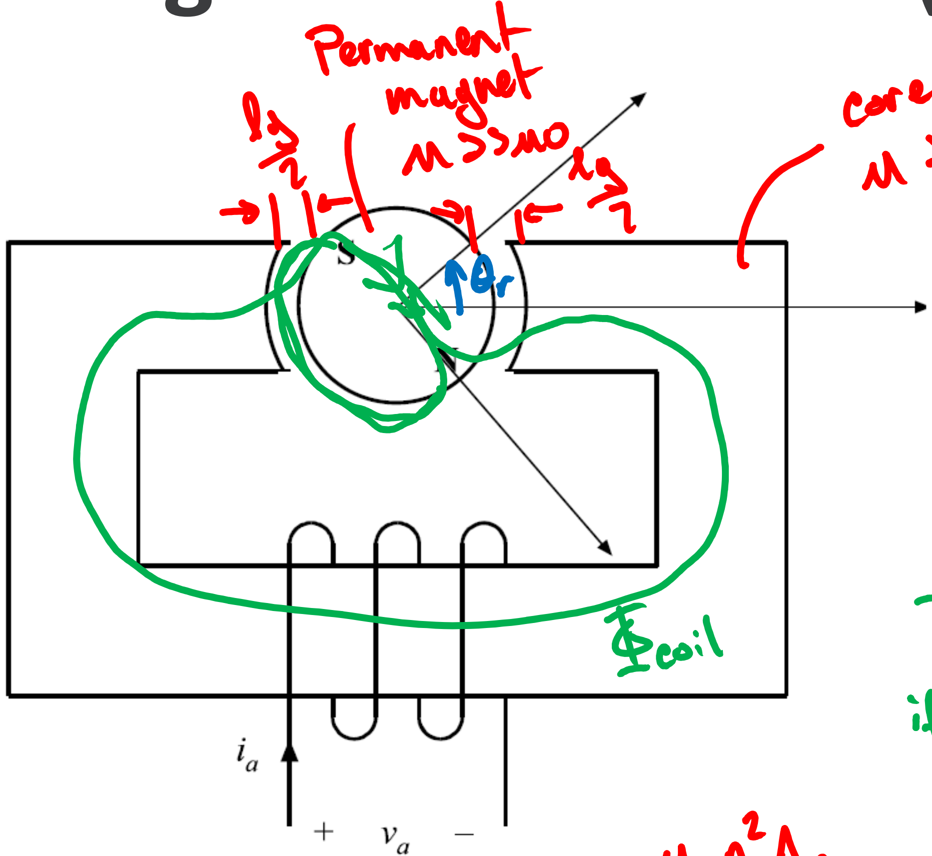
Single Phase Motor (Simplified)



Assume magnet produces a maximum flux Φ_m

$$\Phi_{coil} = f(\theta_r)\,\Phi_m$$

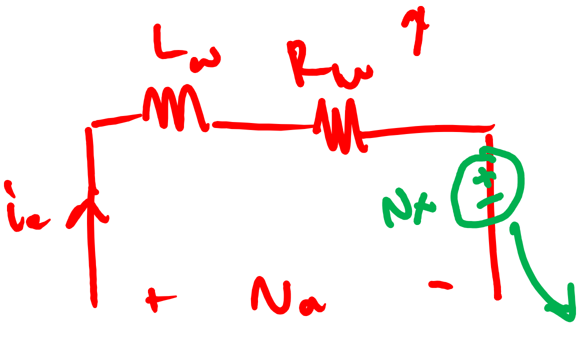
$$-1 < f(\theta_r) < 1$$

if magnet spins $\theta_r \Rightarrow \theta_r(t)$

winding induced voltage

$$v_x = n\frac{d\Phi_{coil}}{dt}$$

$$v_x = n\Phi_m \frac{d}{dt} f(\theta_r)$$



back EMF

$$L_w = \frac{\mu_0 n^2 A_c}{l_g}$$

$$R_w = \rho \frac{n\,MLT}{A_w}$$

Electromechanical Conversion

$v_x = n \Phi_m \frac{d}{dt} f(\theta_r)$

$\lambda_m = n \Phi_m$ = "flux linkage"

