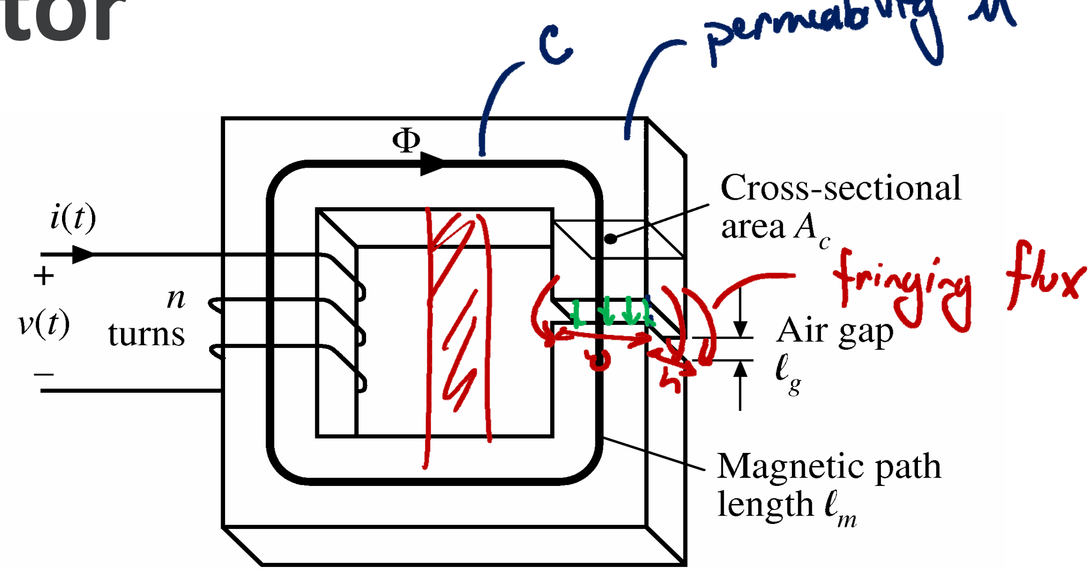
Example: Gapped Inductor

Approximations

(1) - (3) still apply

(4) At the air gap all Φ stays within "A_c"

- Neglect fringing flux
  (good approx if $l_g \ll w$ & h)



Faraday's

$$v(t) = n\frac{d\Phi(t)}{dt} = n A_c \frac{dB(t)}{dt}$$

Ampere's

$$I_{encl} = \oint_c H \cdot dl = H_c(l_m - l_g) + H_g l_g = n\,i(t)$$

$$= \frac{B}{\mu}(l_m - l_g) + \frac{B}{\mu_0} l_g = n\,i(t) \rightarrow B(t) = \frac{n\,i(t)}{\frac{l_m - l_g}{\mu} + \frac{l_g}{\mu_0}}$$

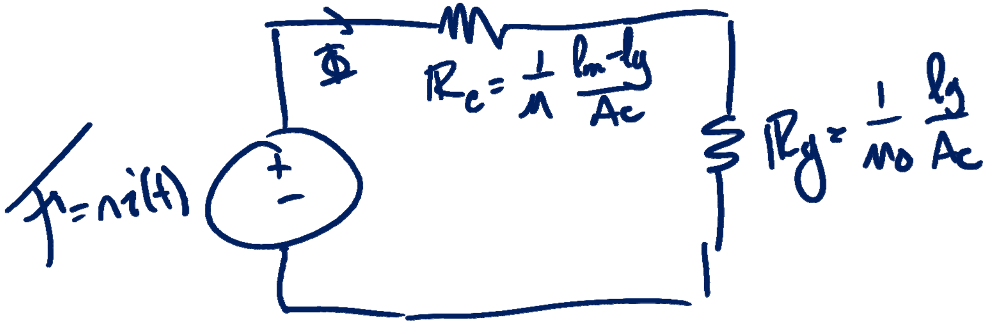
$l_m \gg l_g$
$\downarrow$
$l_m - l_g \approx l_m$

$$v(t) = n A_c \frac{d}{dt}\left(\frac{n\,i(t)}{\frac{l_m}{\mu} + \frac{l_g}{\mu_0}}\right) = \underbrace{\frac{n^2 A_c}{\frac{l_m}{\mu} + \frac{l_g}{\mu_0}}}_{L} \frac{di(t)}{dt}$$

