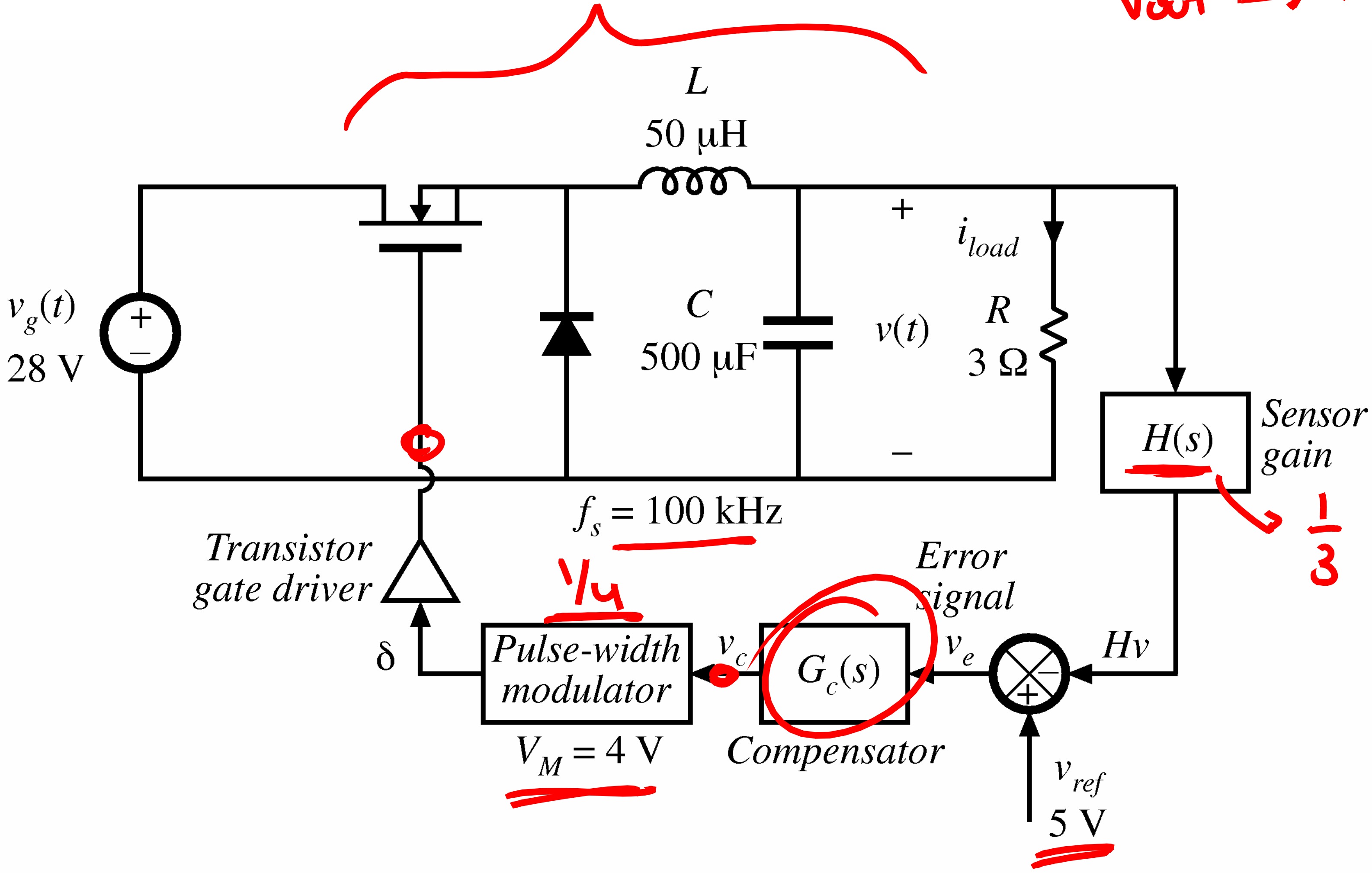
Example Design of Buck Compensator

Vout → 15V

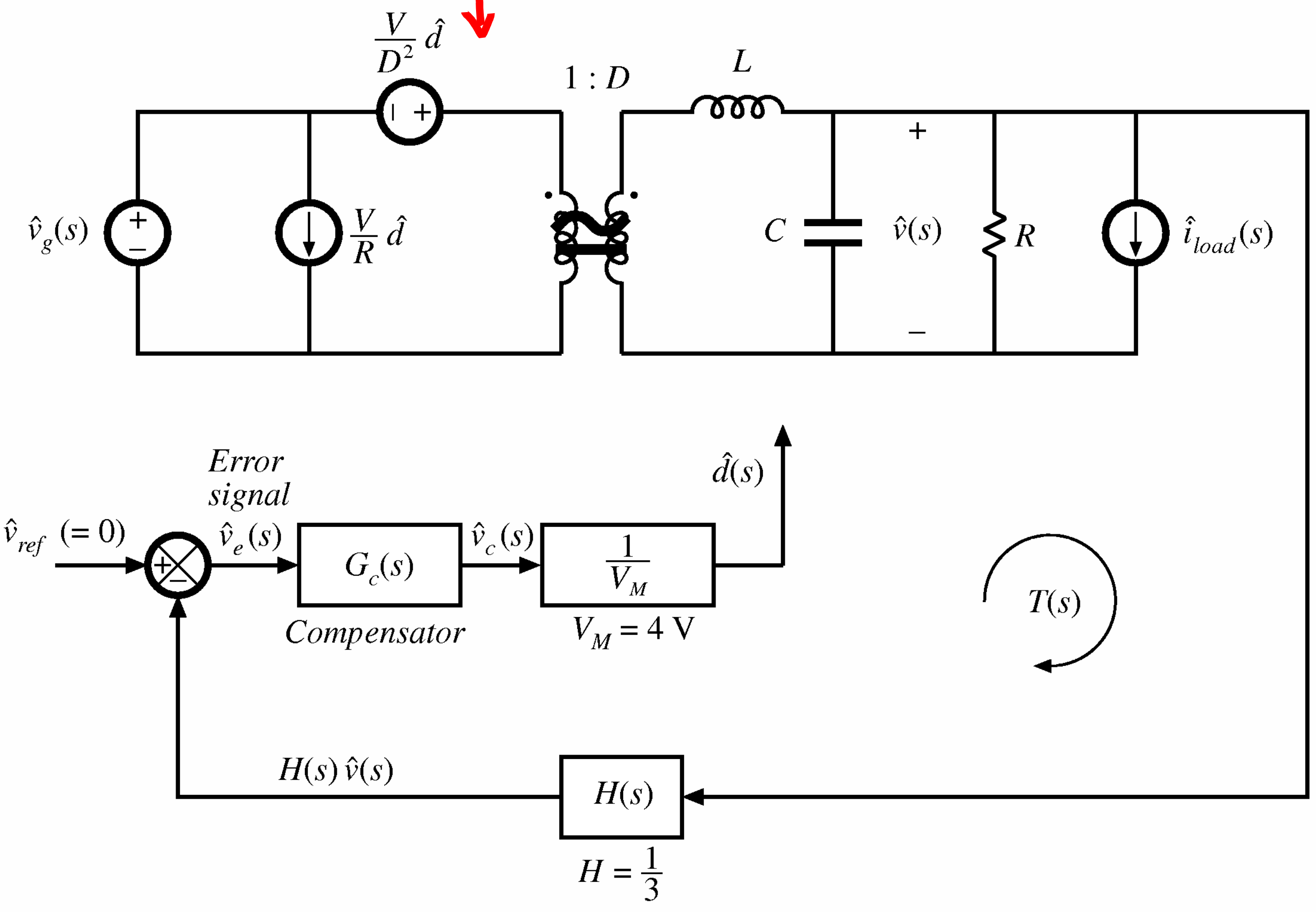


DC (Quiescent) Operating Point

Input voltage	$\underline{V_g = 28\text{V}}$
Output	$\underline{V = 15\text{V}}, \underline{I_{load} = 5\text{A}, R = 3\Omega}$
Quiescent duty cycle	$D = \underline{15/28 = 0.536}$
Reference voltage	$V_{ref} = 5\text{V}$
Quiescent value of control voltage	$V_c = \underline{DV_M} = 2.14\text{V}$
Gain $H(s)$	$H = V_{ref}/V = 5/15 = 1/3$

