• Begin by solving important waveforms throughout converter assuming lossless operation
Power Stage Losses

Low-Frequency Losses
- $R_{on}$

Frequency-Dependent Losses
- $C_{oss}$
- Overlap
- $P_g$
- $T_{d\ cond.}$
- $C_d$
- Reverse-Recovery
- Skin Effect
- Core Loss
- Fringing
- Proximity
- Dielectric Losses

MOSFETS

Body Diodes
- $V_F$
- $R_d$

Inductor
- $R_{dc}$

Capacitors
- ESR
LOW FREQUENCY CONDUCTION LOSSES
• Considering only power stage losses (gate drive neglected)
• MOSFET operated as power switch
• Intrinsic body diode behaviors considered using normal diode analysis
Datasheet Interpretation

- On resistance extracted from datasheet waveforms
- Significantly dependent on $V_{gs}$ amplitude, temperature
• MOSFET conduction losses due to \((r_{ds})_{on}\) depend given as

\[
P_{\text{cond,FET}} = I_{di,rms}^2 (r_{ds})_{on}
\]
MOSFET Conduction Losses

Pulsating waveform with linear ripple, Fig. A.6:

\[ r_{ms} = I \sqrt{D} \sqrt{1 + \frac{1}{3} \left( \frac{\Delta i}{I} \right)^2} \]  

(A.6)

Fig. A.6

• RMS values of commonly observed waveforms appendix from Power Book
Capacitor Loss Model

- Operation well below resonance
- All loss mechanisms in a capacitor are generally lumped into an empirical loss model
- Equivalent Series Resistance (ESR) is *highly* frequency dependent
-Datasheets may give effective impedance at a frequency, or loss factor:

\[ \delta = \frac{\pi}{2} - \theta \]

\[ D = \tan(\delta) \]
DC Inductor Resistance

- DC Resistance given by
  \[ R_{DC} = \rho \frac{l_b}{A_w} \]
- At room temp, \( \rho = 1.724 \cdot 10^{-6} \Omega \cdot \text{cm} \)
- At 100°C, \( \rho = 2.3 \cdot 10^{-6} \Omega \cdot \text{cm} \)
- Losses due to DC current:
  \[ P_{cu,DC} = I_{L,rms}^2 R_{DC} \]
Inductor Conduction Losses

DC plus linear ripple, Fig. A.2:

\[ \text{rms} = I \sqrt{1 + \frac{1}{3} \left( \frac{\Delta i}{I} \right)^2} \]  \hspace{2cm} (A.2)

- Conduction losses dependent on RMS current through inductor
Switching Loss
Switching Loss Modeling

$V_{gs1}$

$V_{gs2}$

$V_{sw}$

$t$

$i_l$

$i_{gs1}$ > $\phi$

4Hs body diode conduction

6Hs body diode
Dead Time Selection

$P_{sw}$

$dt$

$t$

$vgs$

$vgs_1$

$vgs_2$

huge cross-conduction losses

safer

gradual increase - body diode conduction

$C_1$

$C_2$
Types of Switching Loss

1. Gate Charge Loss
2. Overlap Loss
3. Capacitive Loss
4. Body Diode Conduction
5. Reverse Recovery
6. Parasitic Inductive Losses
7. Anomalous Losses
Gate Drive Losses

- Gate charge is supplied through driver resistance during switch turn-on
- Gate charge is dissipated in gate driver on switch turn-off

\[ E_{\text{loss}} = q_{\text{gate}} V_{DD} \]

\[ P_{\text{sw},g} = E_{\text{loss}} f_s \]

*comes from \( V_{DD} \), not from \( V_g \)
Gate Charge Loss

9 Typ. gate charge

\( V_{GS} = f(Q_{gate}); \ I_D = 5.2 \text{ A pulsed} \)

Parameter: \( V_{DD} \)

\[ @ 10V, \ Q_g = 17nC \]

\[ P_g = Q_g V_{cc} f_s \]

during switching
Overlap Loss

\[ P_{\text{overlap}} = \frac{1}{2} I_L V \frac{t_{\text{sw}}}{T_s} \]

Energy Lost

\[ E_{\text{ov}} = \frac{1}{2} (I_L V_{\text{ov}})(t_{\text{ov}}) \]

