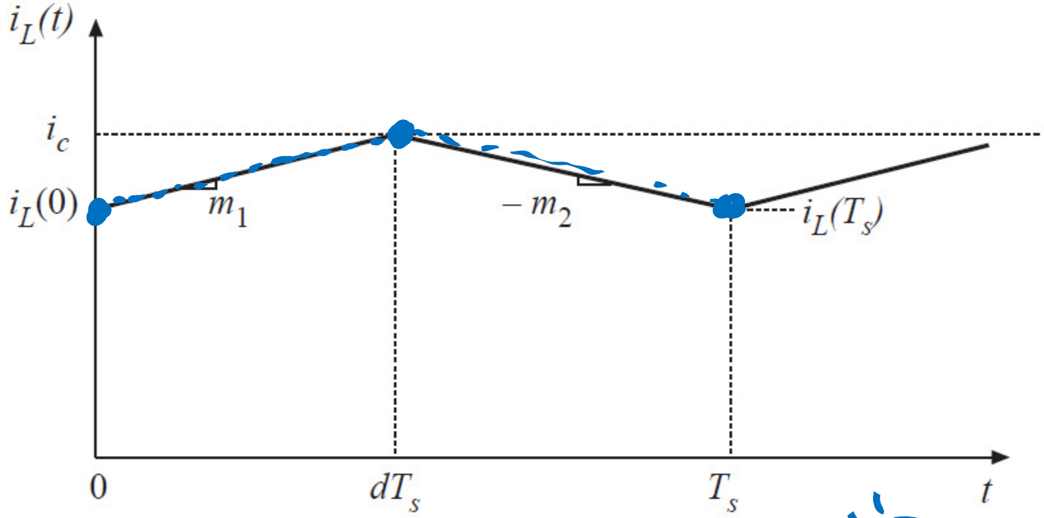
CPM Oscillations for D>0.5

- The current programmed controller is inherently unstable for $D > 0.5$, regardless of the converter topology
- Controller can be stabilized by addition of an artificial ramp

