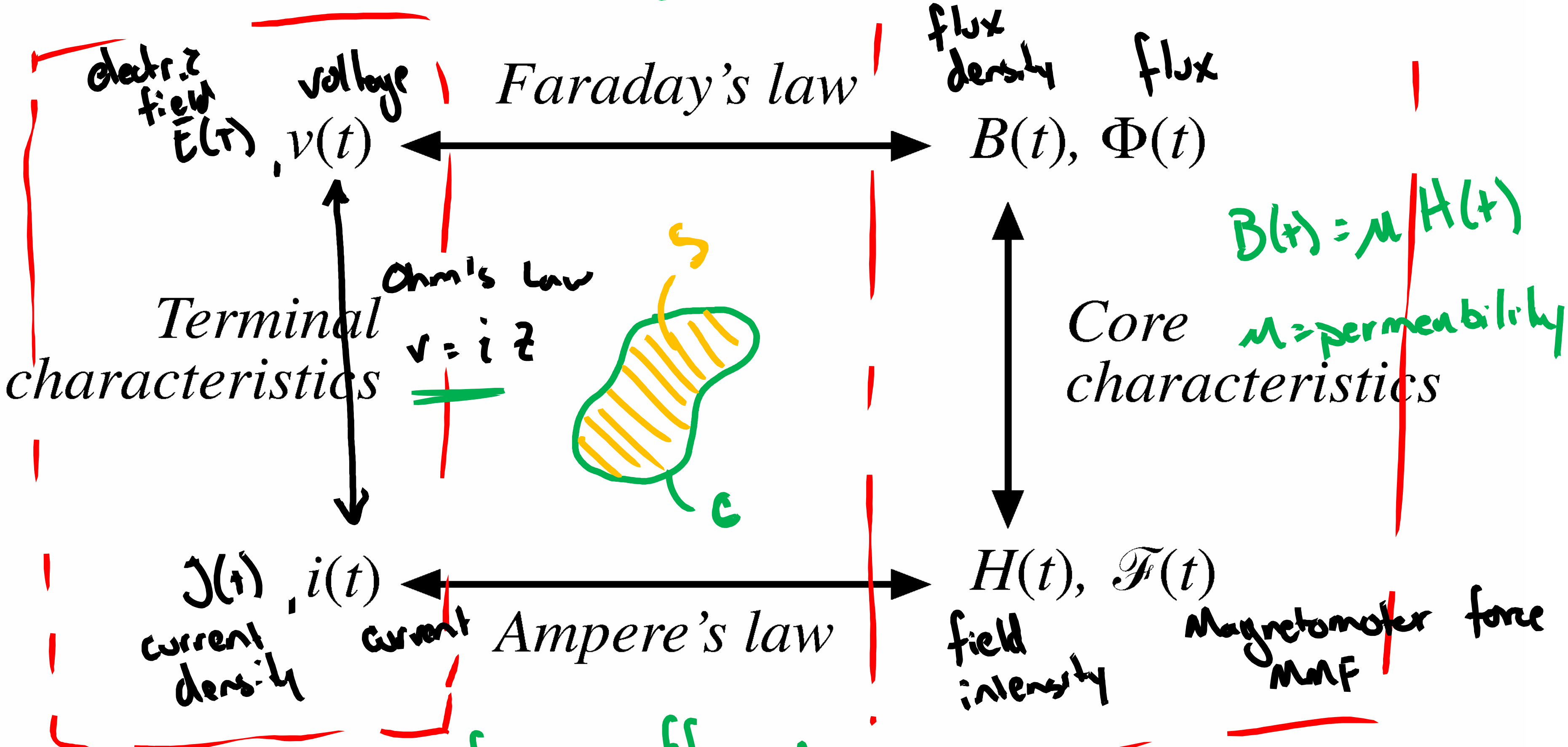
# Basic Magnetics Relationships

$$\oint_C E \cdot dl = \iint_S \frac{dB}{dt}\, dA$$

Electric

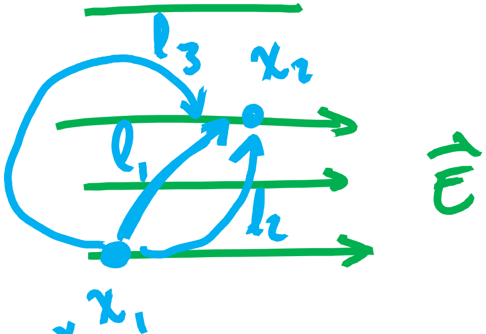
Magnetic




$$\oint_C H\, dl = \iint_S J\, dA$$

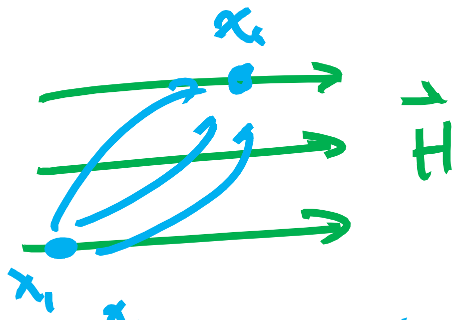
# Electric/Magnetic Duals

Electric | Magnetic



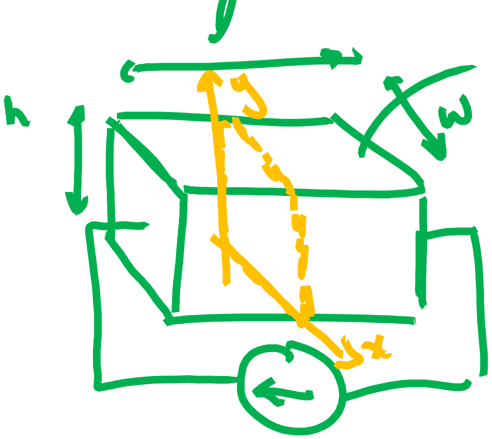

$E \leftrightarrow H$

$V \leftrightarrow \mathcal{F}$

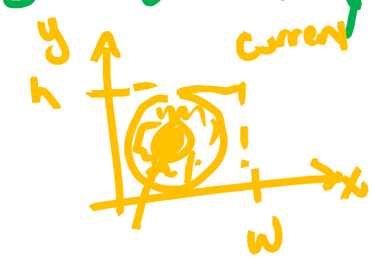



$$\int_{x_1}^{x_2} \vec{E}\, dl = V_{x_2,x_1}$$

$$\int_{x_1}^{x_2} H\, dl = \mathcal{F}_{x_2,x_1}$$

