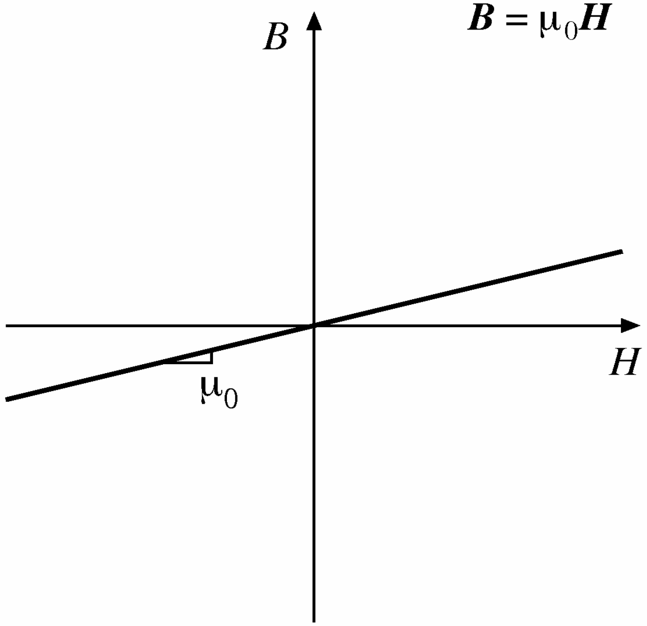
Core Material Characteristics

in a core $\mu >> \mu_0$

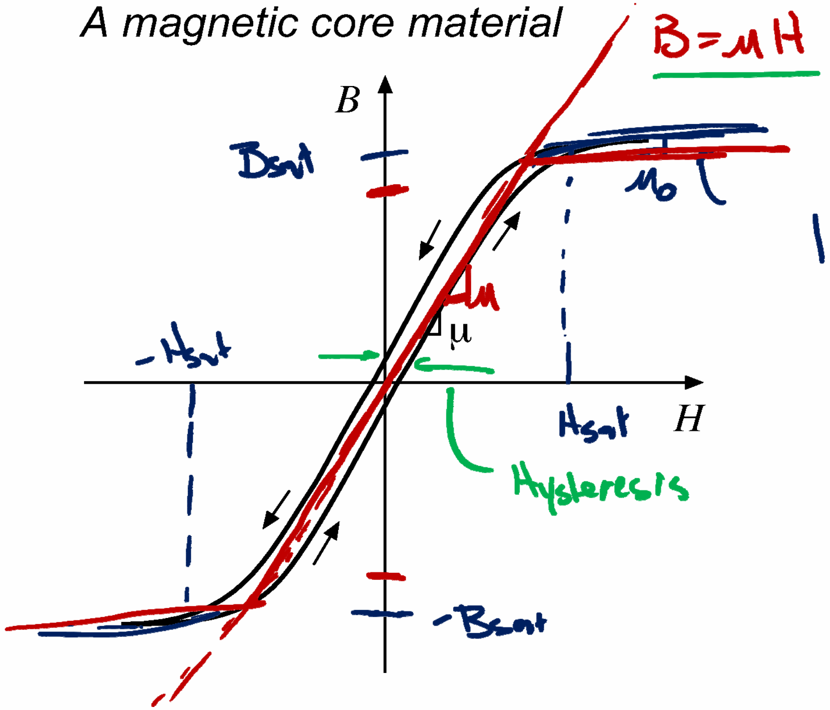
Free space



μ_0 = permeability of free space

= $4\pi \cdot 10^{-7}$ Henries per meter

A magnetic core material



|B| ≥ Bsat
material saturates

Highly nonlinear, with hysteresis and saturation

Units

Table 12.1. Units for magnetic quantities

quantity	*MKS*	*unrationalized cgs*	*conversions*
core material equation	$B = \mu_0 \mu_r H$	$B = \mu_r H$	
B	Tesla	Gauss	$1\mathrm{T} = 10^4\mathrm{G}$
H	Ampere / meter	Oersted	$1\mathrm{A/m} = 4\pi \cdot 10^{-3}\ \mathrm{Oe}$
Φ	Weber	Maxwell	$1\mathrm{Wb} = 10^8\ \mathrm{Mx}$ $1\mathrm{T} = 1\mathrm{Wb} / \mathrm{m}^2$