AC Power Stage Model

Table 7.1 ↓



System Block Diagram

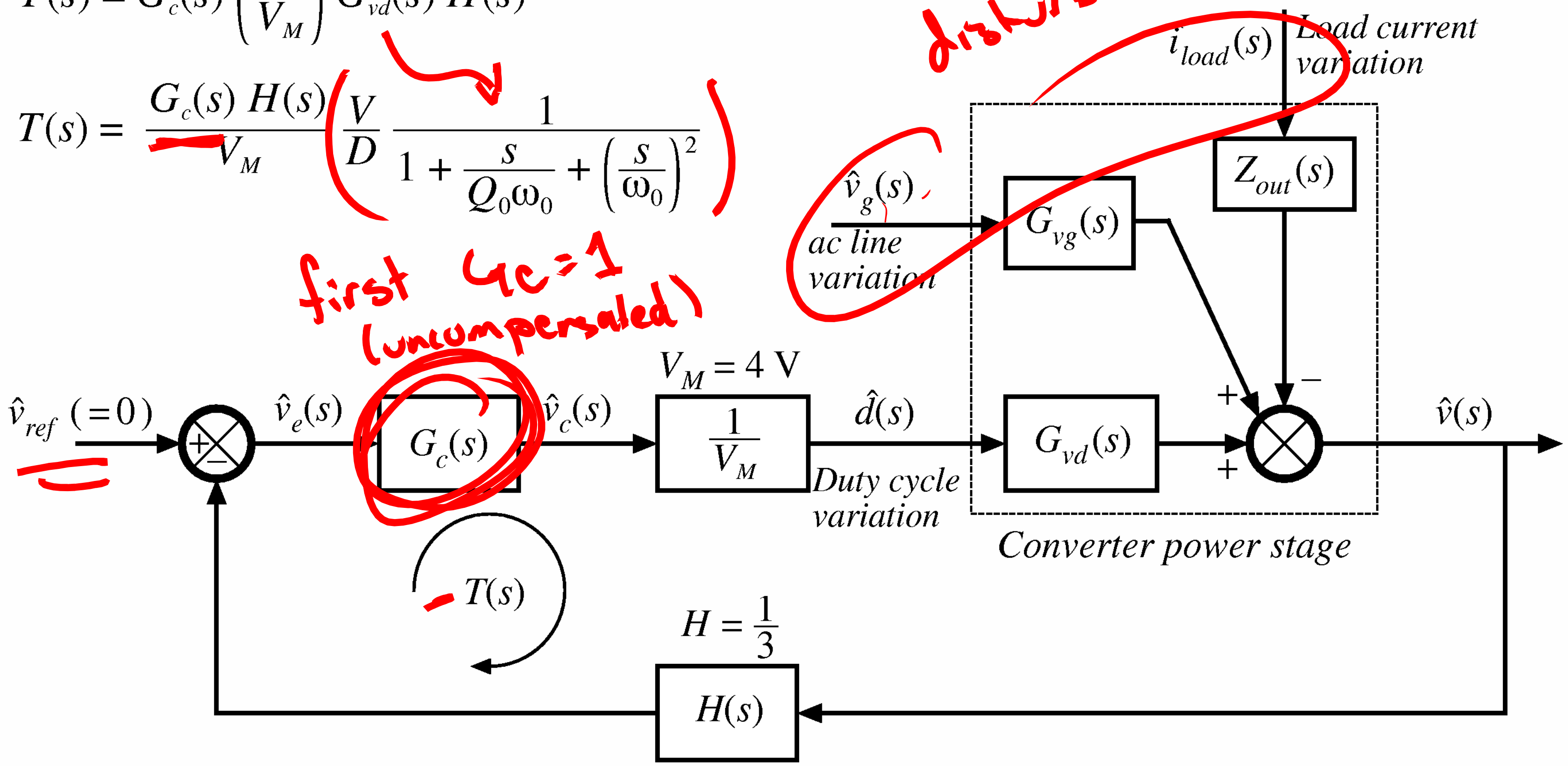
$$\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1+T} + \hat{v}_g G_{vg} \frac{1}{1+T} - \hat{i}_{load} Z_{out} \frac{1}{1+T}$$

$$\rightarrow T(s) = G_c(s)\left(\frac{1}{V_M}\right) G_{vd}(s)\, H(s)$$

$$T(s) = \frac{G_c(s)\, H(s)}{V_M} \left(\frac{V}{D} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \right)$$

first $G_c = 1$ (uncompensated)

