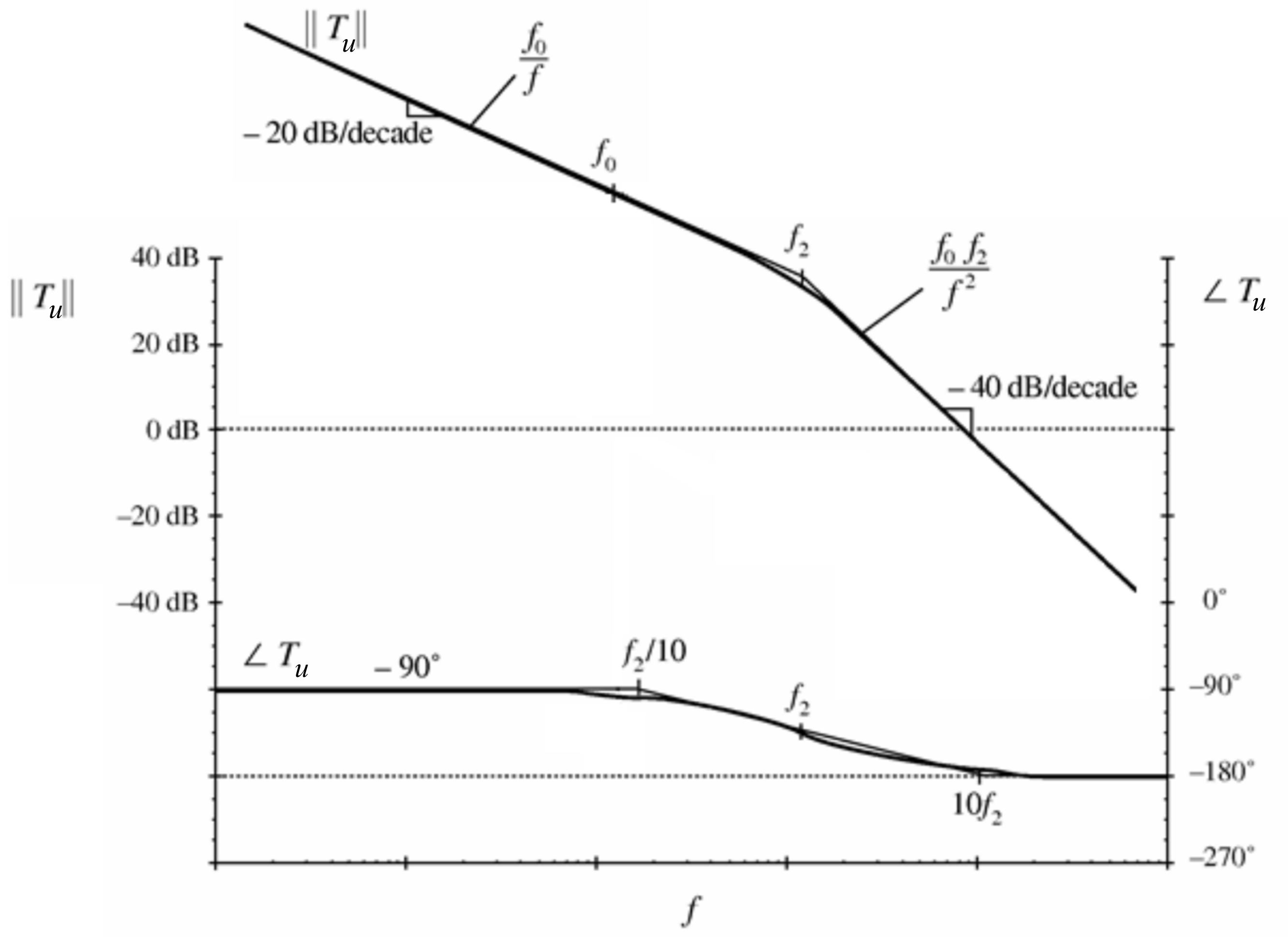
9.5 – Compensator Design

Design Approach

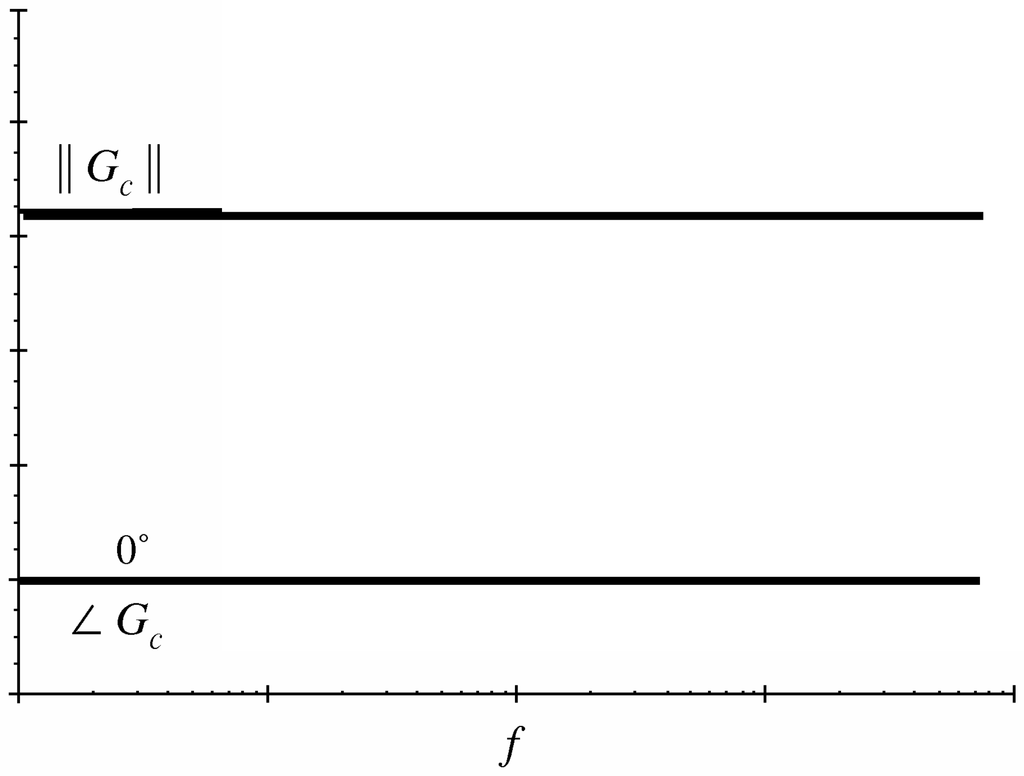
- Assume $G_c(s) = 1$, and plot the resulting uncompensated loop gain $T_u(s)$
- Examine uncompensated loop gain to determine the needs of the compensator
  - Is low-frequency loop gain amplitude $\|T(0)\|$ large enough to result in **low steady-state error**?
  - Is φ_m sufficient for stability and requirements **on ringing/overshoot**?
  - Is f_c high enough for a sufficiently **fast response**?
- Construct compensator to address shortcomings of $T_u(s)$
  - Use “toolbox” of compensators