if $\frac{l_g}{\mu_0} \gg \frac{l_m}{\mu}$ $\rightarrow$

$$L = \frac{n^2 A_c \mu_0}{l_g}$$

Gapped Inductor Magnetic Circuit



$$\Phi = \frac{ni}{\mathcal{R}_c + \mathcal{R}_g}$$

$$v(t) = n\frac{d\Phi}{dt} = n\frac{d}{dt}\left(\frac{ni}{\mathcal{R}_c + \mathcal{R}_g}\right)$$

$$v(t) = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}\frac{di(t)}{dt}$$

$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g} = \frac{n^2 A_c}{\frac{l_m - l_g}{\mu} + \frac{l_g}{\mu_0}} \approx \frac{n^2 A_c}{\frac{l_m}{\mu} + \frac{l_g}{\mu_0}} \approx \frac{n^2 A_c \mu_0}{l_g}$$

Effect of Air Gap

$$L = \frac{n^2 A_c}{\frac{l_g}{\mu_0} + \frac{l_m}{\mu}} = \frac{n^2 A_c}{\mathcal{R}_c + \mathcal{R}_g}$$

$$B(t) = \frac{1}{nA_c} \int v\,dt$$

$$i(t) = \frac{1}{L} \int v\,dt$$

$$B(t) = \frac{1}{nA_c} L \cdot i(t)$$

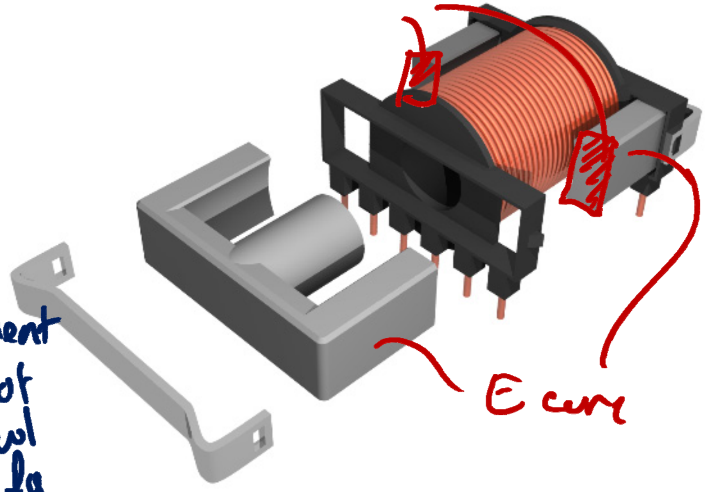
$$B_{sat} = \frac{1}{nA_c} \frac{n^2 A_c}{\mathcal{R}_c + \mathcal{R}_g} I_{sat}$$

$$I_{sat} = B_{sat} \frac{(\mathcal{R}_c + \mathcal{R}_g)}{n}$$

Adding Air Gap

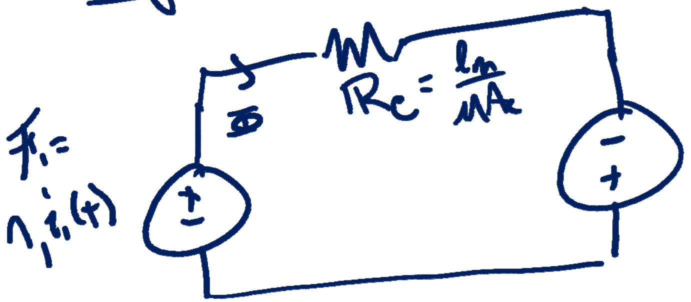
(1) increase I_{sat}
(2) Decrease L
(3) Reduce L's dependence on μ