disturbances



Plotting Uncompensated Loop Gain

(1) φ_m too low → ringing, overshoot → PD compensator

(2) LF gain too low → significant SS error → PI

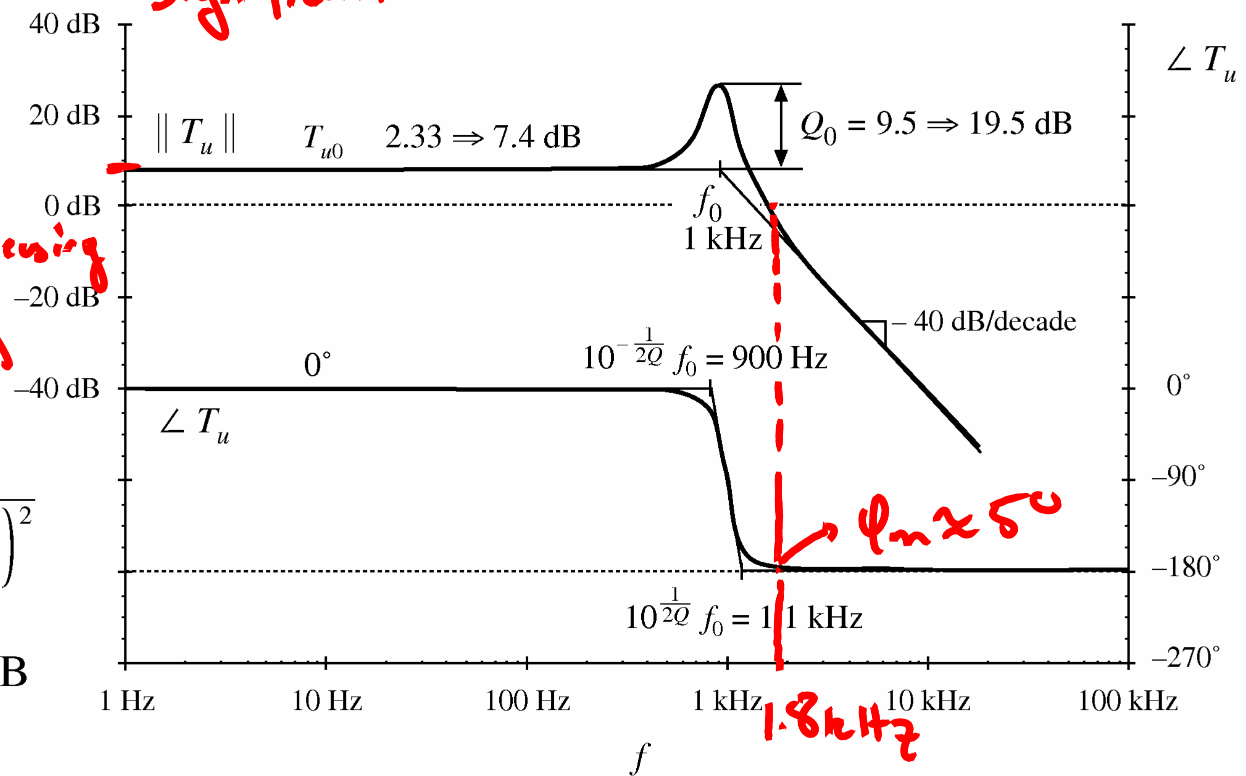
(3) $f_c \approx$ ok

→ can improve bandwidth by increasing (P)

With $G_c = 1$, the loop gain is

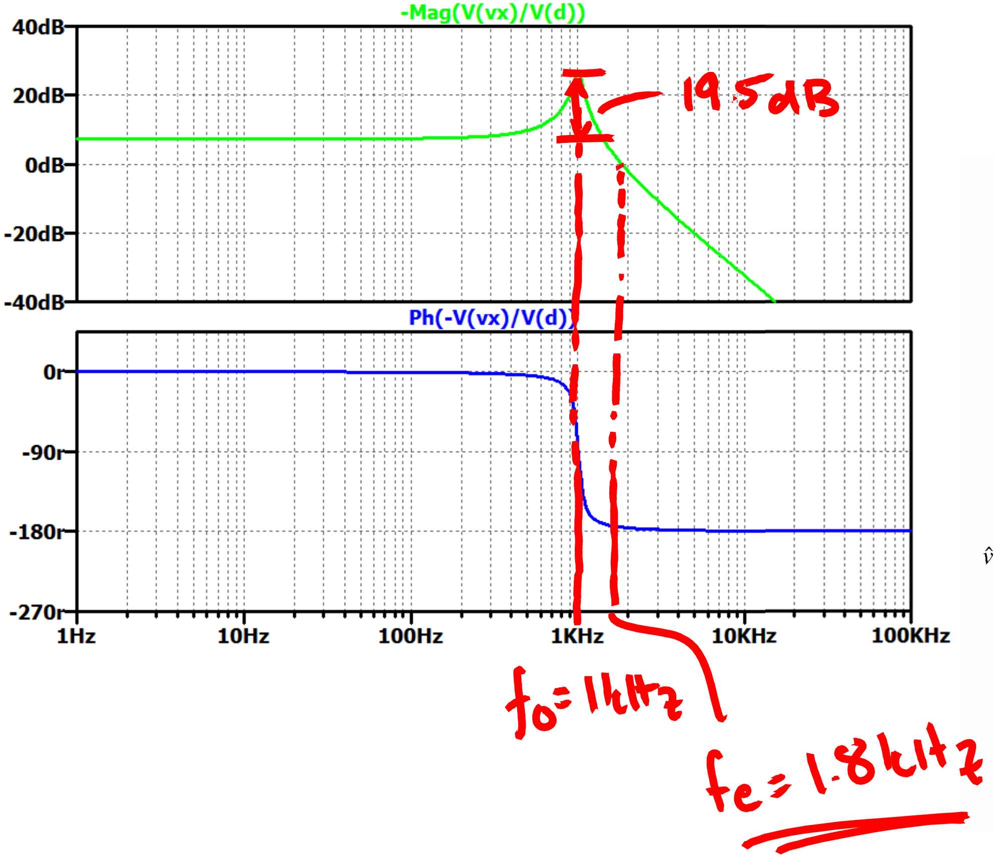
$$T_u(s) = T_{u0}\,\frac{1}{1+\frac{s}{Q_0\omega_0}+\left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H\,V}{D\,V_M} = 2.33 \Rightarrow 7.4\text{dB}$$