Example: Uncompensated Loop Gain

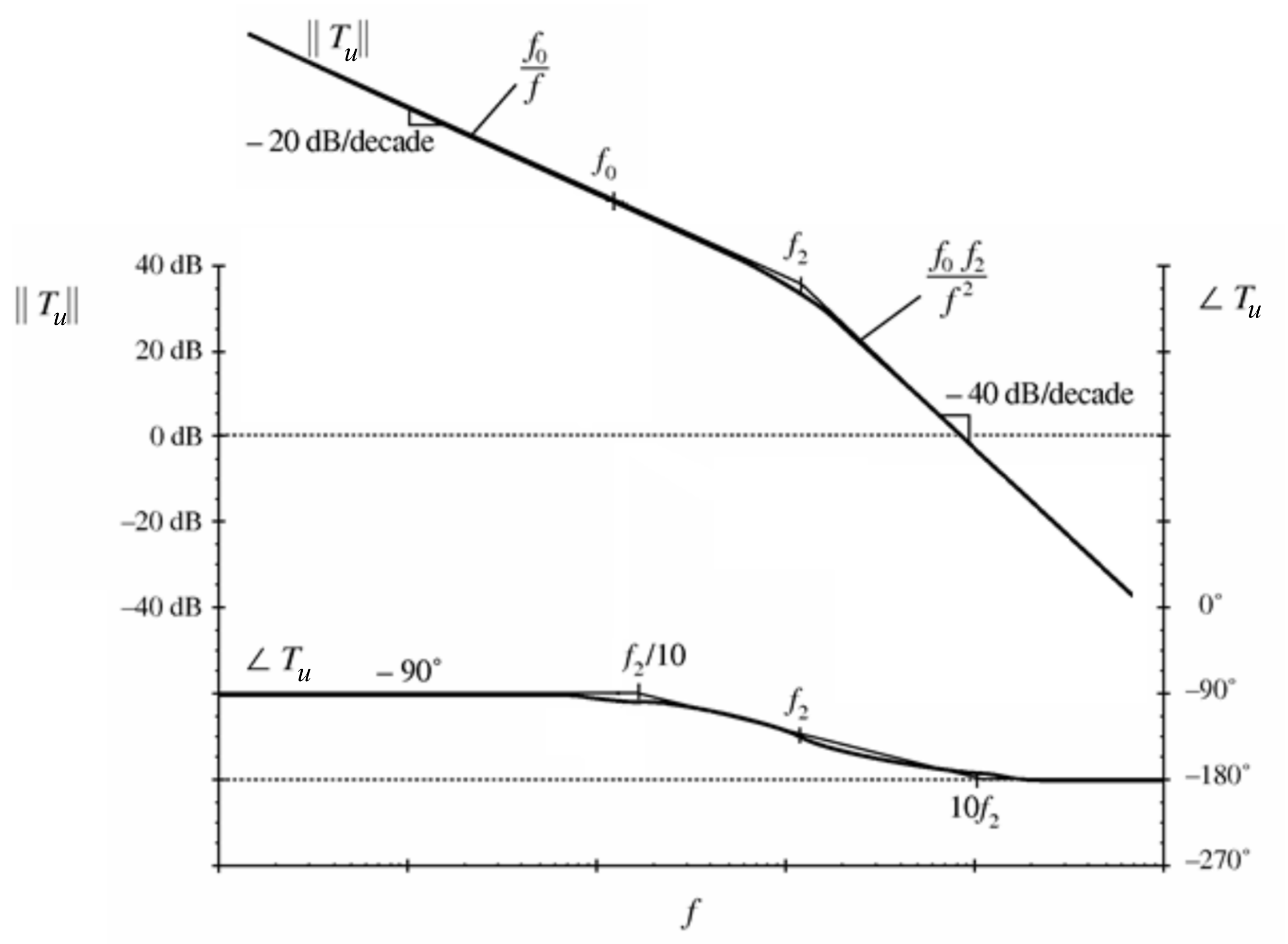


Proportional (P) Compensator

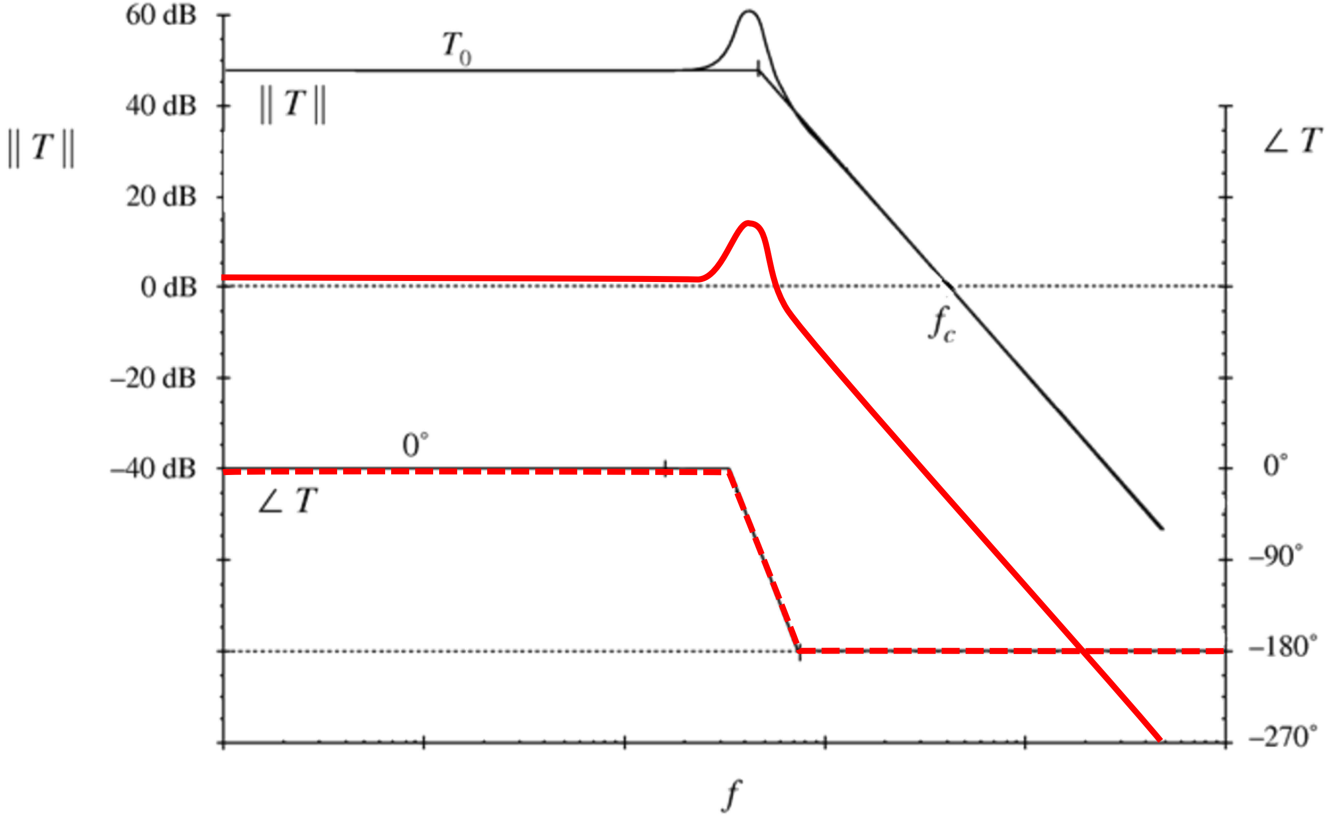