Overlap

\[ V_{\text{ov}} = \frac{1}{2} (V_{\text{gs}} - V_{\text{ds}}) \]

Energy Charge

\[ Q_{\text{ch}1} = \int I_{\text{ov}}(t) \, dt \]

Gate Charge

\[ Q_{\text{gate}} = \int I_{\text{ov}}(t) \, dt \]

Typical Gate Charge

\[ V_{\text{GS}} = f(Q_{\text{gate}}); \quad I_c = 5.2 \text{ A pulsed} \]

Parameter: \( V_{\text{DD}} \)

\[ 0 \text{ Typ. gate charge} \]

\[ t_{\text{sw}} = t_{\text{ov}} \]

\[ t_{\text{on}} = t_{\text{ov}} \]

\[ t_{\text{off}} = t_{\text{ov}} \]

\[ I_{\text{ov}} = I_{\text{ov}} \]

\[ V_{\text{ov}} = V_{\text{ov}} \]

\[ P_{\text{ov}} = P_{\text{ov}} \]

\[ t_{\text{ov}} = t_{\text{ov}} \]

\[ t_{\text{on}} = t_{\text{ov}} \]

\[ t_{\text{off}} = t_{\text{ov}} \]
Lump Switched Node Capacitance

- Consider a single equivalent capacitor at switched node which combines energy storage due to all four semiconductor devices.
Diode Loss Model

- Example loss model includes resistance and forward voltage drop extracted from datasheet.
Diode Reverse Recovery

- FET body diodes may turn on during dead time intervals
- Significant reverse recovery losses possible

\[ E_{on,rr} = \left( \left( I_L - \Delta i_L \right) t_{rr} + Q_{rr} \right) V_{bus} \]
Reverse Recovery - Datasheet

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Characteristics</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Reverse recovery time</td>
<td>( t_{rr} )</td>
<td>- 140</td>
<td>ns</td>
</tr>
<tr>
<td>( V_R = 400V, I_F = 45A, )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{di_F}{dt} = 1000A/\mu s ), ( T_j = 25^\circ C )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_R = 400V, I_F = 45A, )</td>
<td></td>
<td>- 185</td>
<td></td>
</tr>
<tr>
<td>( \frac{di_F}{dt} = 1000A/\mu s ), ( T_j = 125^\circ C )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_R = 400V, I_F = 45A, )</td>
<td></td>
<td>- 195</td>
<td></td>
</tr>
<tr>
<td>( \frac{di_F}{dt} = 1000A/\mu s ), ( T_j = 150^\circ C )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reverse recovery charge</td>
<td>( Q_{rr} )</td>
<td>- 1400</td>
<td>nC</td>
</tr>
<tr>
<td>( V_R = 400V, I_F = 45A, )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{di_F}{dt} = 1000A/\mu s ), ( T_j = 25^\circ C )</td>
<td></td>
<td></td>
<td></td>
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<td>( V_R = 400V, I_F = 45A, )</td>
<td></td>
<td>- 2650</td>
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<tr>
<td>( \frac{di_F}{dt} = 1000A/\mu s ), ( T_j = 125^\circ C )</td>
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<td>( V_R = 400V, I_F = 45A, )</td>
<td></td>
<td>- 2900</td>
<td></td>
</tr>
<tr>
<td>( \frac{di_F}{dt} = 1000A/\mu s ), ( T_j = 150^\circ C )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 Typ. reverse recovery time
\( t_{rr} = f \left( \frac{di_F}{dt} \right) \)
parameter: \( V_R = 400V, T_j = 125^\circ C \)
Reverse Recovery – Rough Approximations

- \( E_{rr} \approx \frac{E_{rr_{\text{datasheet}}}}{I_{F_{\text{datasheet}}}} \frac{I_F}{V_{DC_{\text{datasheet}}}} \)

- \textbf{Rough} approximation with \( I_F \ll I_{max} \)
INDUCTOR AC LOSSES
Skin Effect in Copper Wire

- Current profile at high frequency is exponential function of distance from center with characteristic length $\delta$
AC Resistance

\[ A_{w,\text{eff}} = \pi r_w^2 - \pi (r_w - \delta)^2 \]

\[ R_{ac} = \rho \frac{l_b}{A_{w,\text{eff}}} \]
Skin Depth

\[ \delta = \sqrt{\frac{\rho}{\pi \mu f}} \]