$f_c = 1.8$ kHz, $\varphi_m = 5°$

LTSpice Simulation – AC, Uncompensated



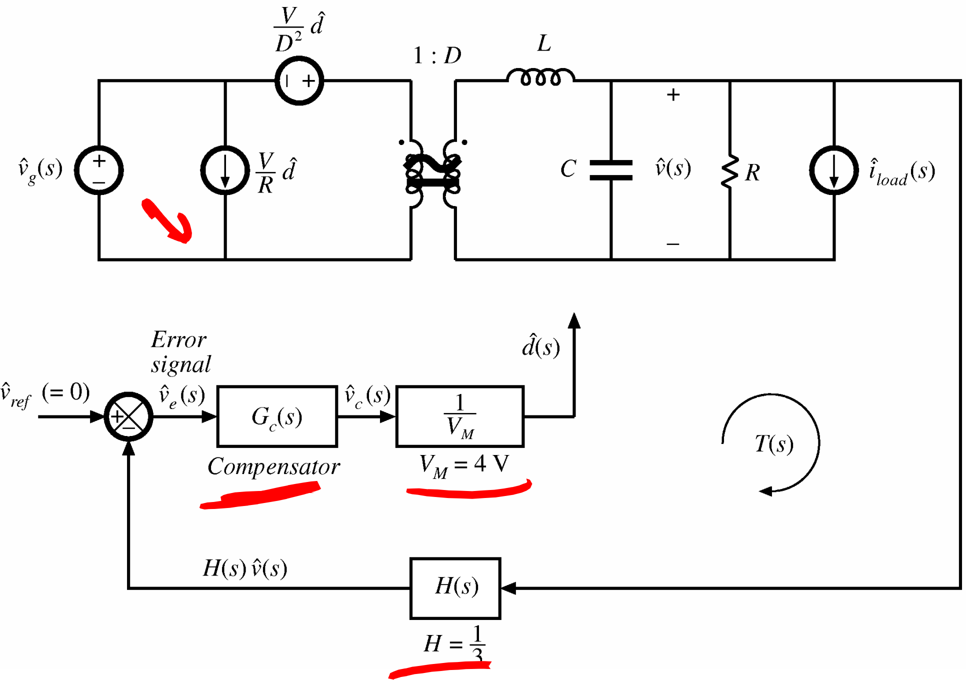
19.5dB

f_0 = 1kHz

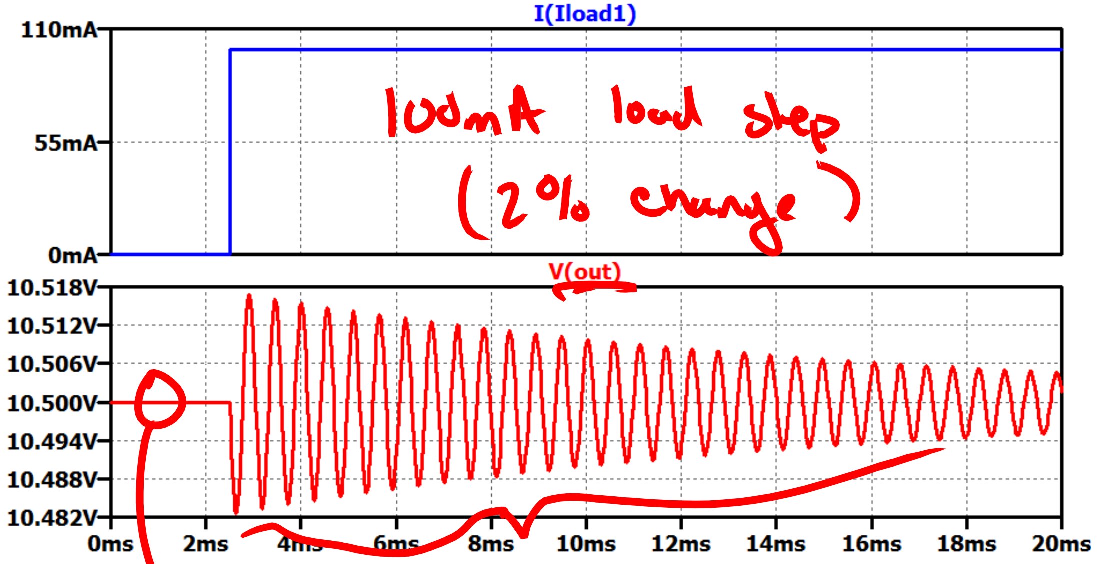
f_c = 1.8kHz

.param Vg = 28 V = 15 R = 3 D = .536
.param Vref = 5 H = 1/3 Vm = 4
.param L = 50u C = 500u

.lib myParts.lib
.ac dec 1000 1 1Meg



Transient Simulation, Uncompensated

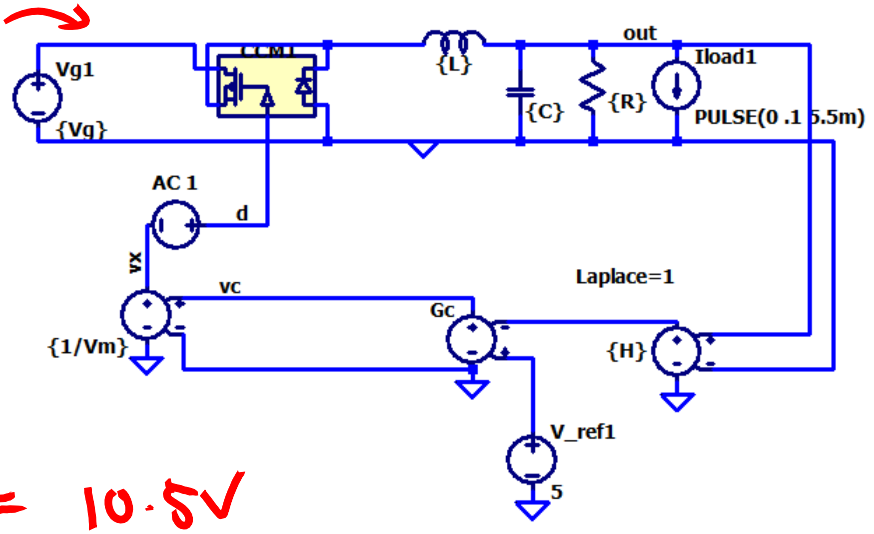


100mA load step (2% change)

```
.lib switch.lib
.lib myParts.lib
.tran 0 20m 3m
```

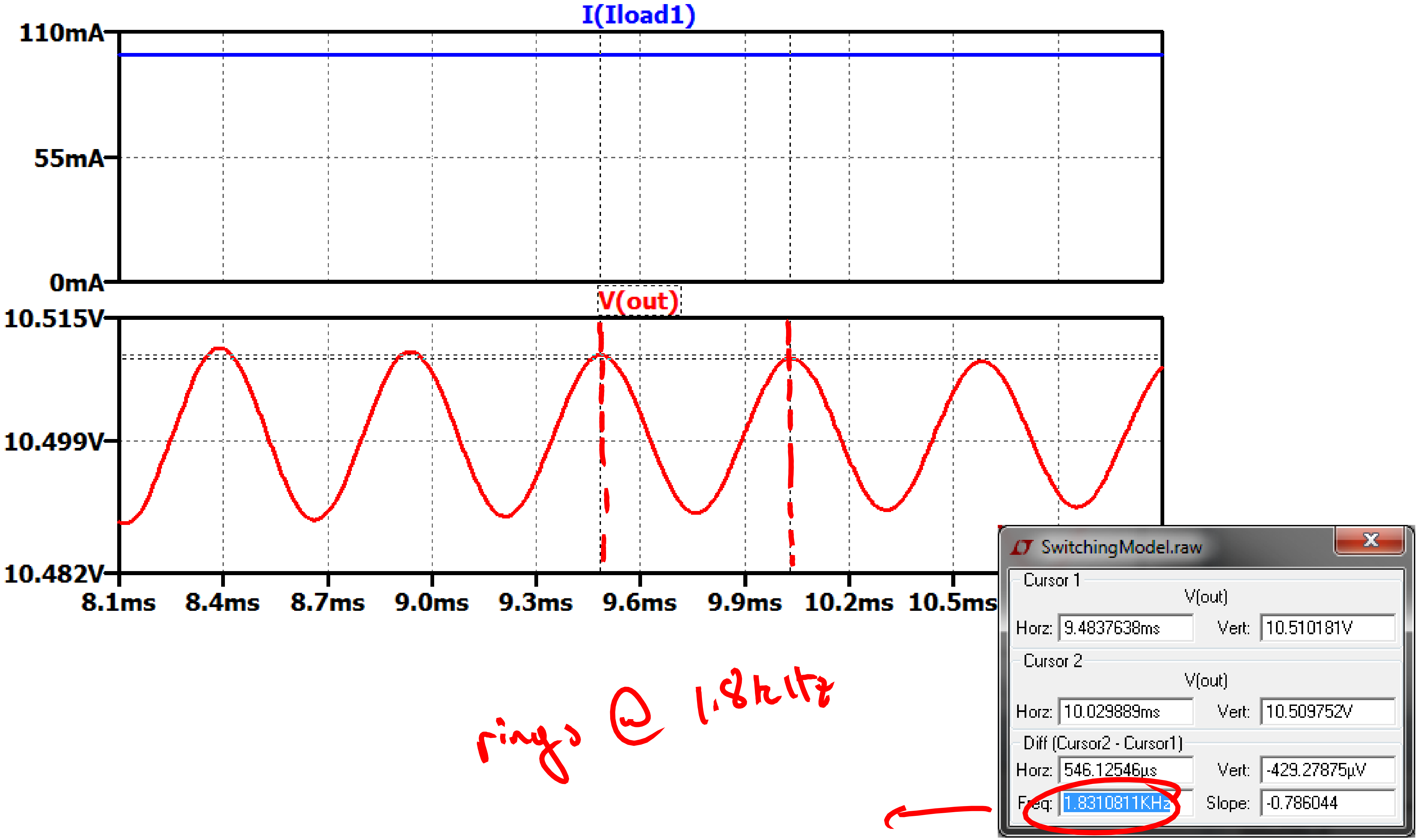
```
.param Vg = 28  V = 15  R = 3  D = .536
.param Vref = 5  H = 1/3  Vm = 4
.param L = 50u C = 500u

.ic V(out) = 15 I(L1) = 5 V(vc) = {D*Vm}
```



$$v = \frac{V_{ref}}{H} \frac{\|T_0\|}{1+\|T_0\|} = (15V)\frac{2.3}{1+2.3} \simeq 10.5V$$