$$G_c(s) = G_{c0}$$



Stabilization by (P) Compensator

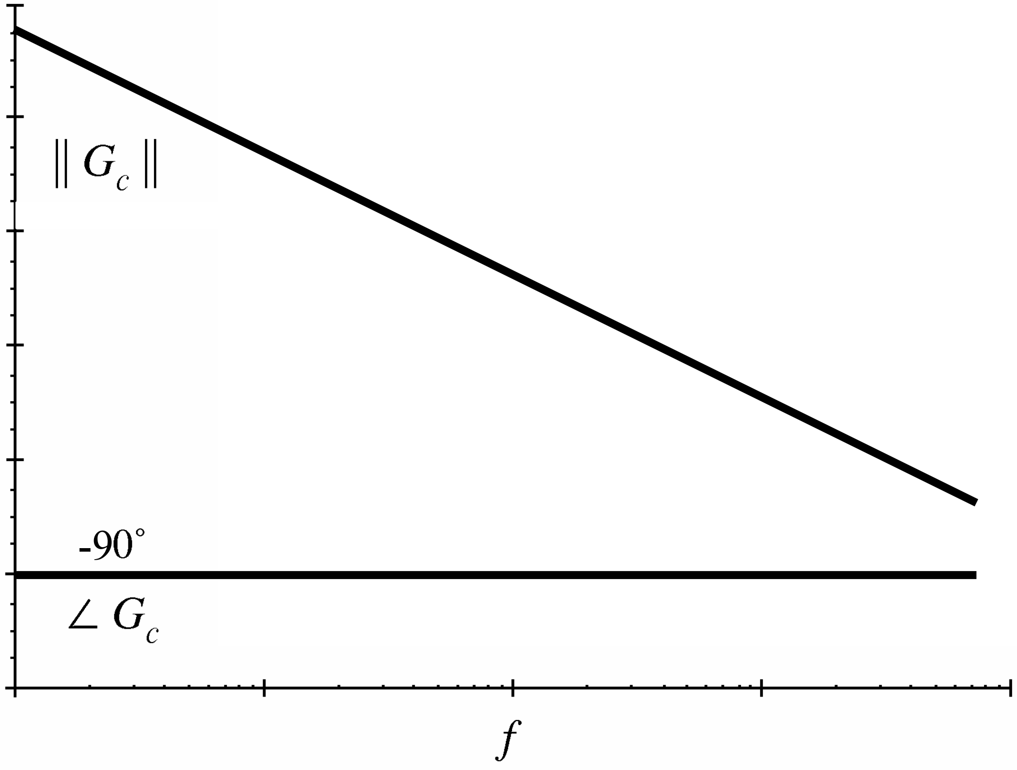


Another Example

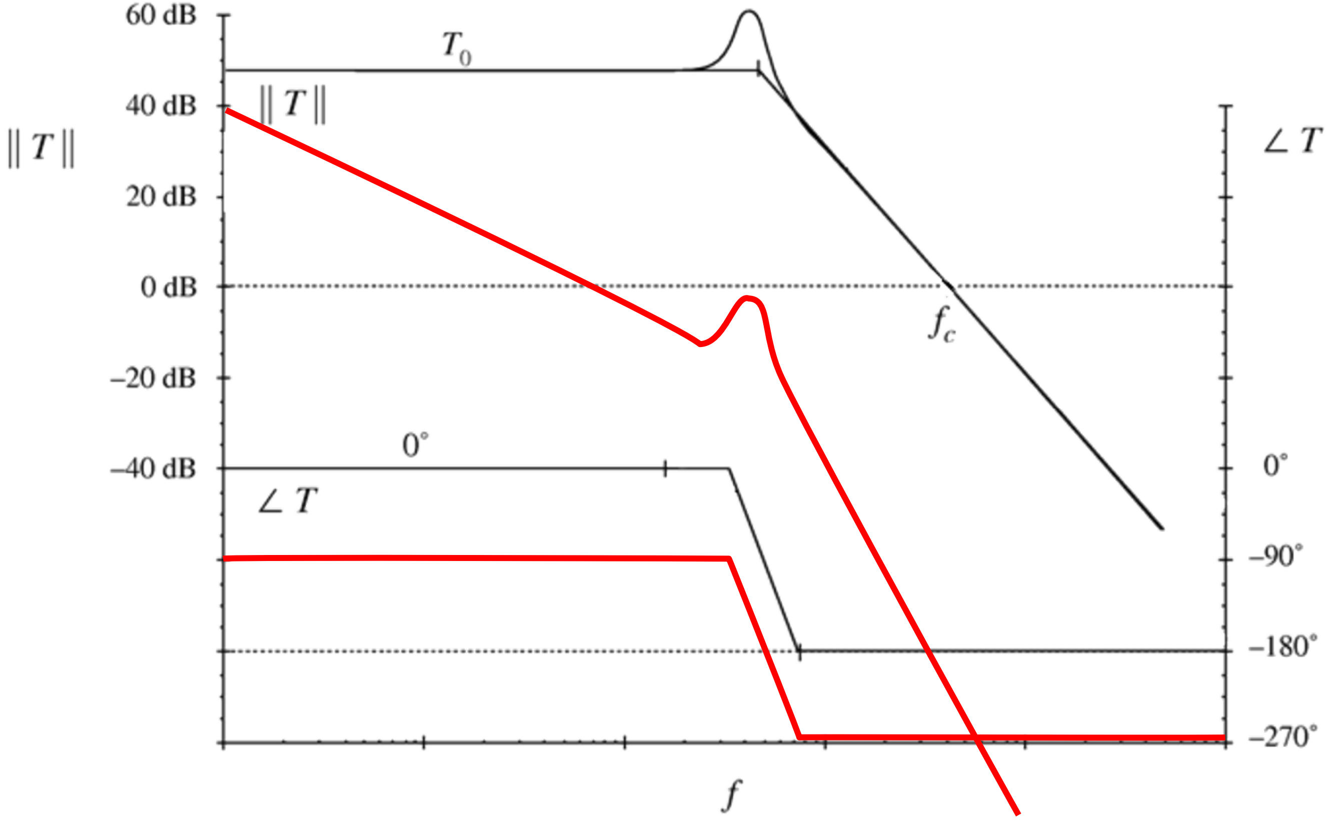


