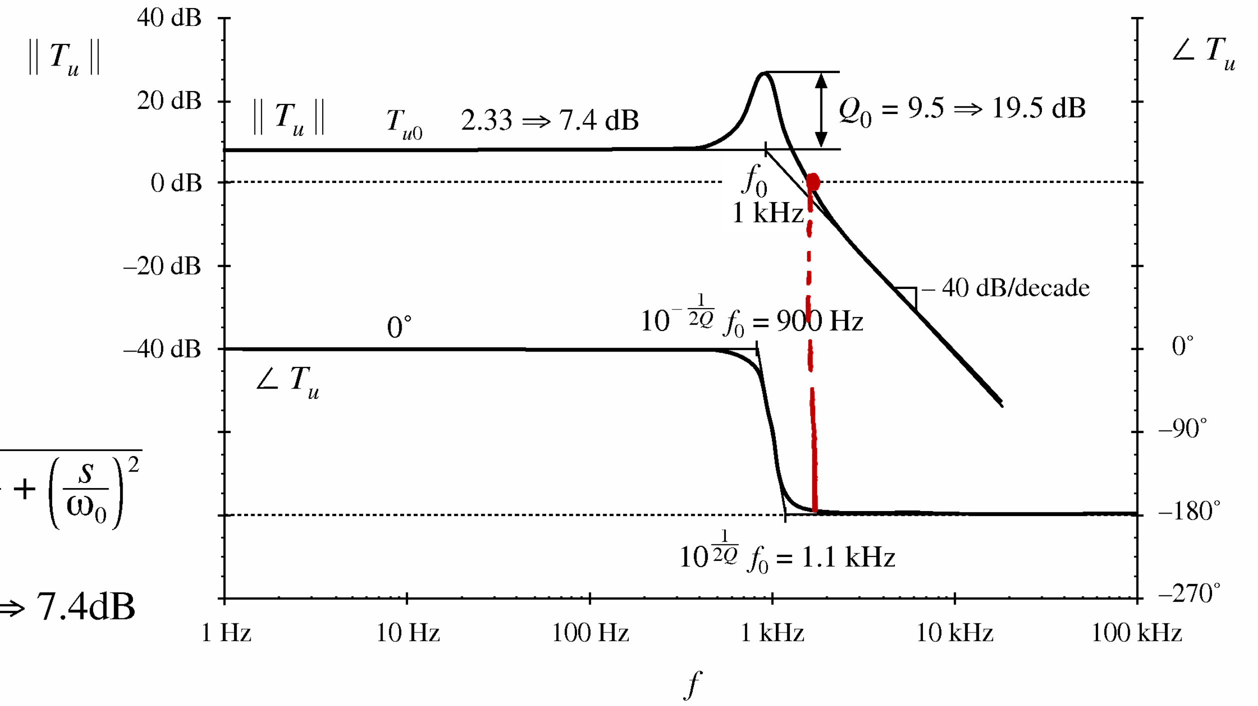


Plotting Uncompensated Loop Gain

With $G_c = 1$, the loop gain is

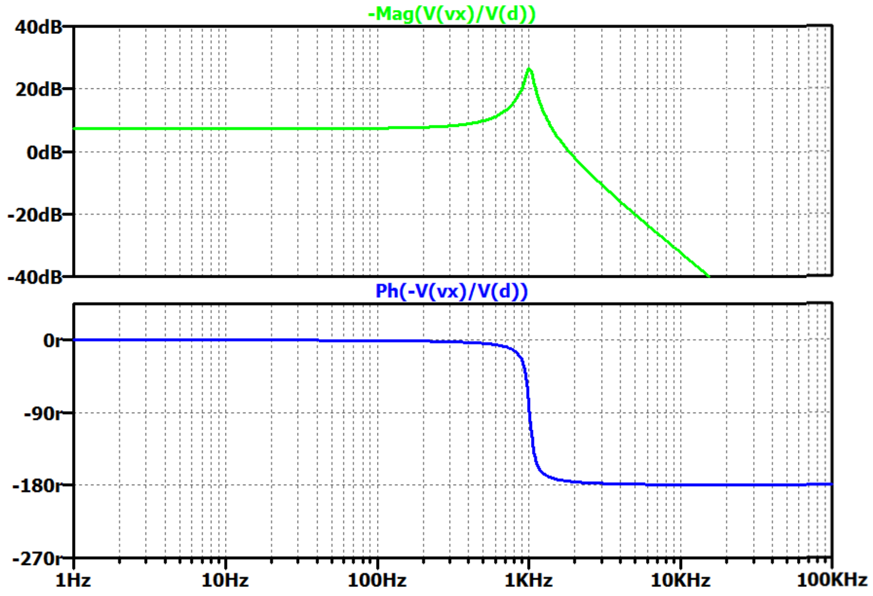
$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H\ V}{D\ V_M} = 2.33 \Rightarrow 7.4\text{dB}$$



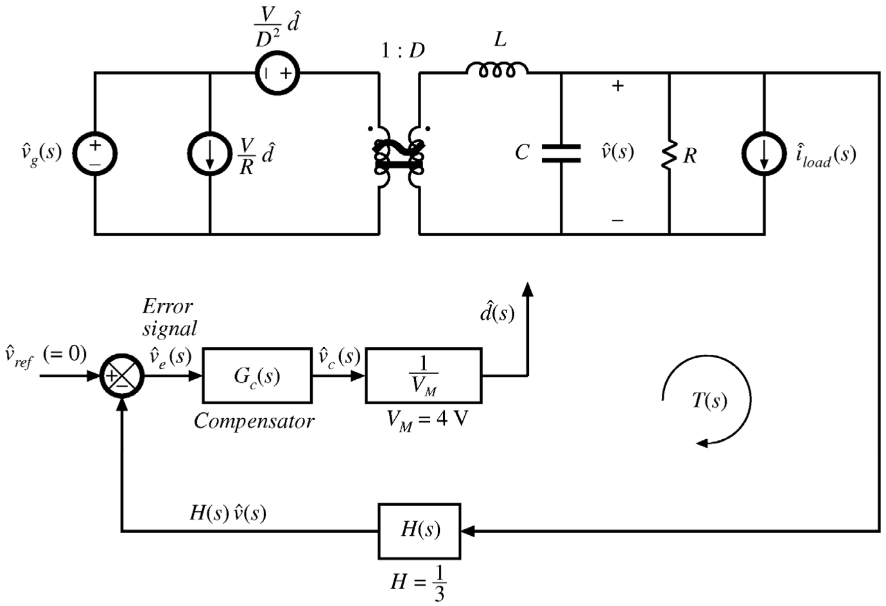
$f_c = 1.8$ kHz, $\varphi_m = 5^\circ$

LTSpice Simulation – AC, Uncompensated



.lib myParts.lib
.ac dec 1000 1 1Meg

.param Vg = 28 V = 15 R = 3 D = .536
.param Vref = 5 H = 1/3 Vm = 4
.param L = 50u C = 500u



Transient Simulation, Uncompensated