Ringing Frequency



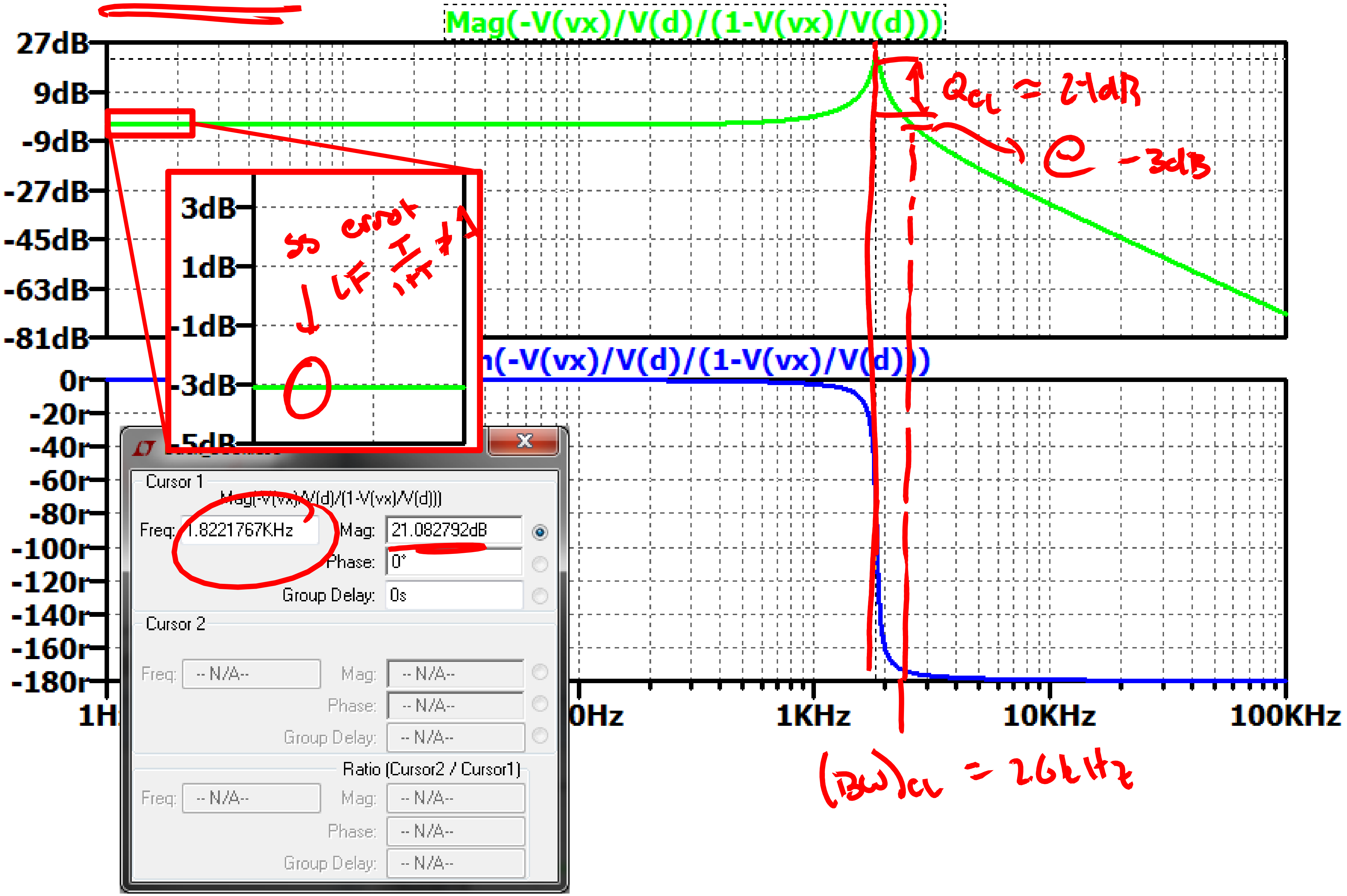
Cursor 1
V(out)
Horz: 9.4837638ms Vert: 10.510181V

Cursor 2
V(out)
Horz: 10.029889ms Vert: 10.509752V

Diff (Cursor2 - Cursor1)
Horz: 546.12546µs Vert: -429.27875µV
Freq: 1.8310811KHz Slope: -0.786044

rings @ 1.8kHz

T/(1+*T*)



Cursor 1
Mag(-V(vx)/V(d)/(1-V(vx)/V(d)))
Freq: 1.8221767KHz
Mag: 21.082792dB ●
Phase: 0° ○
Group Delay: 0s ○

Cursor 2
Freq: -- N/A--
Mag: -- N/A-- ○
Phase: -- N/A-- ○
Group Delay: -- N/A-- ○

Ratio (Cursor2 / Cursor1)
Freq: -- N/A--
Mag: -- N/A--
Phase: -- N/A--
Group Delay: -- N/A--

5% error

↓ LF

$\frac{T}{1+T} \approx 1$

$Q_{CL} \approx 21dB$

@ -3dB

$(BW)_{CL} = 2kHz$

Summary: Uncompensated Behavior

- Significant steady-state error
  - Need to increase low-frequency gain
- Barely stable; significant ringing
  - Need to increase ϕ_m
- Speed: ok
  - f_c = 1.8 kHz
  - $(BW)_{CL}$ = 2.6 kHz
  - OK for $f_s \approx$ 10 kHz or above