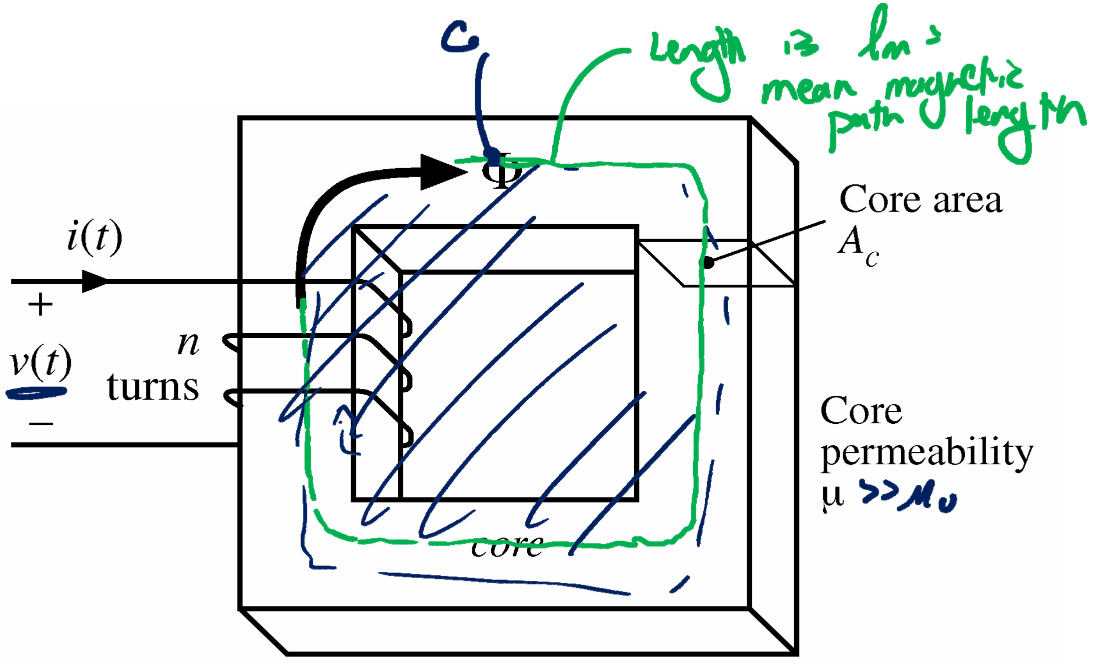
Inductor Example

Simplifying Assumptions/Approximations

(1) $\mu \gg \mu_0$ & all of the flux stays within the core

(2) B & H are uniform everywhere in the core

(3) $B = \mu H$ within the core (check saturation later)



Faraday

$$v_{turn} = \frac{d\Phi}{dt} \approx \frac{dB(t)}{dt} A_c$$

$$\boxed{v(t) = n A_c \frac{dB(t)}{dt}}$$ for n turns

Ampere's

$$\mathcal{F} = \oint_c H \cdot dl = I_{encl} = \boxed{n\, i(t) \approx H(t)\, l_m} = \frac{B(t)}{\mu} l_m$$

$$v(t) = n A_c \frac{d}{dt}\left(\frac{n\mu}{l_m} i(t)\right) \longrightarrow \boxed{v(t) = \frac{n^2 \mu A_c}{l_m} \frac{di(t)}{dt}}$$