Guess $f(\theta_r) = \sin\theta_r$

$v_x = \lambda_m \cos\theta_r \frac{d\theta_r}{dt}$

Assume $\frac{d\theta_r}{dt} = \text{const} \Rightarrow \omega_r$

$v_x = \omega_r \lambda_m \cos\theta_r$

Look at $P_a = v_a \cdot i_a$

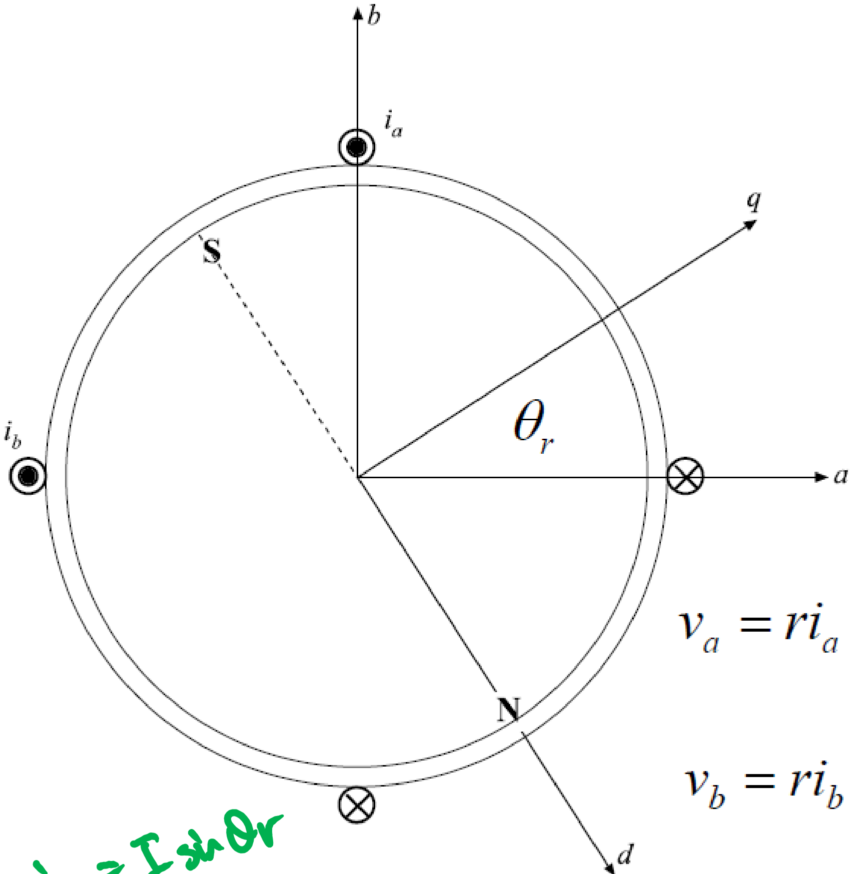
$P_a = i_a^2 R_w + i_a L_w \frac{di_a}{dt} + i_a \omega_r \lambda_m \cos\theta_r$

- $i_a^2 R_w$ → conduction loss
- $i_a L_w \frac{di_a}{dt}$ → reactive power
- $i_a \omega_r \lambda_m \cos\theta_r$ → mechanical power

torque goes to zero due to $\cos\theta_r$

$P_{mech} = \tau_r \omega_r = i_a \omega_r \lambda_m \cos\theta_r \rightarrow \tau_r = i_a \lambda_m \cos\theta_r$

2-Pole, 2-Phase PMSM



Two-pole, two-phase PMSM terminal characteristics in stator reference frame

$$\lambda_a(\theta_r) = \lambda_M \sin(\theta_r)$$

$$\lambda_b(\theta_r) = -\lambda_M \cos(\theta_r)$$

same

$$v_a = ri_a + \frac{d\lambda_a}{dt} = ri_a + L\frac{di_a}{dt} + \lambda_M \omega_r \cos(\theta_r)$$

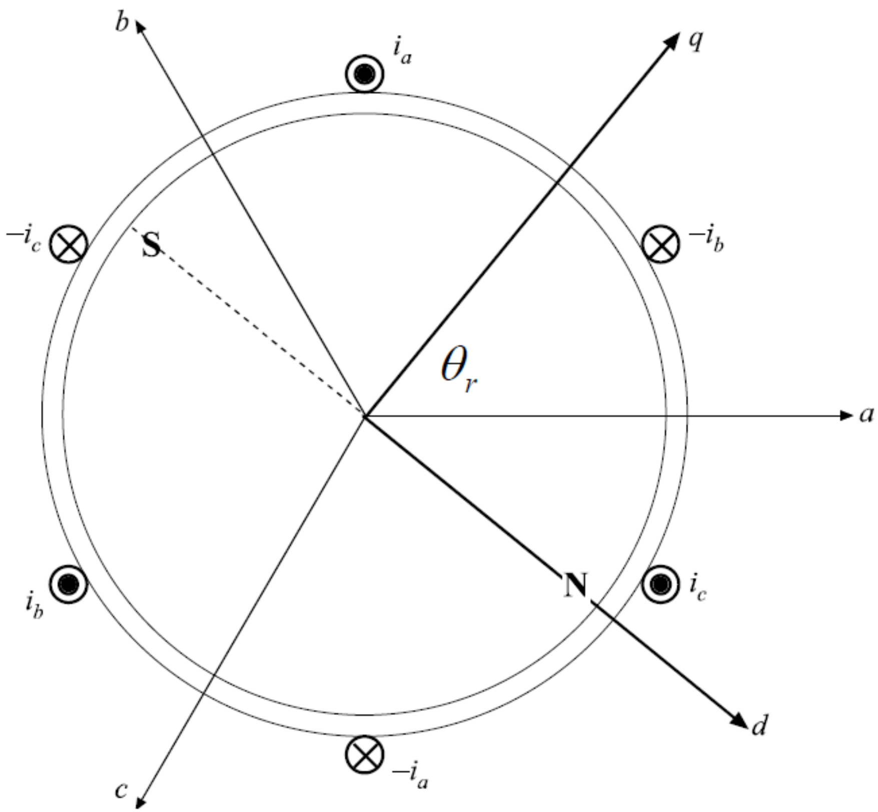
$$v_b = ri_b + \frac{d\lambda_b}{dt} = ri_b + L\frac{di_b}{dt} + \lambda_M \omega_r \sin(\theta_r)$$