Compensator Design

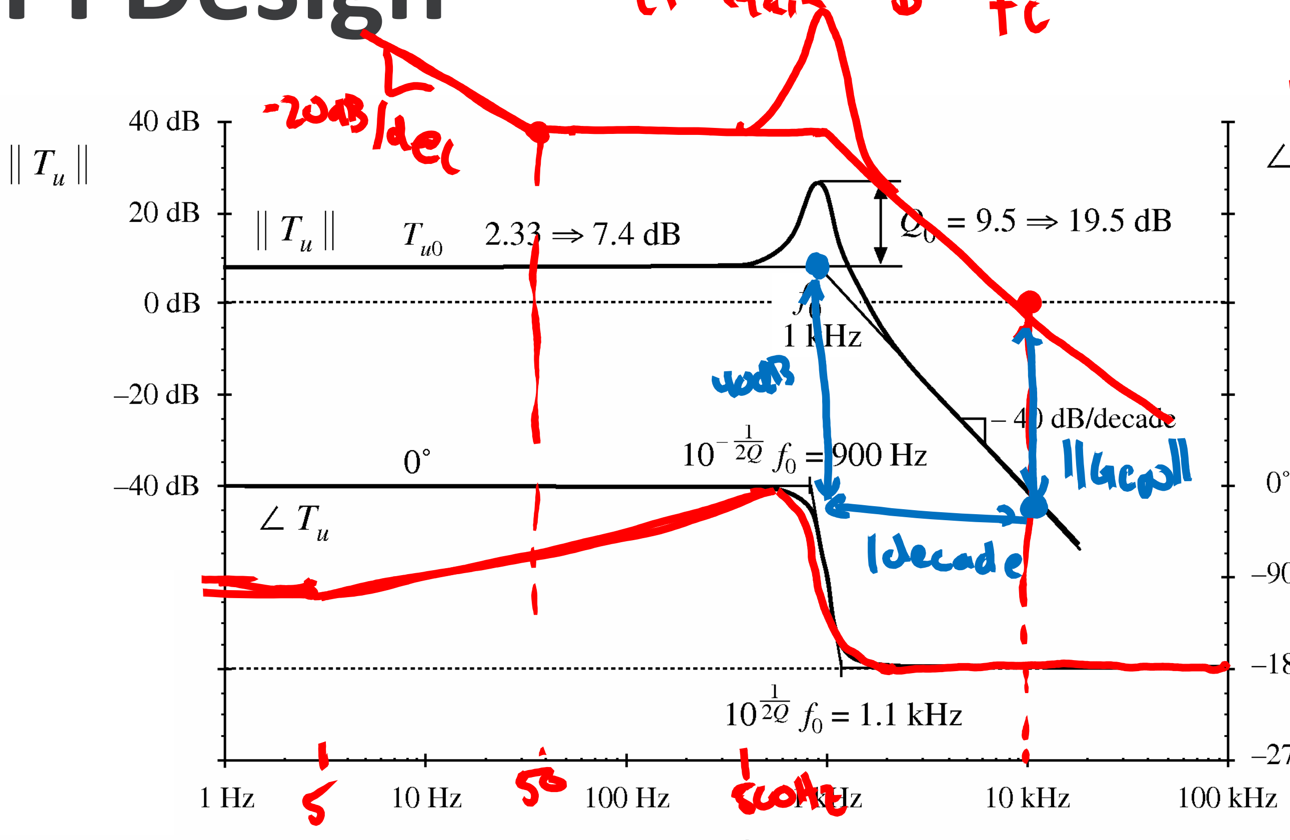
- As an example, try to
  - Increase f_c to 10 kHz
  - Increase ϕ_m to 76° (Q_{CL}=0.5)
  - Increase $\|T_0\|$ to ∞

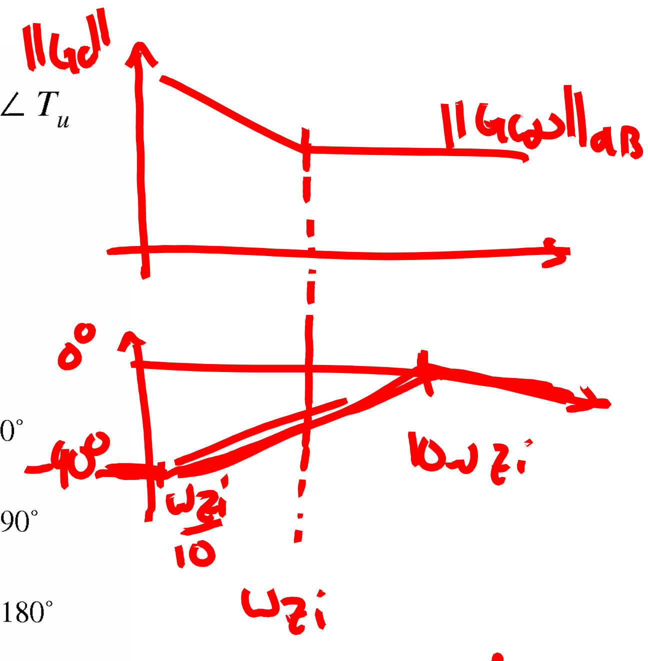
- Note: Book Chooses f_c = 5 kHz and ϕ_m =52° (Q=0.5)

PI Design

→ LF Gain & f_c

$G_c = G_{c\infty}\left(1 + \frac{\omega_{zi}}{s}\right)$





Need $f_{zi} < \frac{f_c}{10}$

select f_{zi} = 50 Hz

$\|G_{c\infty}\|_{dB}$ = 40dB − 7.4 dB = 32.6 dB = 42.6

PI compensator

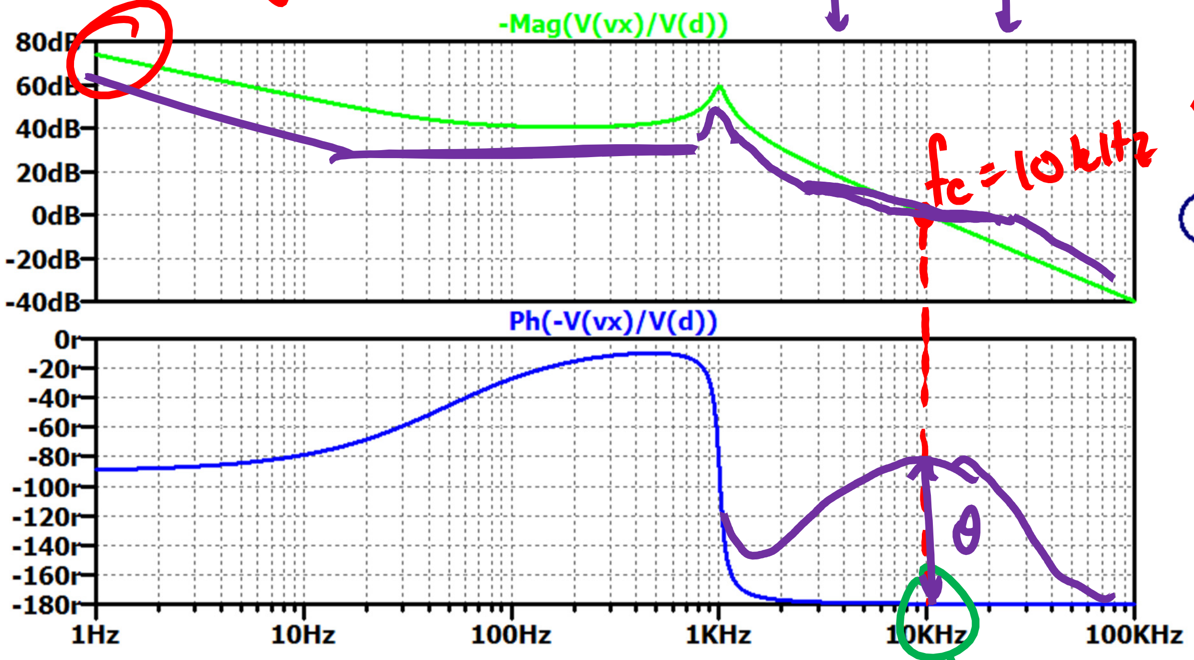
$$\boxed{G_{c,PI} = 42.6\left(1 + \frac{2\pi\, 50\text{Hz}}{s}\right)}$$