**Fig. 13.23** Penetration depth \( \delta \), as a function of frequency \( f \), for copper wire.
Proximity Effect

- In foil conductor closely spaced with $h \gg \delta$, flux between layers generates additional current according to Lentz’s law.

\[ P_1 = I_{L, rms}^2 R_{ac} \]

- Power loss in layer 2:

\[ P_2 = I_{L, rms}^2 R_{ac} + (2I_{L, rms})^2 R_{ac} \]

\[ P_2 = 5P_1 \]

- Needs modification for non-foil conductors

See Fundamentals of Power Electronics, Section 13.4
Simulation Example

- AWG#30 copper wire
  - Diameter $d = 0.294$ mm
  - $d = \delta$ at around 50 kHz
- 1:1 transformer
  - Primary and secondary are the same, 30 turns in 3 layers
- Sinusoidal currents,
  $$I_{1\text{rms}} = I_{2\text{rms}} = 1 \text{ A}$$

Numerical field and current density solutions using FEMM (Finite Element Method Magnetics), a free 2D solver, http://www.femm.info/wiki/HomePage
Frequency: 1 kHz

Flux density

Current Density
Frequency: 100 kHz

Flux density

Current Density

Total copper losses 1.8 larger than at 1 kHz
Frequency: 1 MHz

Total copper losses 20 times larger than at 1 kHz
Frequency: 10 MHz

Very significant proximity effect
Total copper losses = 65 times larger than at 1 KHz
Fringing

- Near air gap, flux may bow out significantly, causing additional eddy current losses in nearby conductors.
Physical Origin of Core Loss

- Magnetic material is divided into “domains” of saturated material
- Both Hysteresis and Eddy Current losses occur from domain wall shifting

Reinert, J.; Brockmeyer, A.; De Doncker, R.W.; , "Calculation of losses in ferro- and ferrimagnetic materials based on the modified Steinmetz equation,"
Inductor Core Loss

- Governed by Steinmetz Equation:

\[ P_v = K_{fe} f_s^\alpha (\Delta B)^\beta \text{ [mW/cm}^3\text{]} \]

- Parameters \( K_{fe}, \alpha, \) and \( \beta \) extracted from manufacturer data

\[ P_{fe} = P_v A_c l_m \text{ [mW]} \]

- \( \Delta B \propto \Delta i_L \rightarrow \text{small losses with small ripple} \)
Steinmetz Parameter Extraction

Fig. 6 Specific power loss as a function of peak flux density with frequency as a parameter.

Fig. 7 Specific power loss for several frequency/flux density combinations as a function of temperature.
Ferroxcube Curve Fit Parameters

Power losses in our ferrites have been measured as a function of frequency \((f\text{ in Hz})\), peak flux density \((B\text{ in T})\) and temperature \((T\text{ in °C})\). Core loss density can be approximated \(^{(2)}\) by the following formula:

\[
P_{\text{core}} = C_m \cdot f^x \cdot B_{\text{peak}}^y \cdot (ct_0 - ct_1 T + ct_2 T^2) \quad [3]
\]

\[
= C_m \cdot C_T \cdot f^x \cdot B_{\text{peak}}^y \quad [\text{mW/cm}^3]
\]