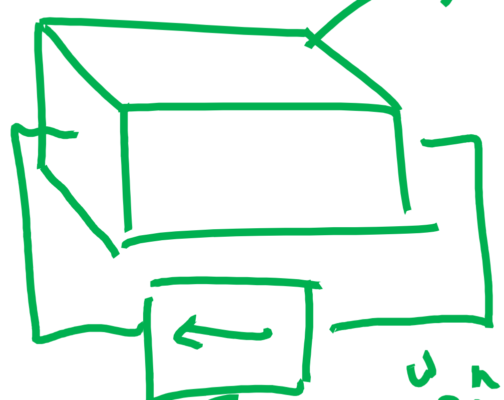



$\sigma \rightarrow$ conductivity




$J$ = current density

$\mu$ permeability



I

$\Phi$

$$\int_0^w \int_0^h J\, dy\, dx = I$$

$J \leftrightarrow B$

$I \leftrightarrow \Phi$

$R \leftrightarrow \mathcal{R}$

$$\int_0^w \int_0^h B\, dy\, dx = \Phi$$

resistance

$$R = \frac{1}{\sigma}\frac{l}{hw}$$

reluctance

$$\mathcal{R} = \frac{1}{\mu}\frac{l}{hw}$$

# Faraday's Law

$$V_{turn} = \oint_c E \cdot dl = \iint_s \frac{dB}{dt} dA = \frac{d\Phi(t)}{dt} = \frac{dB(t)}{dt} A_c$$