PI Simulation

Large LF gain ✓

fz

fp



fc = 10 kHz ✓

$\varphi_m \approx 0°$ X

```
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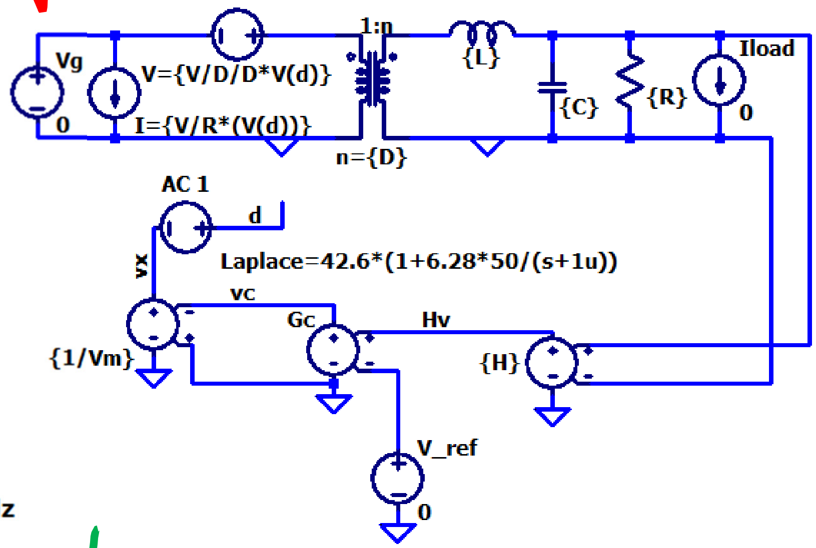
.param Vref = 5  H = 1/3  Vm = 4
.param L = 50u C = 500u
.lib myParts.lib
.ac dec 1000 1 1Meg
```



PD Design

$$f_z = f_c \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}}$$

$$f_p = f_c \sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}}$$

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$

$\varphi_{m\_now} = 0^0$

$\varphi_m = \theta + \varphi_{m\_now} = 76^0$

need $\theta = 76^0$

$f_c = 10\text{kHz}$

$\downarrow$

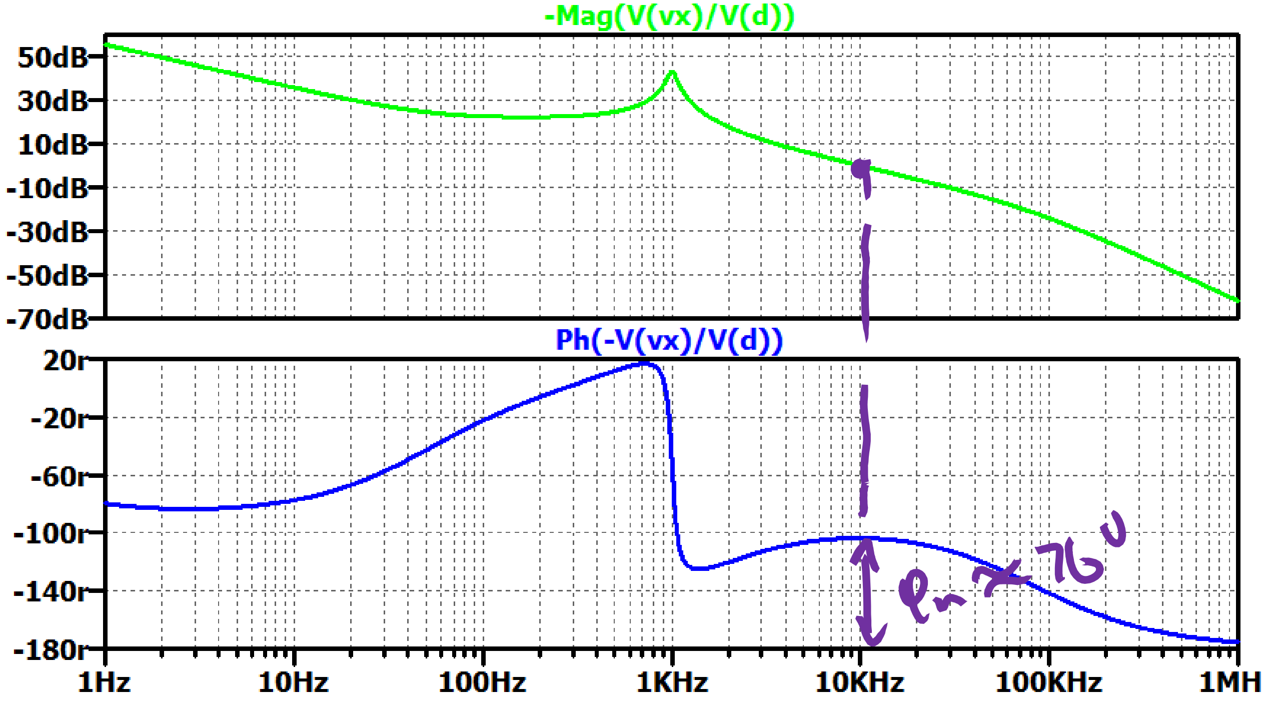
$f_z = 1.23\text{ kHz}$

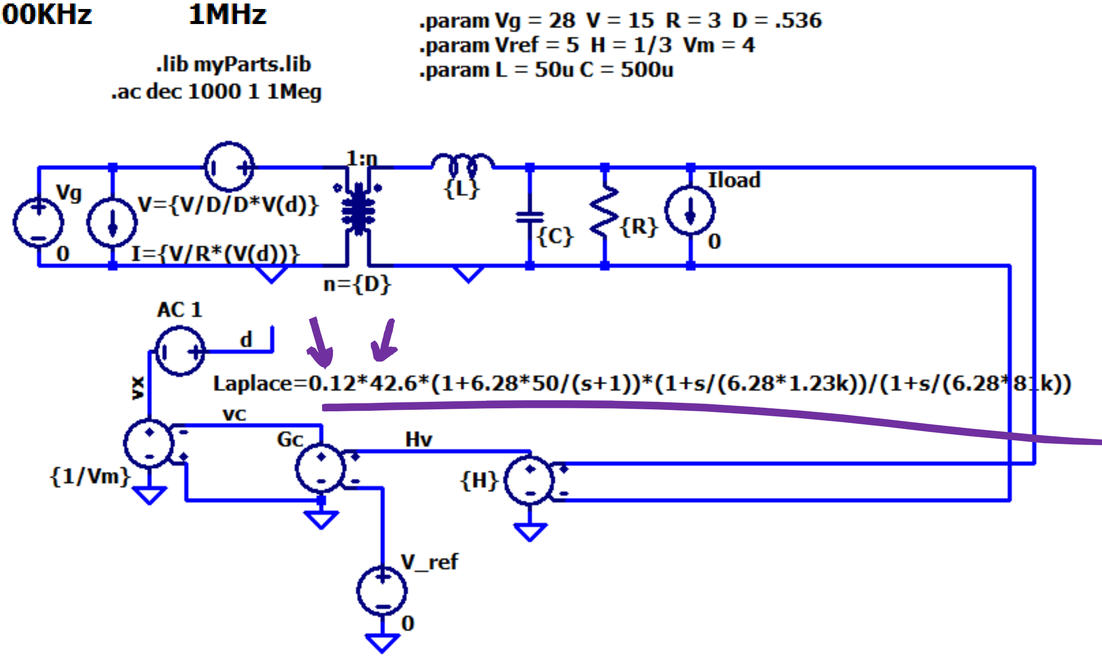
$f_p = 81\text{kHz}$

$G_{c0} = 0.12$

$$G_{cPD} = G_{c0} \frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_p}\right)}$$