<table>
<thead>
<tr>
<th>ferrite</th>
<th>(f) (kHz)</th>
<th>(C_m)</th>
<th>(x)</th>
<th>(y)</th>
<th>(ct_2)</th>
<th>(ct_1)</th>
<th>(ct_0)</th>
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<tr>
<td>3C30</td>
<td>20-100</td>
<td>7.13 \times 10^{-3}</td>
<td>1.42</td>
<td>3.02</td>
<td>3.65 \times 10^{-4}</td>
<td>6.65 \times 10^{-2}</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>100-200</td>
<td>7.13 \times 10^{-3}</td>
<td>1.42</td>
<td>3.02</td>
<td>4.1 \times 10^{-4}</td>
<td>6.8 \times 10^{-2}</td>
<td>3.8</td>
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<td>1.46</td>
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<td>1.65 \times 10^{-4}</td>
<td>3.1 \times 10^{-2}</td>
<td>2.45</td>
</tr>
<tr>
<td>3C94</td>
<td>20-200</td>
<td>2.37 \times 10^{-3}</td>
<td>1.46</td>
<td>2.75</td>
<td>1.65 \times 10^{-4}</td>
<td>3.1 \times 10^{-2}</td>
<td>2.45</td>
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<tr>
<td></td>
<td>200-400</td>
<td>2.1 \times 10^{-9}</td>
<td>2.6</td>
<td>2.75</td>
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<td>3.1 \times 10^{-2}</td>
<td>2.45</td>
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<tr>
<td>3F3</td>
<td>100-300</td>
<td>0.25 \times 10^{-3}</td>
<td>1.63</td>
<td>2.45</td>
<td>0.79 \times 10^{-4}</td>
<td>1.05 \times 10^{-2}</td>
<td>1.26</td>
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<tr>
<td></td>
<td>300-500</td>
<td>2.1 \times 10^{-5}</td>
<td>1.8</td>
<td>2.5</td>
<td>0.77 \times 10^{-4}</td>
<td>1.05 \times 10^{-2}</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>500-1000</td>
<td>3.6 \times 10^{-9}</td>
<td>2.4</td>
<td>2.25</td>
<td>0.67 \times 10^{-4}</td>
<td>0.81 \times 10^{-2}</td>
<td>1.14</td>
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<tr>
<td>3F4</td>
<td>500-1000</td>
<td>12 \times 10^{-4}</td>
<td>1.75</td>
<td>2.9</td>
<td>0.95 \times 10^{-4}</td>
<td>1.1 \times 10^{-2}</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>1000-3000</td>
<td>1.1 \times 10^{-11}</td>
<td>2.8</td>
<td>2.4</td>
<td>0.34 \times 10^{-4}</td>
<td>0.01 \times 10^{-2}</td>
<td>0.67</td>
</tr>
</tbody>
</table>

*Table 1: Fit parameters to calculate the power loss density*
\[ P_{NSE} = \left( \frac{\Delta B}{2} \right)^{2-\alpha} \frac{k_N}{T} \int_{0}^{T} \left| dB \right|^{\alpha} dt \]

\[ k_N = \frac{k}{(2\pi)^{\alpha-1} \int_{0}^{2\pi} \left| \cos \theta \right|^{\alpha} d\theta} \]

Simple Formula for Square-wave voltages:

\[ P_{NSE} = k_N (2f)^{\alpha} (\Delta B)^{\beta} \left( D^{1-\alpha} + (1-D)^{1-\alpha} \right) \] (10)

where \( f \) is the operating frequency;
\( \Delta B/2 \) is the peak induction;
\( D \) is the duty ratio of the square wave voltage.

Note: The second and third harmonics are dominant at moderate values of duty ratio \( D \). For extreme values of \( D \) (95%), a higher value of \( \alpha \) could give better matching to the actual losses.

INDUCTOR DESIGN
Inductor Design

Freedoms:
1. Core Size and Material
2. Number of turns and wire gauge
3. Length of Air Gap

Constraints:
1. Obtain Designed $L$
2. Prevent Saturation
3. Minimize Losses
Minimization of Losses

• For given core, number of turns can be used to index possible designs, with air gap solved after (and limited) to get correct inductance

• A minimum sum of the two exists and can be solved

• Design always subject to constraint $B_{\text{max}} < B_{\text{sat}}$
Spreadsheet Design

- Use of spreadsheet permits simple iteration of design
- Can easily change core, switching frequency, loss constraints, etc.
Matlab (Programmatic) Design

- Matlab, or similar, permits more powerful iteration and plotting/insight into design variation
Closed-Form Design Methods

- Fundamentals of Power Electronics Ch 13-15
  - Step-by-Step design methods
  - Simplified, and may require additional calculations
**$K_g$ and $K_{gfe}$ Methods**

- Two closed-form methods to solve for the optimal inductor design *under certain constraints/assumptions*
- Neither method considers losses other than DC copper and (possibly) steinmetz core loss
- Both methods particularly well suited to spreadsheet/iterative design procedures