Inductor Current Waveform in CCM

In steady-state
$i_L(0) = i_L(T_s)$



$i_L(T_s) = i_L(0) + m_1 d T_s - m_2 d' T_s$

in steady-state

$0 = m_1 D \cancel{T_s} - m_2 D' \cancel{T_s}$

$$\boxed{\frac{m_2}{m_1} = \frac{D}{D'}}$$

Inductor current slopes m_1 *and* $-m_2$

buck converter

$$m_1 = \frac{v_g - v}{L} \qquad -m_2 = -\frac{v}{L}$$

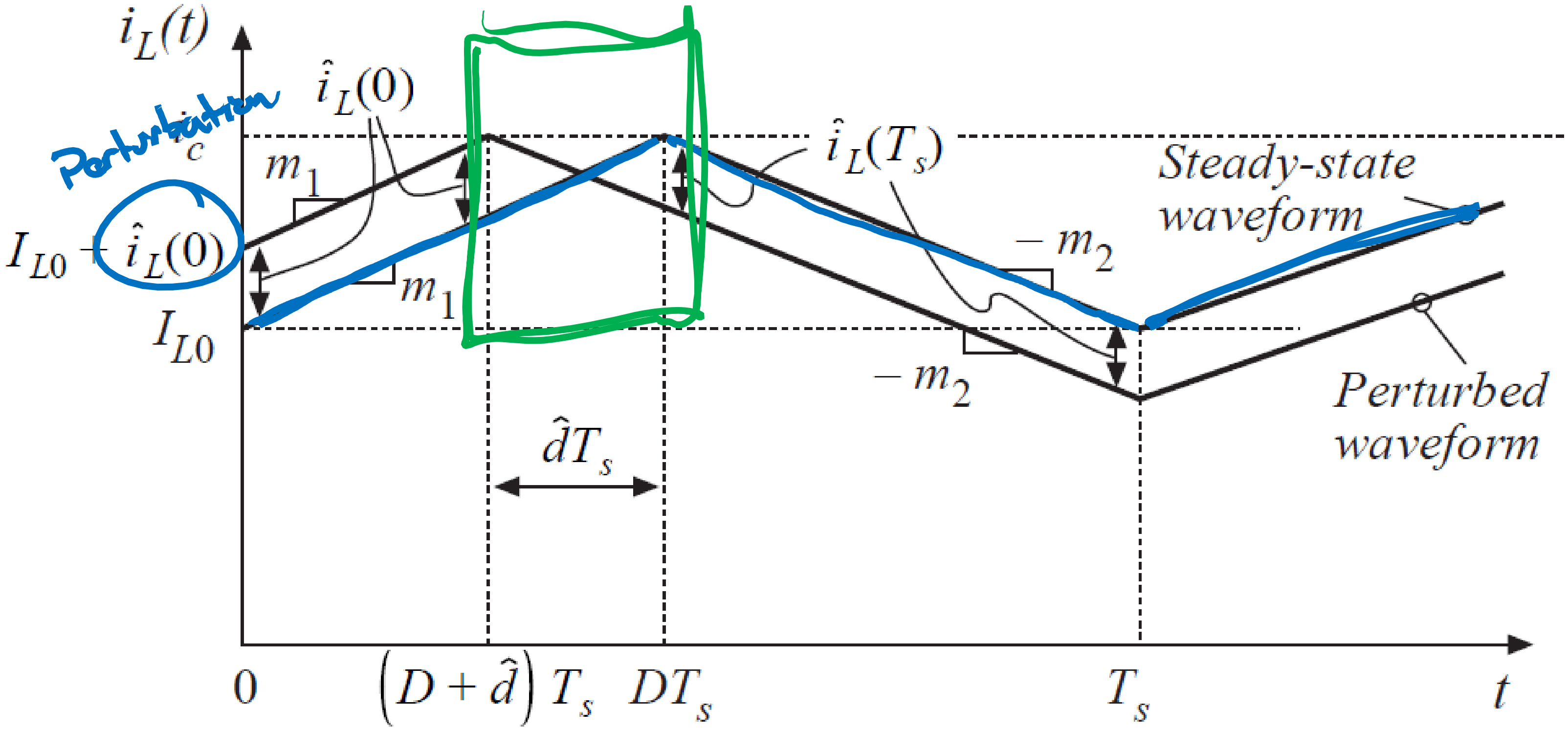
boost converter

$$m_1 = \frac{v_g}{L} \qquad -m_2 = \frac{v_g - v}{L}$$

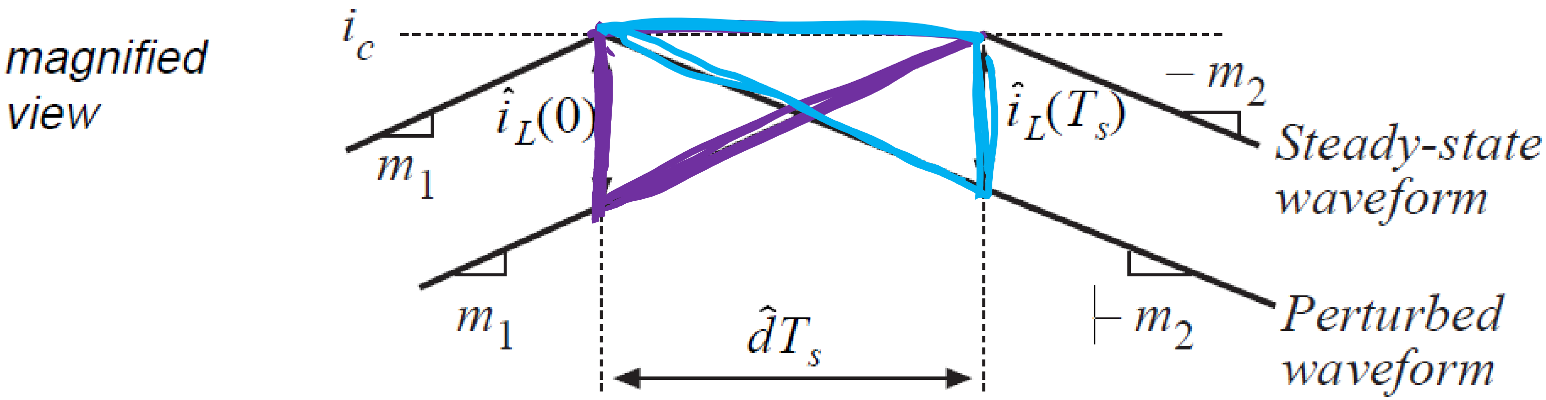
buck–boost converter

$$m_1 = \frac{v_g}{L} \qquad -m_2 = \frac{v}{L}$$

Introducing a Perturbation



Change in Inductor Current Over T_s



$$\left.\begin{aligned}\hat{i}_L(0) &= m_1 \hat{d} T_s \\ \hat{i}_L(T_s) &= -m_2 \hat{d} T_s\end{aligned}\right\} \quad \hat{i}_L(T_s) = \hat{i}_L(0)\frac{-m_2}{m_1}$$