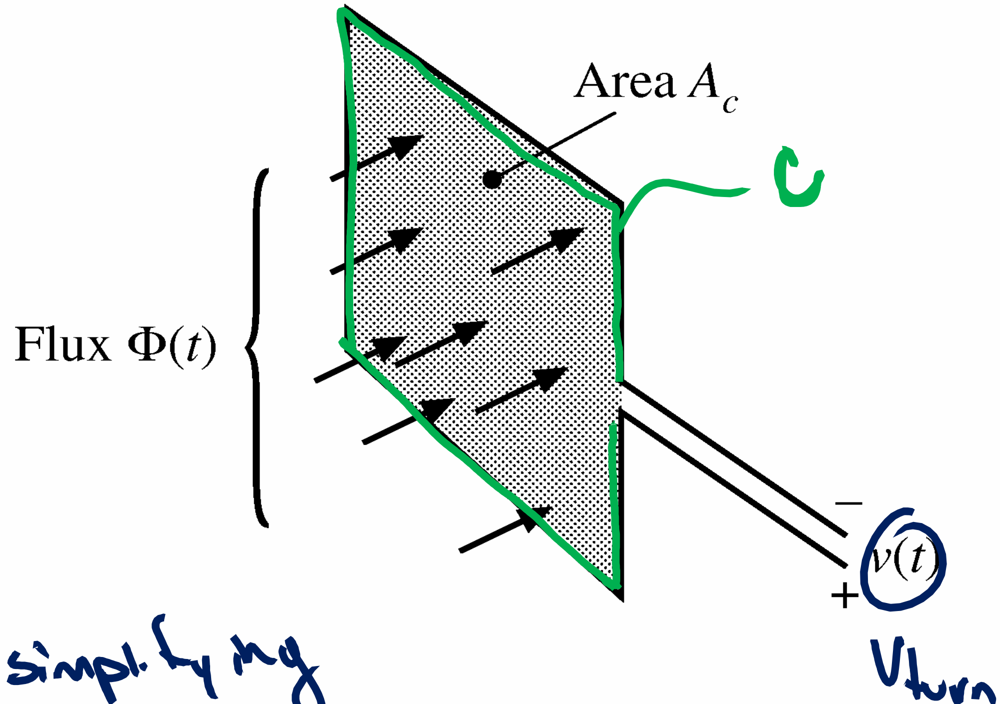
Voltage $v(t)$ is induced in a loop of wire by change in the total flux $\Phi(t)$ passing through the interior of the loop, according to

$$v(t) = \frac{d\Phi(t)}{dt}$$

For uniform flux distribution, $\Phi(t) = B(t)A_c$ and hence

$$\boxed{v(t) = A_c \frac{dB(t)}{dt}}$$

Vturn




simplifying assumption

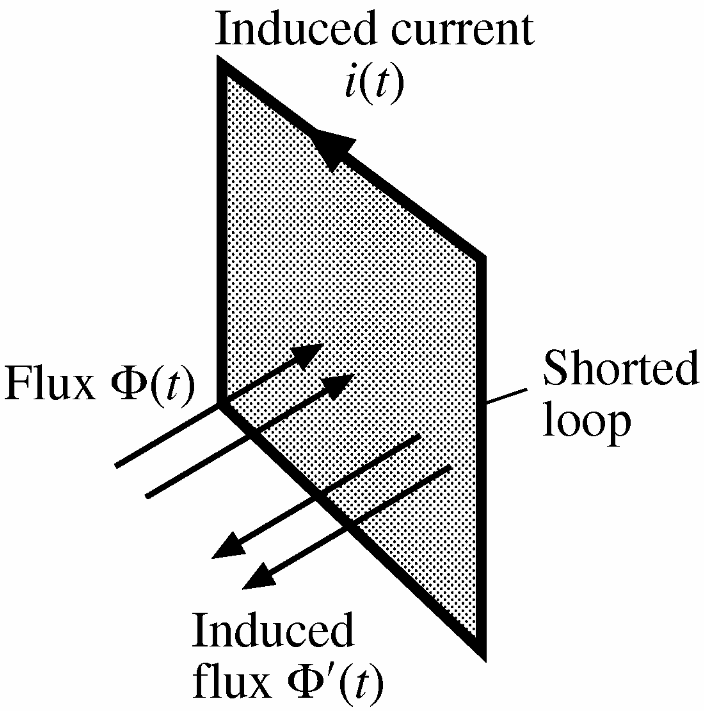
# Lenz's Law

Right hand rule

The voltage $v(t)$ induced by the changing flux $\Phi(t)$ is of the polarity that tends to drive a current through the loop to counteract the flux change.