<table>
<thead>
<tr>
<th></th>
<th>$K_g$</th>
<th>$K_{gfe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Losses</td>
<td>DC Copper (specified)</td>
<td>DC Copper, SE Core Loss (optimized)</td>
</tr>
<tr>
<td>Saturation</td>
<td>Specified</td>
<td>Checked After</td>
</tr>
<tr>
<td>$B_{max}$</td>
<td>Specified</td>
<td>Optimized</td>
</tr>
</tbody>
</table>
$K_g$ Method

- Method useful for filter inductors where $\Delta B$ is small
- Core loss is not included, but may be significant particularly if large ripple is present
- Copper loss is specified through a set target resistance
- The desired $B_{max}$ is given as a constraint
- Method does not check feasibility of design; must ensure that air gap is not extremely large or wire size excessively small
- Simple first-cut design technique; useful for determining approximate core size required
- Step-by-step design procedure included on website
The following quantities are specified, using the units:

- Wire resistivity \( \rho \) (\( \Omega \)-cm)
- Peak winding current \( I_{\text{max}} \) (A)
- Inductance \( L \) (H)
- Winding resistance \( R \) (\( \Omega \))
- Winding fill factor \( K_u \)
- Core maximum flux density \( B_{\text{max}} \) (T)

The core dimensions are expressed in cm:

- Core cross-sectional area \( A_c \) (cm\(^2\))
- Core window area \( W_A \) (cm\(^2\))
- Mean length per turn \( MLT \) (cm)

The equations for the quantities are:

\[
K_g \geq \frac{\rho L^2 I_{\text{max}}^2}{B_{\text{max}}^2 K_u R} \times 10^8 \quad (\text{cm}^5)
\]

\[
\ell_g = \frac{\mu_0 L I_{\text{max}}^2}{B_{\text{max}}^2 A_c} \times 10^4 \quad (\text{m})
\]

\[
n = \frac{L I_{\text{max}}}{B_{\text{max}} A_c} \times 10^4
\]

\[
A_w \leq \frac{K_u W_A}{n} \quad (\text{cm}^2)
\]

\[
R = \frac{\rho n (MLT)}{A_w} \quad (\Omega)
\]
$K_{gfe}$ Method

- Method useful for cases when core loss and copper loss are expected to be significant
- Saturation is not included in the method, rather it must be checked afterward
- Enforces a design where the sum of core and copper is minimized
The following quantities are specified, using the units noted:

- Wire effective resistivity: \( \rho \) (\( \Omega \)-cm)
- Total rms winding current, ref to pri: \( I_{tot} \) (A)
- Desired turns ratios: \( n_2/n_1, \; n_3/n_1, \) etc.
- Applied pri volt-sec: \( \lambda_1 \) (V-sec)
- Allowed total power dissipation: \( P_{tot} \) (W)
- Winding fill factor: \( K_u \)
- Core loss exponent: \( \beta \)
- Core loss coefficient: \( K_{fe} \) (W/cm\(^3\)T\(^\beta\))

Other quantities and their dimensions:

- Core cross-sectional area: \( A_c \) (cm\(^2\))
- Core window area: \( W_A \) (cm\(^2\))
- Mean length per turn: \( MLT \) (cm)
- Magnetic path length: \( \ell_e \) (cm)
- Wire areas: \( A_{w1}, \ldots \) (cm\(^2\))
- Peak ac flux density: \( \Delta B \) (T)
\[ K_{gfe} = \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u(P_{tot})^{((\beta + 2)/\beta)}} 10^8 \]

\[ \Delta B = \left[ 10^8 \frac{\rho \lambda_1^2 I_{tot}^2}{2K_u} \frac{(MLT)}{W_A A_c^3 \ell_m} \frac{1}{\beta K_{fe}} \right]^{\frac{1}{\beta + 2}} \]

\[ n_1 = \frac{\lambda_1}{2\Delta BA_c} 10^4 \quad n_k = n_1 \frac{n_k}{n_1} \]

\[ \alpha_k = \frac{n_k I_k}{n_1 I_{tot}} \quad A_{wk} \leq \frac{\alpha_2 K_u W_A}{n_2} \]

Verify
\( K_{gfe} \) Method: Summary

- Method enforces an operating \( \Delta B \) in which core and copper losses are minimized
- Only takes into account losses from standard Steinmetz equation; not correct unless waveforms are sinusoidal
- Does not consider high frequency losses
- Step-by-step design procedure included on website