$$\hat{i}_L(2T_s) = \hat{i}_L(T_s)\left(\frac{-m_2}{m_1}\right) = \hat{i}_L(0)\left(\frac{-m_2}{m_1}\right)^2$$

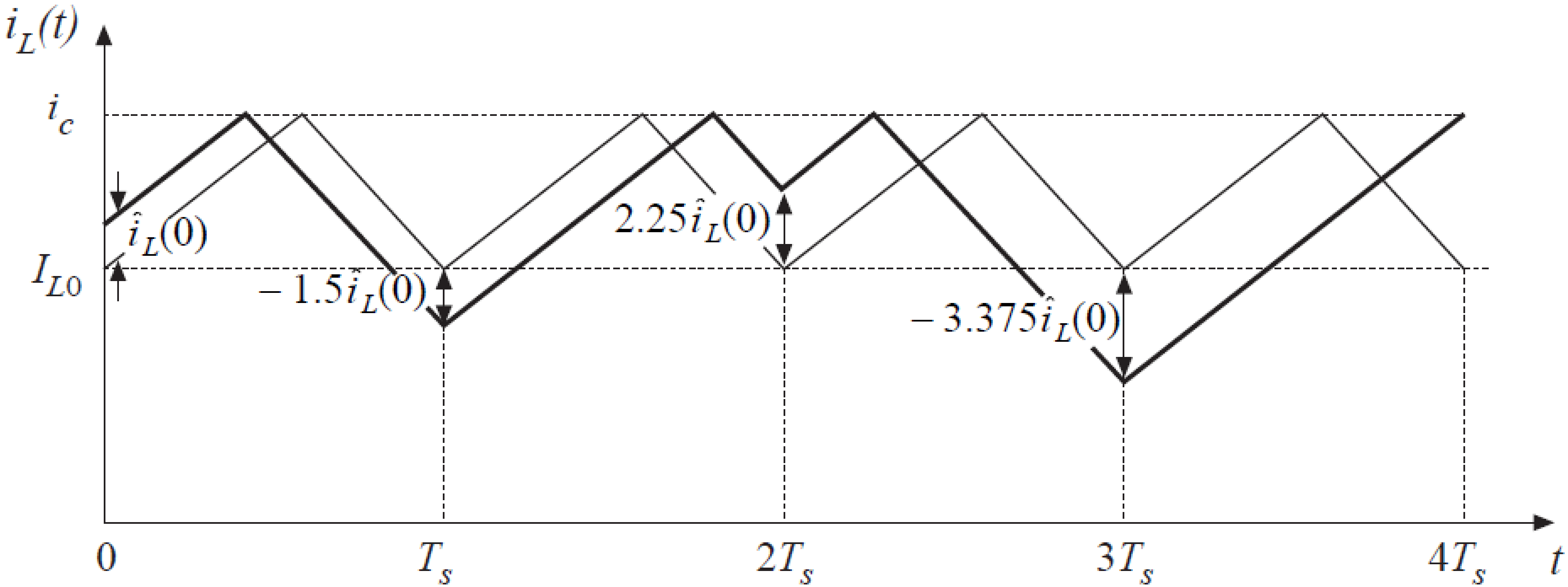
$$\hat{i}_L(nT_s) = \hat{i}_L(0)\left(\frac{-m_2}{m_1}\right)^n$$