same

$$T_m = \lambda_M \left(i_a \cos(\theta_r) + i_b \sin(\theta_r)\right)$$

ex
set $i_a = I \sin\theta_r$
$i_b = I \cos\theta_r$

3-Phase, 2-Pole PMSM



$$\lambda_a(\theta_r) = \lambda_m \sin(\theta_r)$$

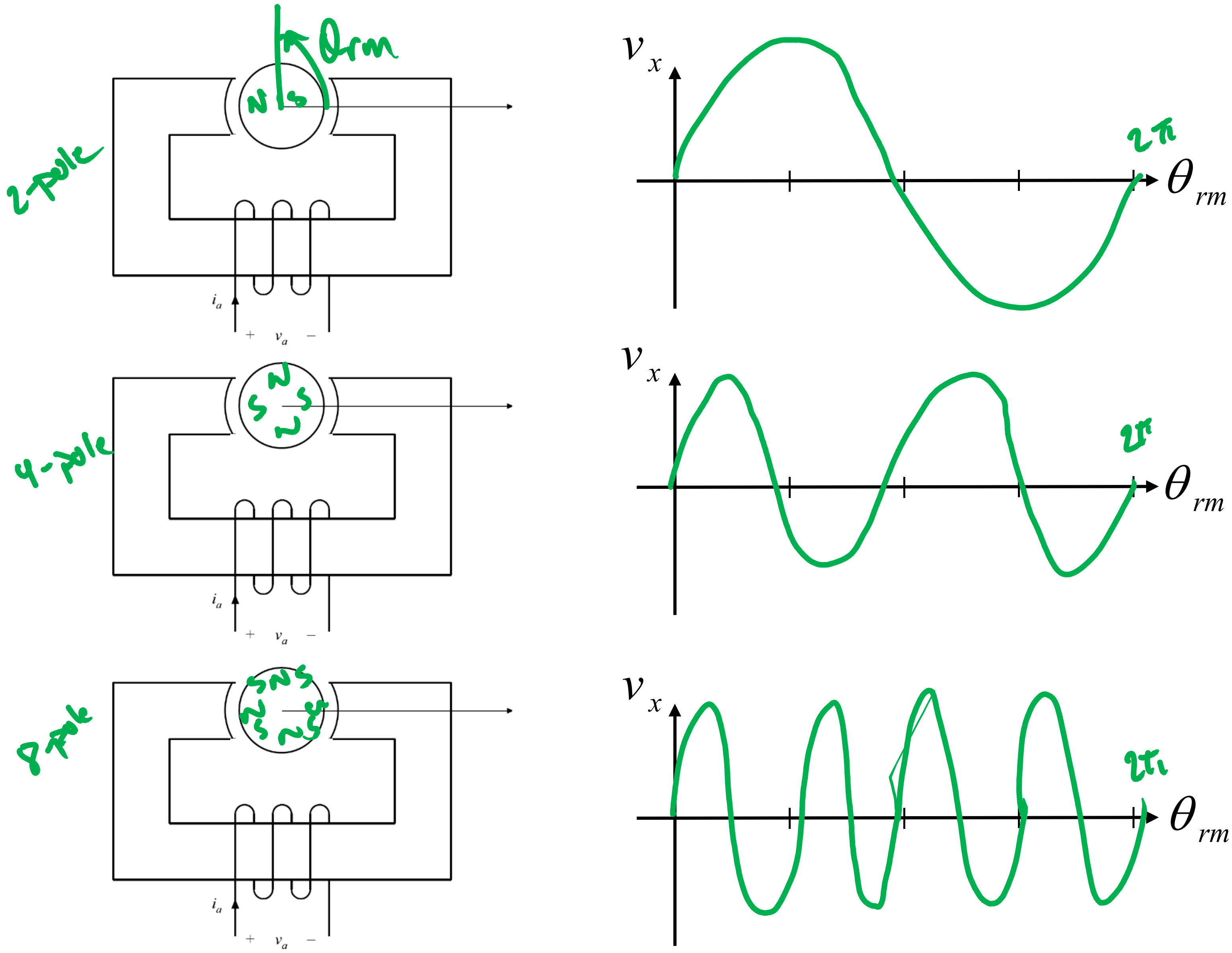
$$\lambda_b(\theta_r) = \lambda_m \sin\left(\theta_r - \frac{2\pi}{3}\right)$$