Integral (I) Compensator

$$G_c(s) = \frac{K}{s}$$



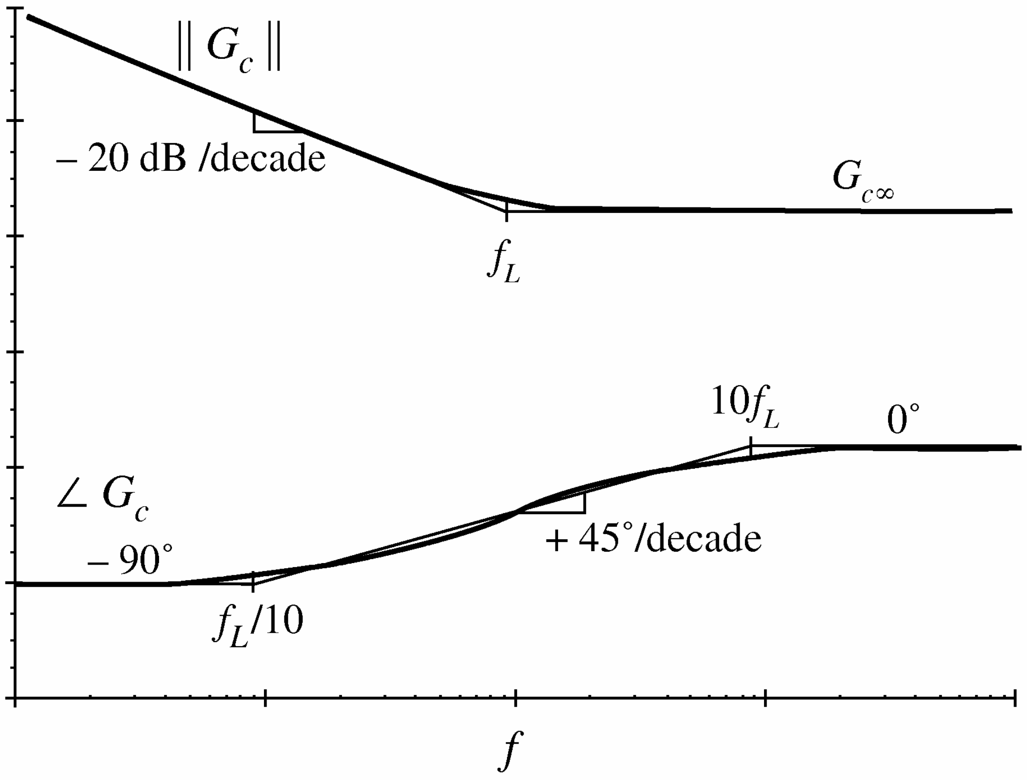
Stabilization by (I) Compensator



Lag (PI) Compensator