$$\alpha = \frac{-m_2}{m_1} = \frac{D}{D'}$$

need $|\alpha| < 1$ for stability

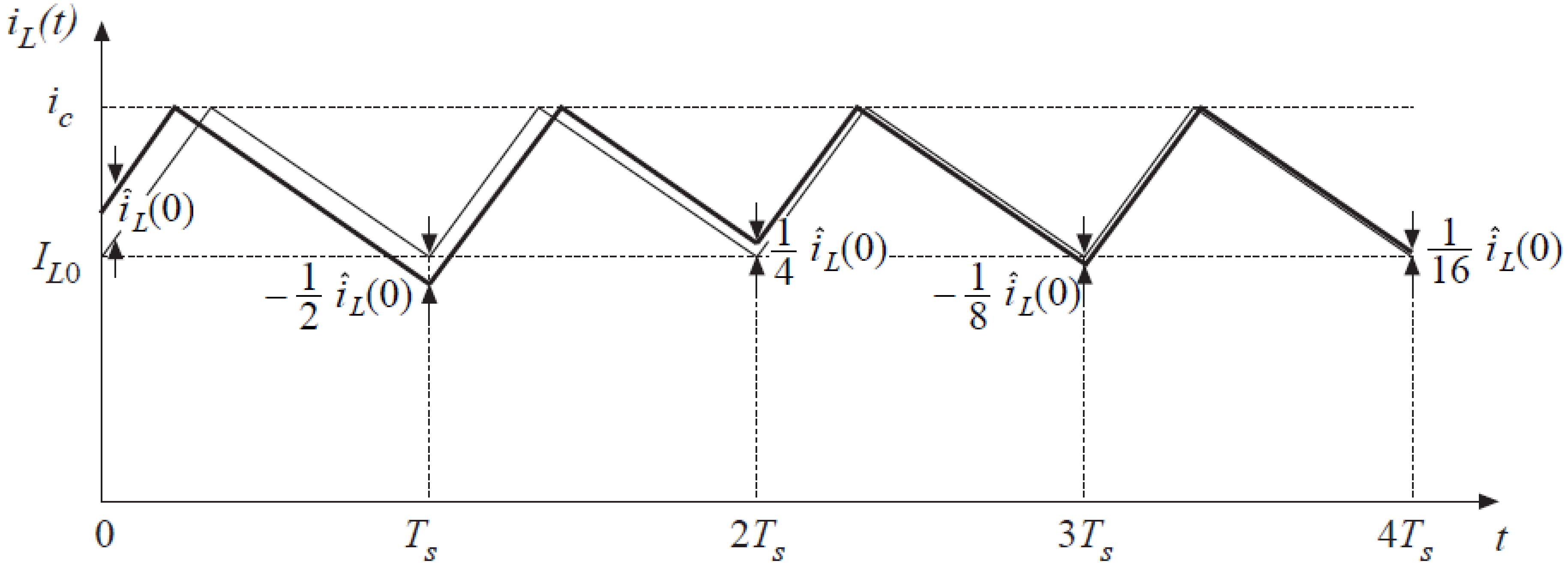
Example: Unstable operation for D=0.6

$$\alpha = -\frac{D}{D'} = \left(-\frac{0.6}{0.4}\right) = -1.5$$

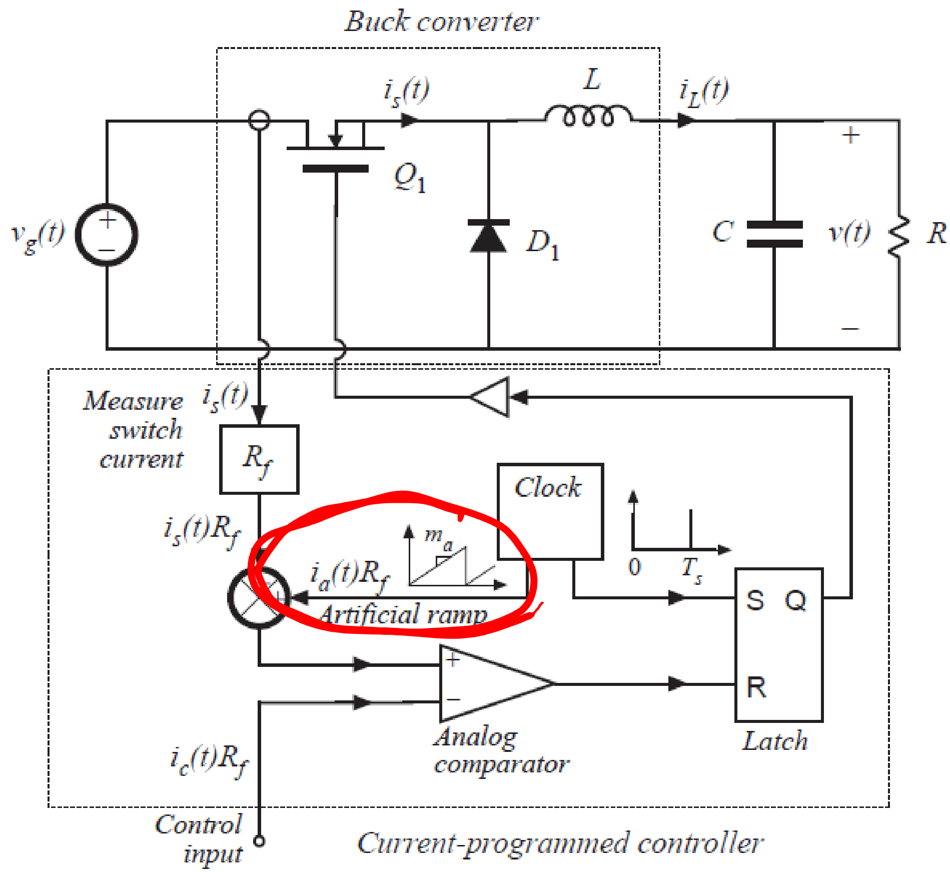


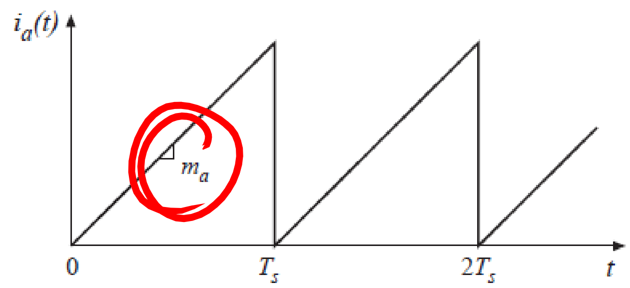
Example: Stable operation for D=1/3

$$\alpha = -\frac{D}{D'} = \left(-\frac{1/3}{2/3}\right) = -0.5$$



Stabilization Through Artificial Ramp





Now, transistor switches off when

$$i_a(dT_s) + i_L(dT_s) = i_c$$

or,

$$i_L(dT_s) = i_c - i_a(dT_s)$$

if $i_a >> i_L$ → PWM control

if $i_a << i_L$ → CPM