$$\boxed{L = \frac{\mu n^2 A_c}{l_m}}$$

Magnetic Circuits

From previous discussion

integral path indep. $\rightarrow$ $V \longleftrightarrow \mathcal{F}$

conservative fields $\rightarrow$ $E \longleftrightarrow H$

$\downarrow$ KVL

$\downarrow$ K$\mathcal{F}$L

$I \longleftrightarrow \Phi$ $\rightarrow$ No point sources

$J \longleftrightarrow B$ $\rightarrow$ Densities

$\downarrow$ KCL

$\downarrow$ KΦL

Electric

Magnetic

V  $\mathcal{F}$

I  Φ

$\frac{1}{\sigma}\frac{l}{A_w} = R$  $\mathcal{R} = \frac{l}{\mu A_c}$

$J = \sigma E$

$\downarrow$

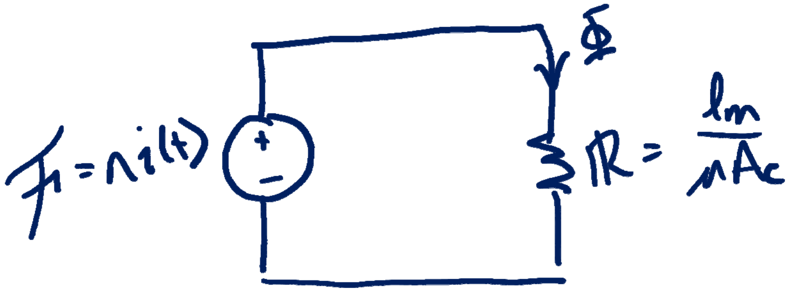
$V = IR$

$B = \mu H$

$\downarrow$

$\mathcal{F} = \Phi \mathcal{R}$

Inductor Magnetic Circuit Model



$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{n\, i(t)}{l_m} \cdot \mu A_c$$

Faraday $v = n \frac{d\Phi}{dt}$

$$v = n \frac{d}{dt}\left(\frac{n \mu A_c}{l_m} i(t) \right)$$

$$\boxed{v = \frac{\mu n^2 A_c}{l_m} \frac{di(t)}{dt}}$$

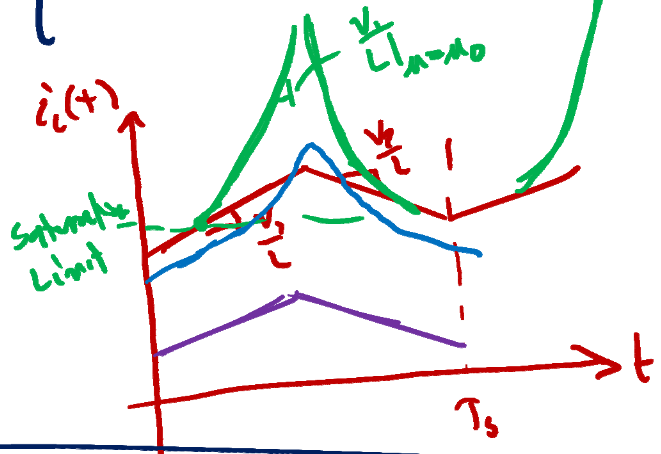
Saturation Limits

Process: Solve assuming the material is not saturated, then check worst-case $|B(t)| < B_{sat}$

typical B_{sat} $\begin{cases} 0.2 - 0.8\ \text{T} & \text{for ferrites} \\ 1.2 \rightarrow 2\ \text{T} & \text{for nanocrystalline} \end{cases}$

from previous $L = \frac{\mu n^2 A_c}{l_m}$

Saturation
$\mu \downarrow \sim 1000\times$
$L \downarrow \sim 1000\times$



$$v(t) = nA_c \frac{dB(t)}{dt} \rightarrow \boxed{B(t) = \frac{1}{nA_c}\int_0^t v(t)\,dt}$$

$$v(t) = L\frac{di(t)}{dt} \rightarrow i(t) = \frac{1}{L}\int_0^t v(t)\,dt$$

$$= B(t) = \frac{1}{nA_c} L\, i(t)$$

$$B(t) = \frac{1}{n\cancel{A_c}} \frac{\mu n^2 \cancel{A_c}}{l_m} i(t)$$

$$\boxed{B(t) = \frac{\mu n}{l_m} i(t)}$$

@ B_{sat}, $\boxed{B_{sat} = \frac{\mu n}{l_m} I_{sat}} \rightarrow I_{sat} = \frac{l_m}{\mu n} B_{sat}$