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

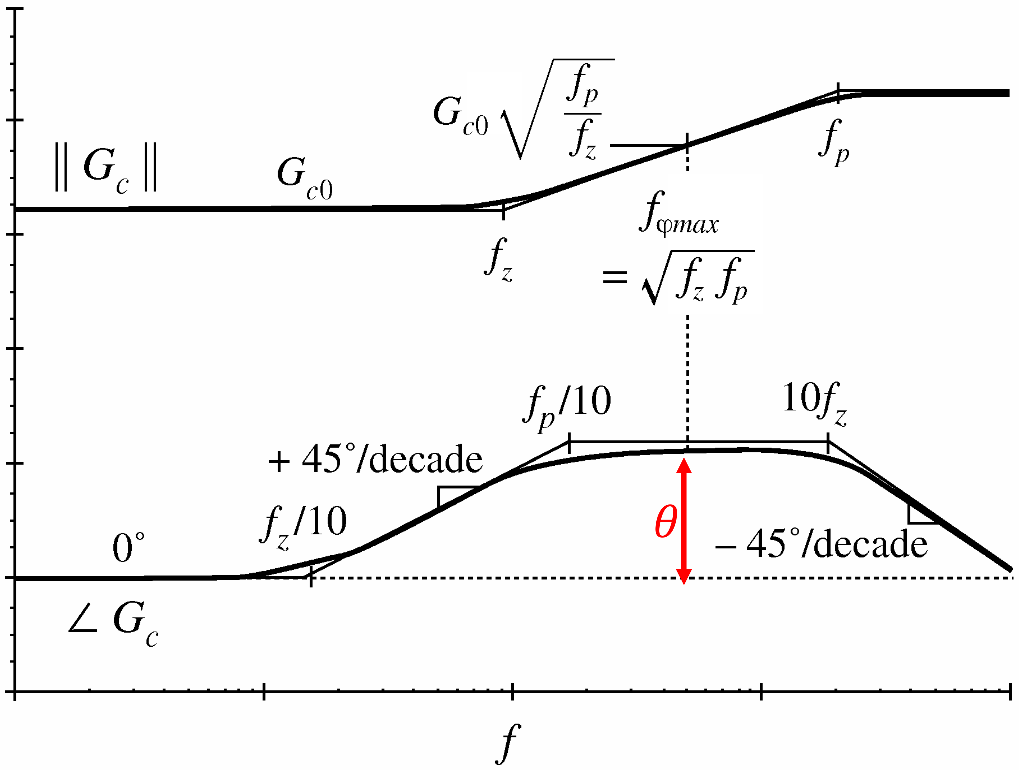
Improves low-frequency loop gain and regulation