But, allows independent setting of each w/ n & l_g

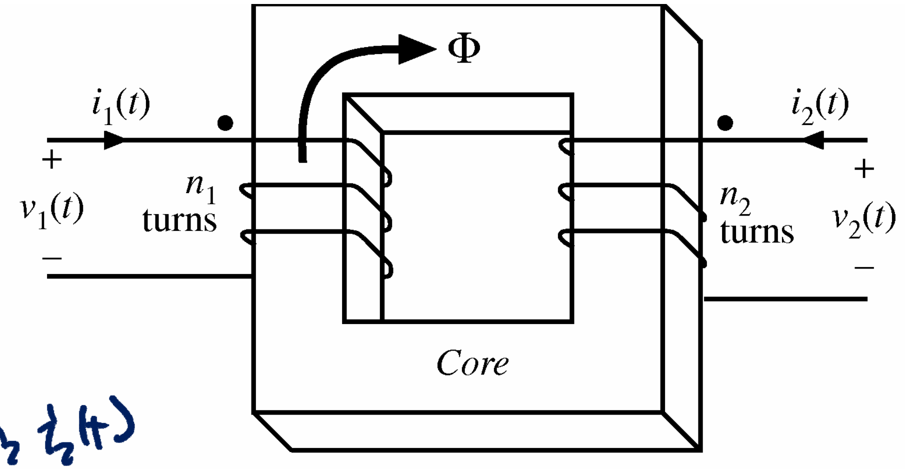


Transformer Example

Magnetic Circuit:



$\mathcal{F}_2 = n_2 i_2(t)$



$$\Phi = \frac{\mathcal{F}_1 + \mathcal{F}_2}{\mathcal{R}_c} \rightarrow \Phi \mathcal{R}_c = n_1 i_1(t) + n_2 i_2(t)$$

if $\mathcal{R}_c$ is very small ($\mu \rightarrow \infty$)

$$0 = n_1 i_1(t) + n_2 i_2(t) \rightarrow \text{ideal XF equation}$$

Faraday's

$$v_1 = n_1 \frac{d\Phi}{dt}$$

$$v_2 = n_2 \frac{d\Phi}{dt}$$

$$\frac{v_1}{n_1} = \frac{v_2}{n_2}$$

Nonideal Transformer

Let $\mathcal{R}_c$ be $> \emptyset$

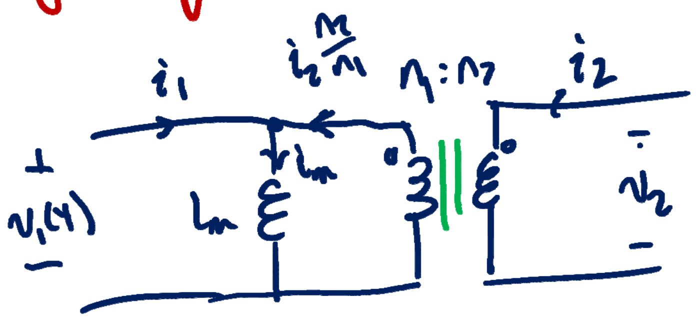
$$v_1 = n_1 \frac{d\Phi}{dt} = n_1 \frac{d}{dt}\left(\frac{n_1 i_1(t) + n_2 i_2(t)}{\mathcal{R}_c}\right)$$

$$\mathcal{R}_c = \frac{l_m}{\mu A_c}$$

$$v_1 = \frac{n_1^2}{\mathcal{R}_c}\frac{di_1}{dt} + \frac{n_1 n_2}{\mathcal{R}_c}\frac{di_2}{dt}$$

$$v_1 = \underbrace{\frac{n_1^2 \mu A_c}{l_m}}\frac{d}{dt}\Big(\underbrace{i_1(t) + \frac{n_2}{n_1} i_2(t)}_{i_m}\Big)$$

$$L_m = \frac{n_1^2 \mu A_c}{l_m} \equiv \text{"Magnetizing Inductance")}$$



XF Saturation:

$i_1(t)$ & $i_2(t)$ do not have any saturation limit

$i_m(t) \to I_{m,sat}$

$$B(t) = \frac{1}{n_1 A_c}\int v_1(t)\,dt$$

$$i_m = \frac{1}{L_m}\int v_1(t)\,dt$$

$$B_{sat} = \frac{1}{n A_c} L_m I_{m,sat}$$

$$\boxed{I_{m,sat} = B_{sat}\, n A_c \frac{1}{L_m}}$$

DC Copper Loss

$$R_{DC} = \rho \frac{l_w}{A_w}$$

l_w ← length of winding

A_w ← Area of winding

ρ ↑ resistivity of copper

$$l_w = n \cdot MLT$$

MLT = "mean length per turn"