*Example: a shorted loop of wire*

- Changing flux $\Phi(t)$ induces a voltage $v(t)$ around the loop
- This voltage, divided by the impedance of the loop conductor, leads to current $i(t)$
- This current induces a flux $\Phi'(t)$, which tends to oppose changes in $\Phi(t)$



# Ampere's Law

$\mathcal{F} = H\ell_m = \oint_C H\cdot dl = \oiint_S J\,dA = I_{enclosed} = i(t)$

The net MMF around a closed path is equal to the total current passing through the interior of the path:

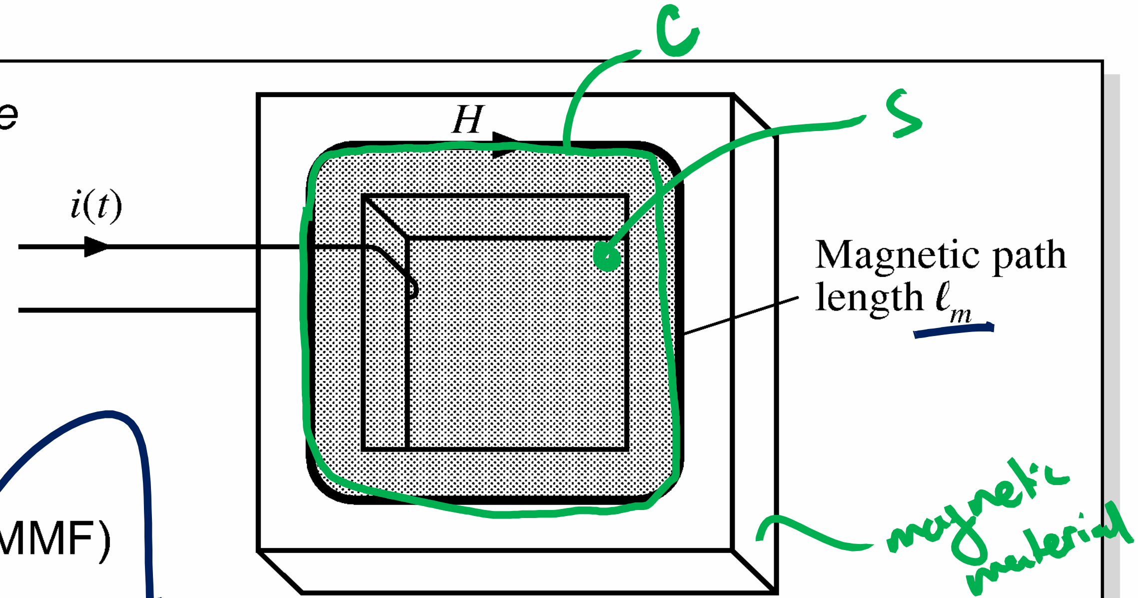
$$\oint_{closed\ path} \boldsymbol{H}\cdot d\boldsymbol{\ell} = \text{total current passing through interior of path}$$

*Example: magnetic core. Wire carrying current $i(t)$ passes through core window.*