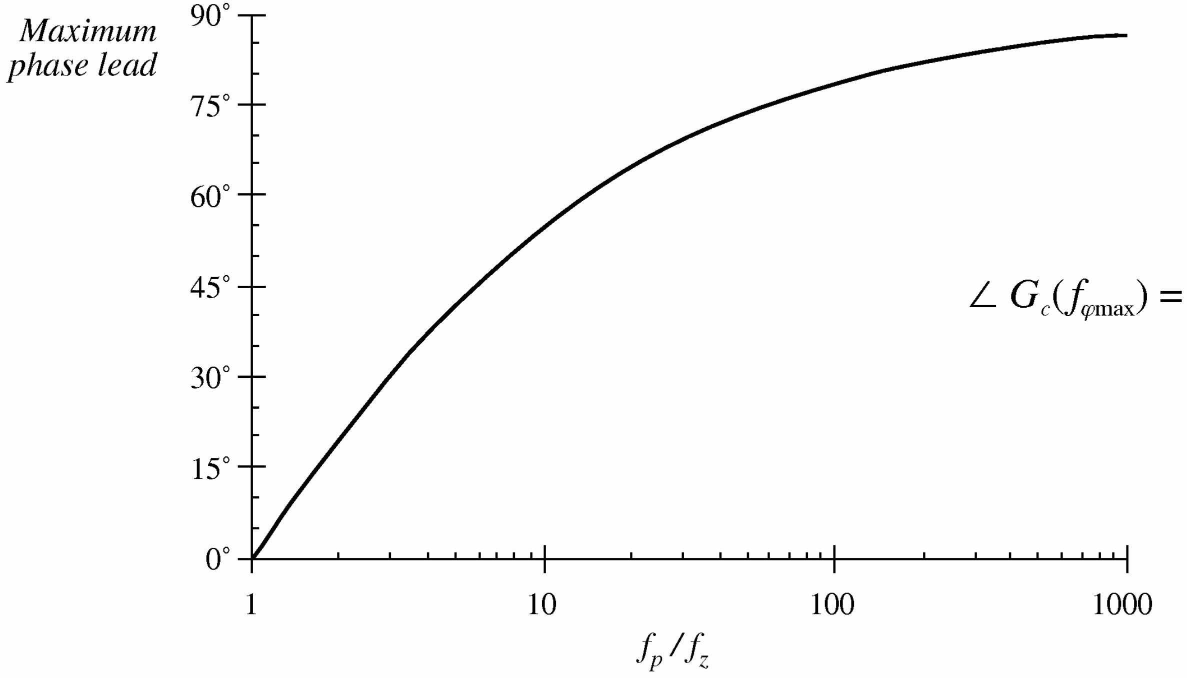
Lead (PD) Compensator

$$G_c(s) = G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

Improves phase margin



Maximum Phase Lead

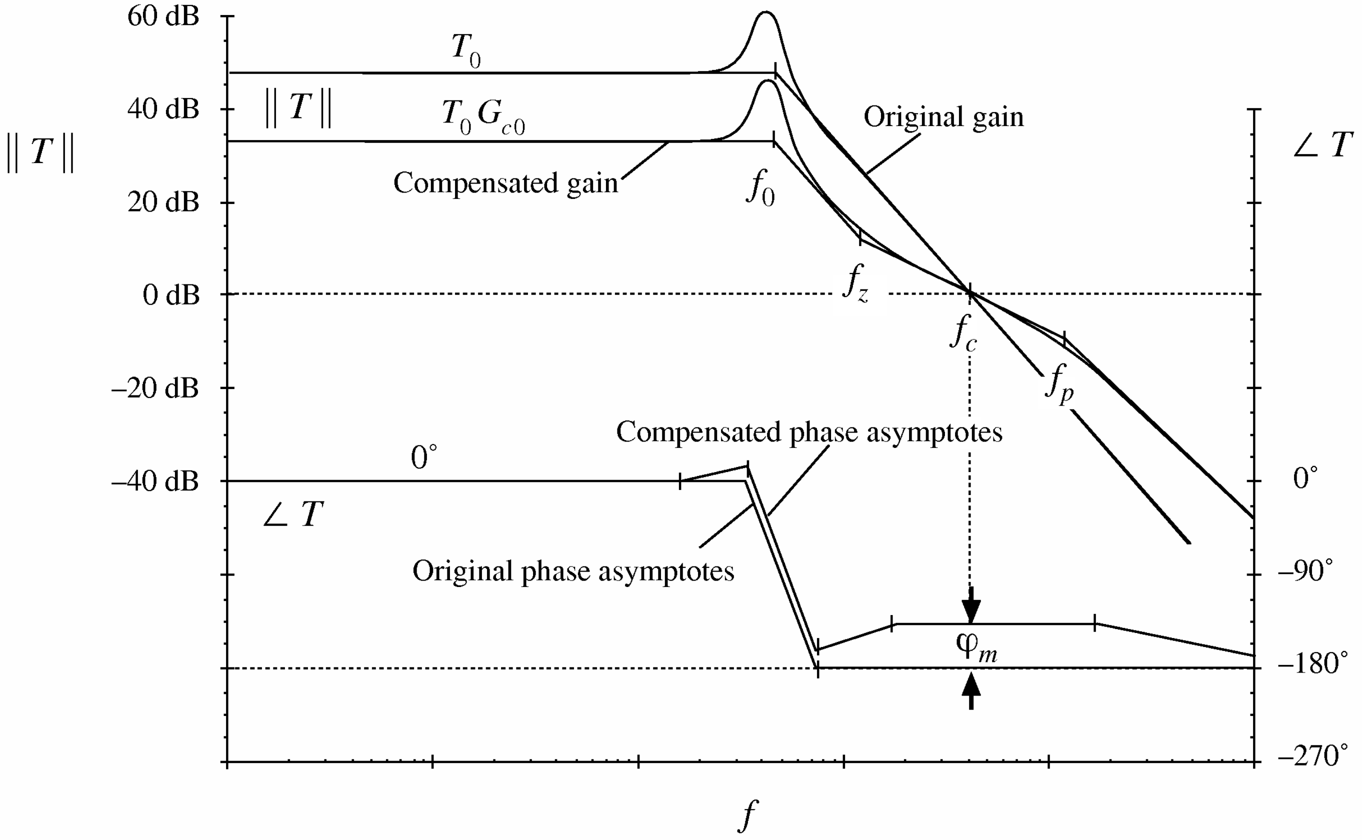


$$f_{\varphi max} = \sqrt{f_z f_p}$$

$$\angle G_c(f_{\varphi max}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2}\right)$$