$$\lambda_c(\theta_r) = \lambda_m \sin\left(\theta_r - \frac{4\pi}{3}\right)$$

$$T_m = i_a \lambda_m \omega_r \cos(\theta_r) + i_b \lambda_m \omega_r \cos\left(\theta_r - \frac{2\pi}{3}\right) + i_c \lambda_m \omega_r \cos\left(\theta_r - \frac{4\pi}{3}\right)$$

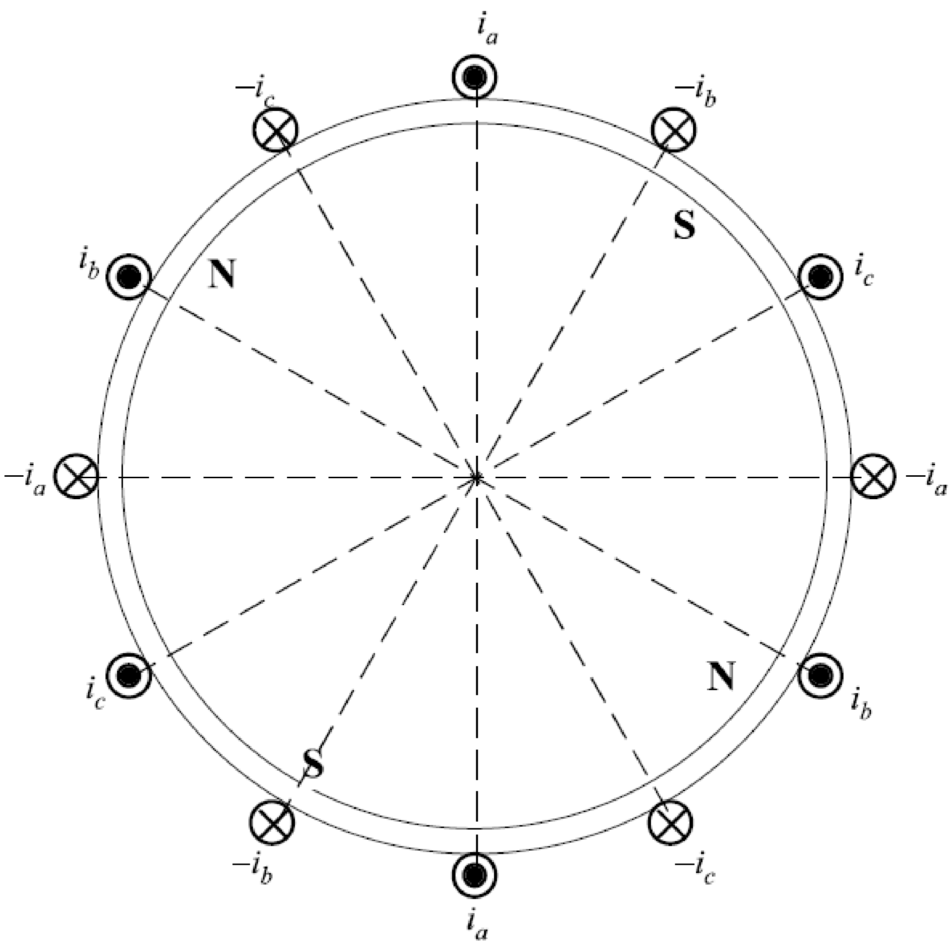
$T = \frac{3}{2} \lambda_m I$

Different Number of Poles



3-Phase, P-Pole PMSM

P = 4 example



Electrical and mechanical angle

$$\theta_r = \frac{P}{2}\theta_{rm}$$

Electrical and mechanical speed

$$\omega_r = \frac{P}{2}\omega_{rm}$$

Max torque per amp

$$T_m \le \lambda_m \frac{P}{2}\frac{3}{2} I$$