PID Simulation





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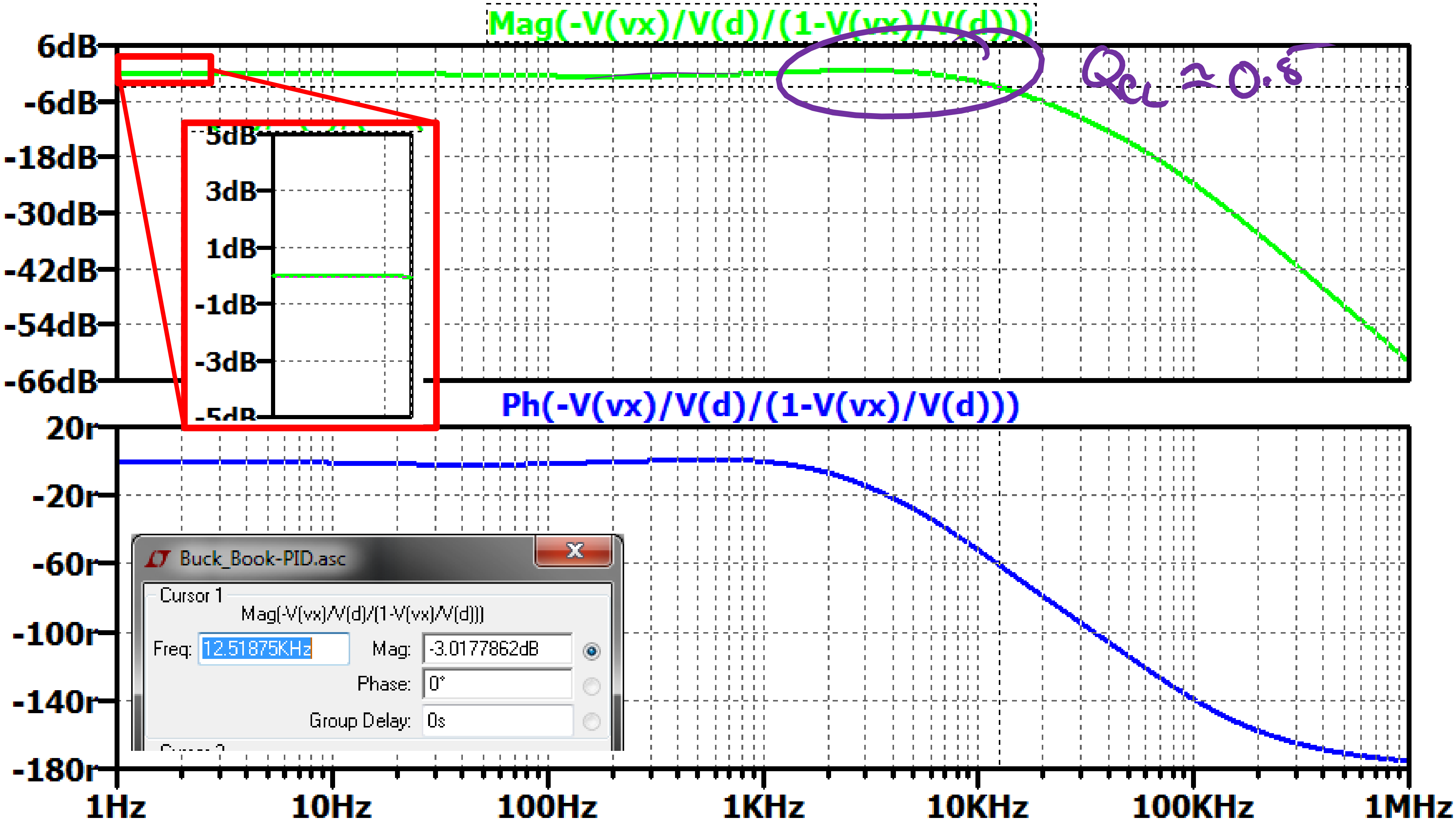
.lib myParts.lib
.ac dec 1000 1 1Meg
Laplace=0.12*42.6*(1+6.28*50/(s+1))*(1+s/(6.28*1.23k))/(1+s/(6.28*81k))
```

T/(1+*T*)



$Q_{CL} \approx 0.8$

Buck_Book-PID.asc

Cursor 1

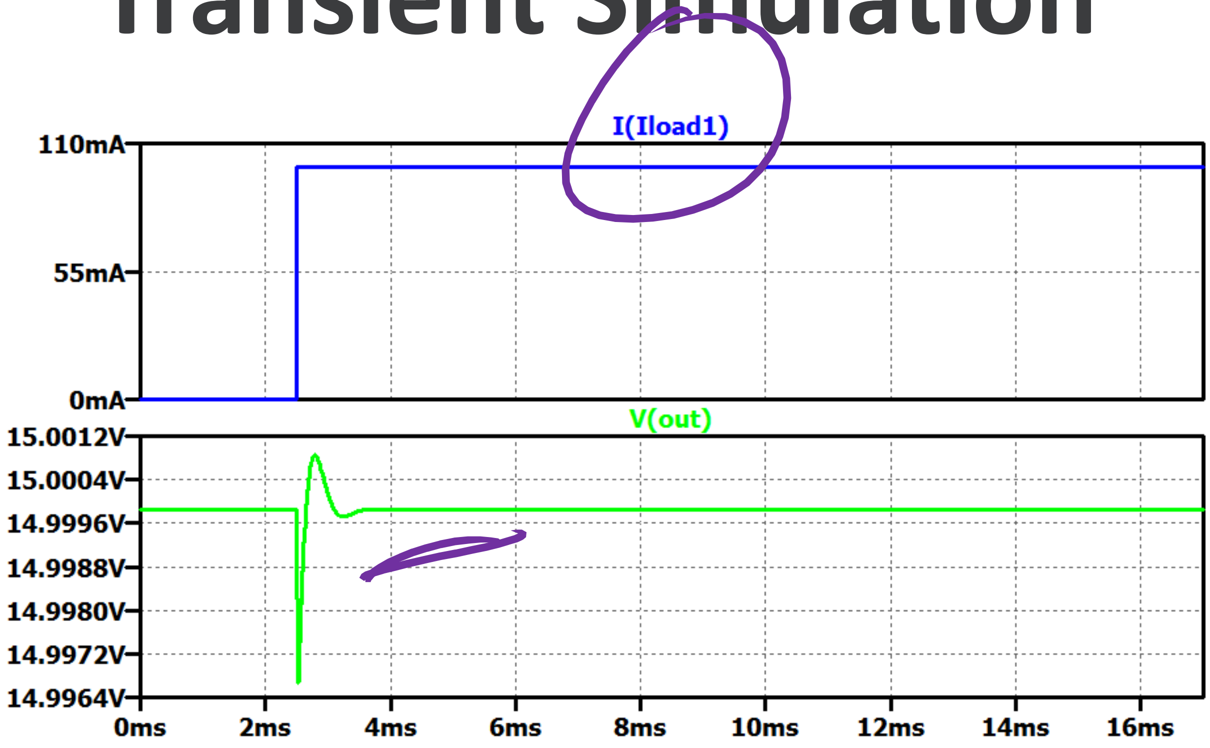
Mag(-V(vx)/V(d)/(1-V(vx)/V(d)))

Freq: 12.51875KHz Mag: -3.0177862dB ●

Phase: 0° ○

Group Delay: 0s ○

Transient Simulation



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