$$\frac{f_p}{f_z} = \frac{1 + \sin(\theta)}{1 - \sin(\theta)}$$

Example Lead Compensator Design



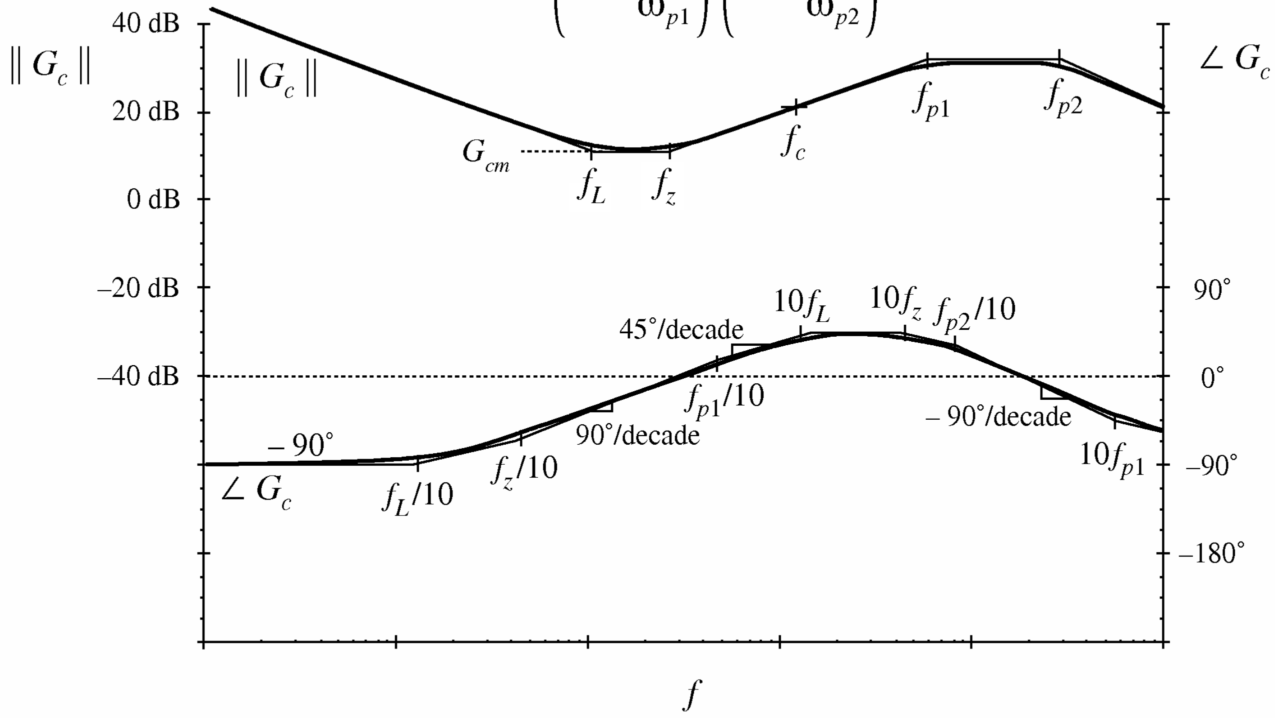
$$f_z = f_{\varphi \max} \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}$$

$$f_p = f_{\varphi \max} \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}}$$

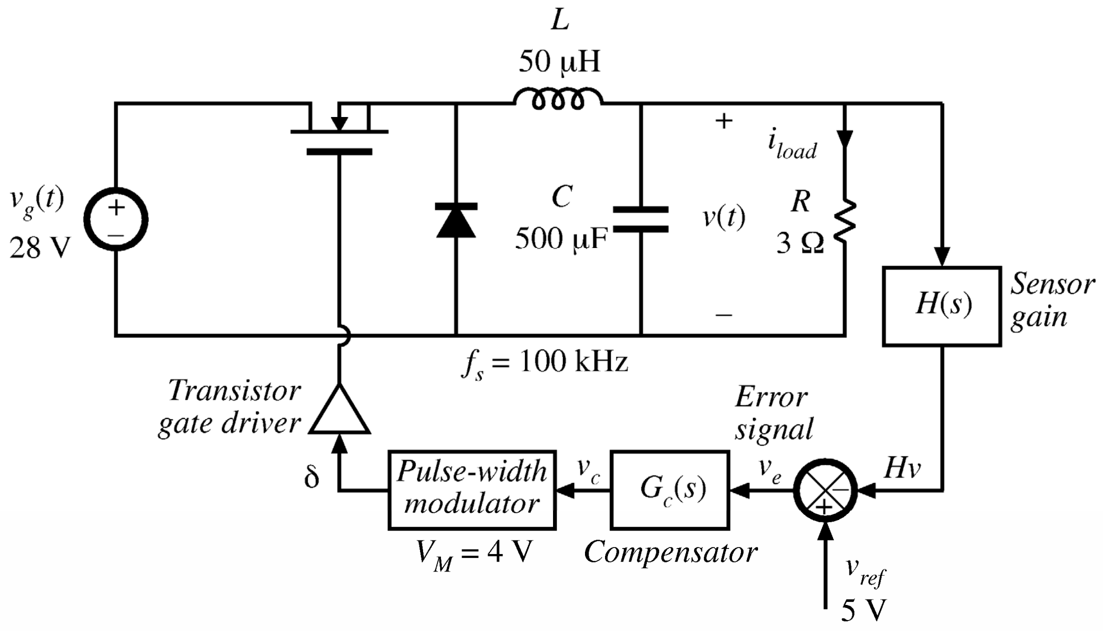
$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$

Combined (PID) Compensator

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{\omega_L}{s}\right)\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$



Example Design of Buck Compensator



Input voltage	$V_g = 28\text{V}$
Output	$V = 15\text{V},\ I_{load} = 5\text{A},\ R = 3\Omega$
Quiescent duty cycle	$D = 15/28 = 0.536$
Reference voltage	$V_{ref} = 5\text{V}$
Quiescent value of control voltage	$V_c = DV_M = 2.14\text{V}$
Gain $H(s)$	$H = V_{ref}/V = 5/15 = 1/3$