Final Value of Inductor Current

First subinterval:

$$\hat{i}_L(0) = -\hat{d}T_s\left(m_1 + m_a\right)$$

Second subinterval:

$$\hat{i}_L(T_s) = -\hat{d}T_s\left(m_a - m_2\right)$$

Net change over one switching period:

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right)$$

After n switching periods:

$$\hat{i}_L(nT_s) = \hat{i}_L((n-1)T_s)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right) = \hat{i}_L(0)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right)^n = \hat{i}_L(0)\,\alpha^n$$

Characteristic value:

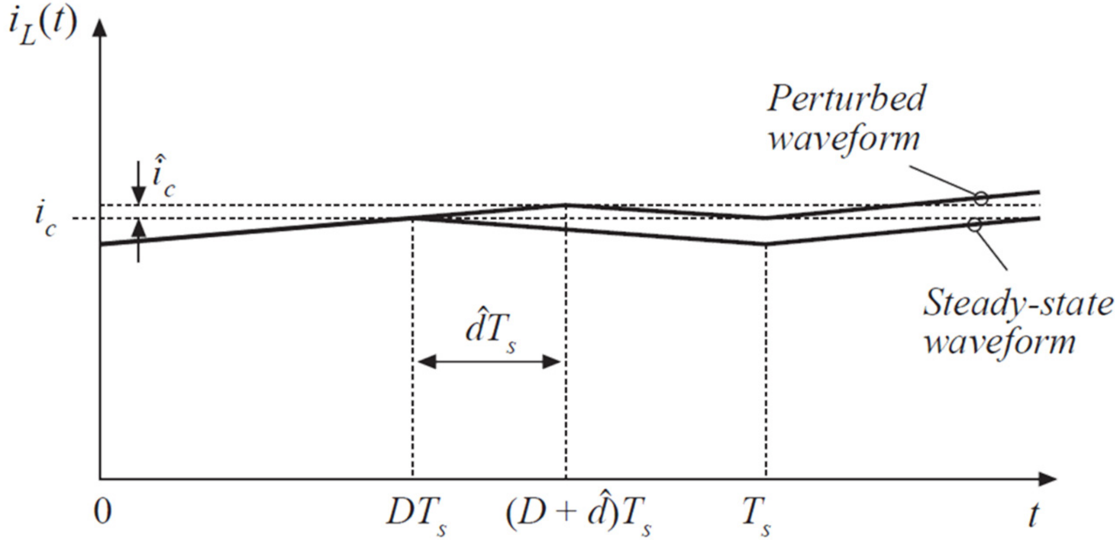
$$\alpha = -\frac{m_2 - m_a}{m_1 + m_a}$$