- Illustrated path follows magnetic flux lines around interior of core
- For uniform magnetic field strength $H(t)$, the integral (MMF) is $H(t)\ell_m$. So

$$\boxed{\mathcal{F}(t) = H(t)\ell_m = i(t)}$$

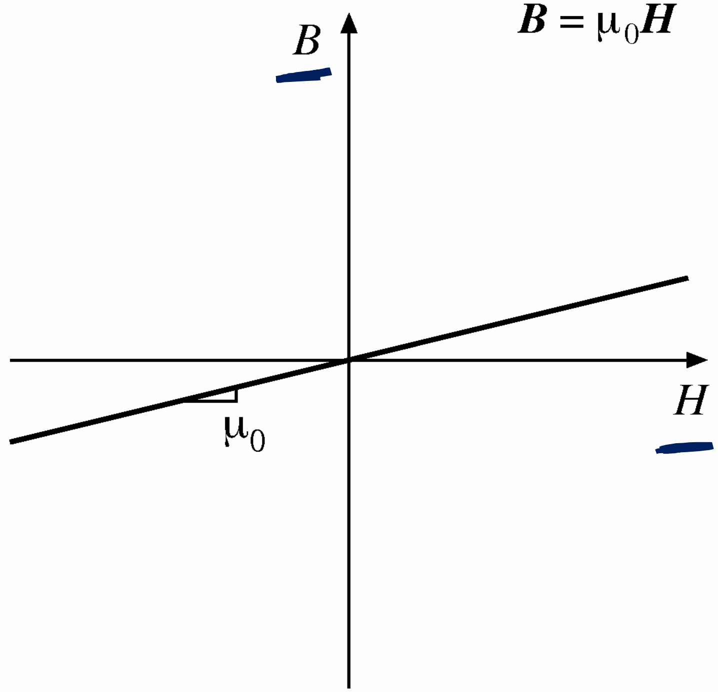
simplifying assumption



# Core Material Characteristics

$B = \mu H$

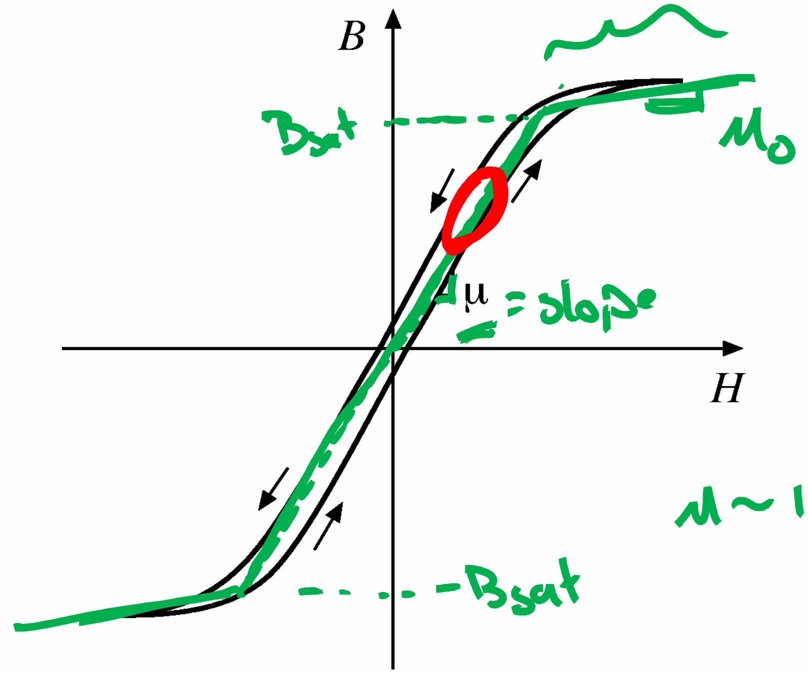
*Free space*




$\mu_0$ = permeability of free space

$\boxed{= 4\pi \cdot 10^{-7}}$ Henries per meter

*A magnetic core material* saturation




$\mu \sim 10 - 10{,}000 \times \mu_0$

Highly nonlinear, with hysteresis and saturation

# Units

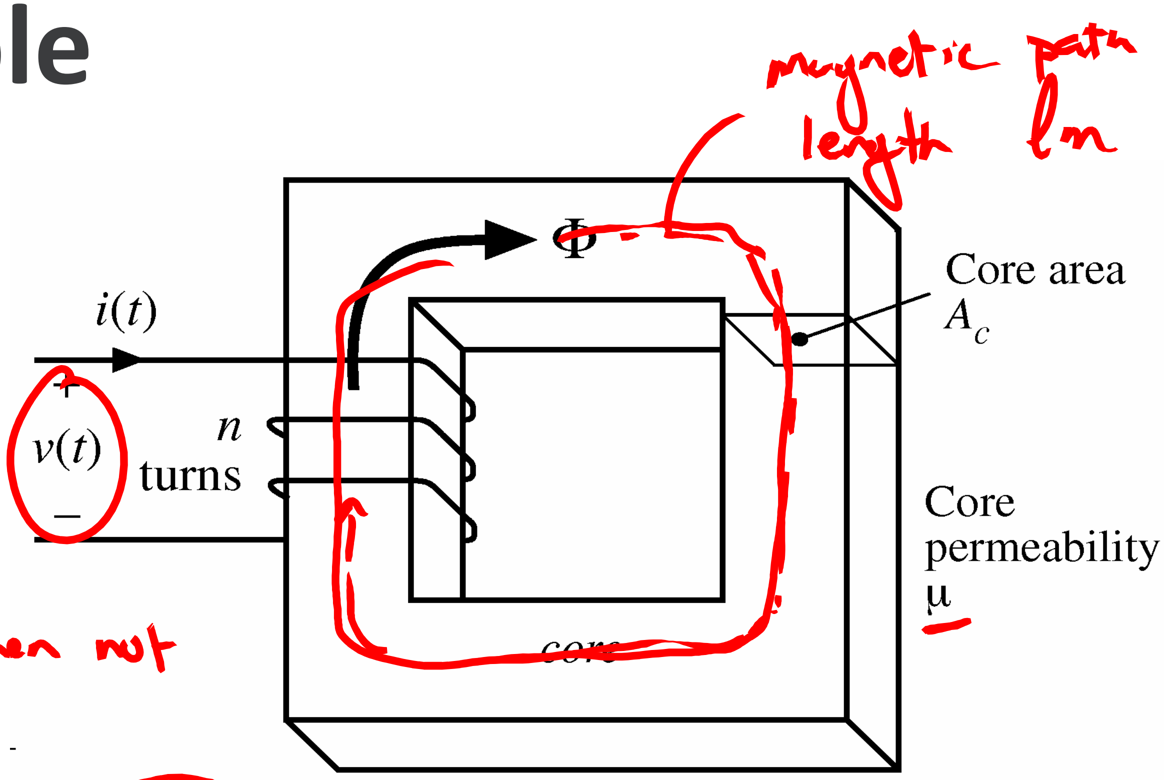
*Table 12.1. Units for magnetic quantities*

| *quantity* | *MKS* | *unrationalized cgs* | *conversions* |
|---|---|---|---|
| core material equation | $B = \mu_0\, \mu_r\, H$ | $B = \mu_r\, H$ | |
| $B$ | Tesla | Gauss | $1\text{T} = 10^4\text{G}$ |
| $H$ | Ampere / meter | Oersted | $1\text{A/m} = 4\pi \cdot 10^{-3}$ Oe |
| $\Phi$ | Weber | Maxwell | $1\text{Wb} = 10^8$ Mx<br>$1\text{T} = 1\text{Wb} / \text{m}^2$ |

# Inductor Example

Simplifying assumptions:

1. $\mu \gg \mu_0$ & flux stays entirely within the core
2. H & B fields are uniform throughout the core
3. $B = \mu H$ in the core (when not saturated)




Faraday:

$$V_{turn} = A_c \frac{dB(t)}{dt}$$

$$\boxed{v(t) = nA_c \frac{dB(t)}{dt}}$$

Ampere:

$$H\,l_m = I_{enclosed}$$

$$\boxed{H\,l_m = n\,i(t)}$$

Core Material:

$$\boxed{B = \mu H} \quad \left(\text{if } \max|B| < B_{sat}\right)$$

$$v(t) = nA_c \frac{d}{dt}\left(\mu \frac{n\,i(t)}{l_m}\right) = \underbrace{\frac{n^2 A_c \mu}{l_m}}_{L} \frac{di(t)}{dt}$$