$$\left|\hat{i}_L(nT_s)\right| \rightarrow \begin{cases} 0 & \text{when } \left|\alpha\right| < 1 \\ \infty & \text{when } \left|\alpha\right| > 1 \end{cases}$$

Artificial Ramp: Additional Notes

$$\alpha = -\frac{1 - \frac{m_a}{m_2}}{\frac{D'}{D} + \frac{m_a}{m_2}}$$

- For stability, require $|\alpha|<1$
- Common choices:
  - $m_a = 0.5\ m_2$ (stable for all duty cycles)
  - $m_a = m_2$ (deadbeat)
- Artificial ramp decreases sensitivity to noise



More Accurate Models

- The simple models of the previous section yield insight into the low- frequency behavior of CPM converters
- Unfortunately, they do not always predict everything that we need to know:
  - Line-to-output transfer function of the buck converter
  - Dynamics at frequencies approaching f_s
- More accurate model accounts for nonideal operation of current mode controller built-in feedback loop
- Converter duty-cycle-controlled model, plus block diagram that accurately models equations of current mode controller
- See Section 12.3 for additional info

More Accurate Model

- Simple model assumes $i_L = i_c$ always
- Accounting for ripple, and artificial ramp weakens this approximation
- Using sampled data modeling

$$\frac{\hat{i}_L}{\hat{i}_C} = \frac{1}{1 + \frac{1}{Q_s}\left(\frac{s}{2\pi\, {f_s}/{2}}\right) + \left(\frac{s}{2\pi\, {f_s}/{2}}\right)^2}$$

if my bandwidth of voltage loop is well below $f_s/2$ then $i_L \approx i_c$ applies

Where

$$Q_s = \frac{2}{\pi\left(\frac{2}{1-\alpha} - 1\right)}$$

F. Dong Tan and R. D. Middlebrook, “A Unified Model for Current Programmed Converters,” IEEE Transactions on Power Electronics, vol. 10, no. 4, July 1995.

Note: Comparison to Datasheet

Application Information (continued)

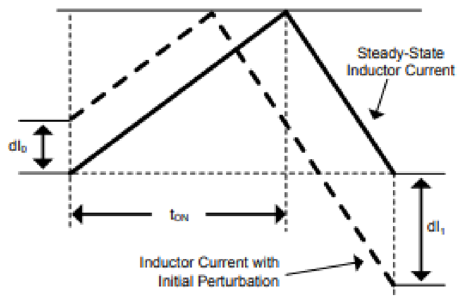


Figure 30. Effect of Initial Perturbation when $dI_1/dI_0 < -1$

dI_1/dI_0 can be calculated as:

$$\frac{dI_1}{dI_0} = 1 - \frac{1}{K} \quad (19)$$

$\alpha = 1 - \frac{1}{K}$

The relationship between dI_1/dI_0 and K factor is illustrated in the graphic below.

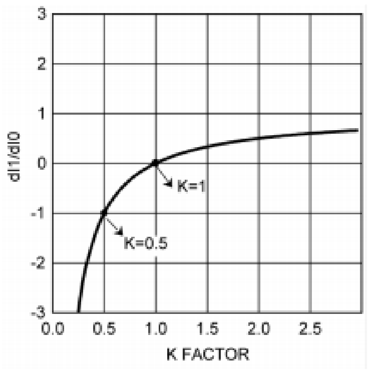


Figure 31. dI_1/dI_0 vs K Factor

The absolute minimum value of K is 0.5. When K<0.5, the amplitude of dI_1 is greater than the amplitude of dI_0 and any initial perturbation results in sub-harmonic oscillation. If K=1, any initial perturbation will be removed in one switching cycle. This is known as one-cycle damping. When $-1<dI_1/dI_0<0$, any initial perturbation will be under-damped. Any perturbation will be over-damped when $0<dI_1/dI_0<1$.