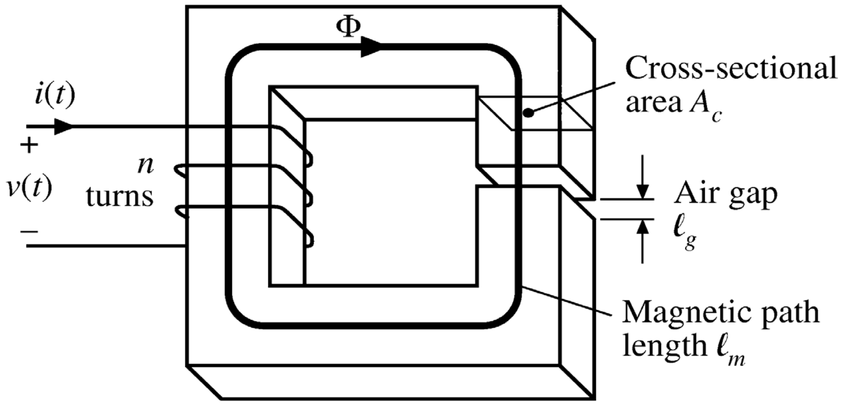
Modeling BLDC Motors

Review of Basic Magnetics

Background knowledge for this lecture, e.g.



Assumptions:

(1) $\mu_c >> \mu_0$ & flux is only within core bounds (even at air gap)

(2) B & H fields are uniform within the core

(3) Core is unsaturated with $B = \mu_c H$

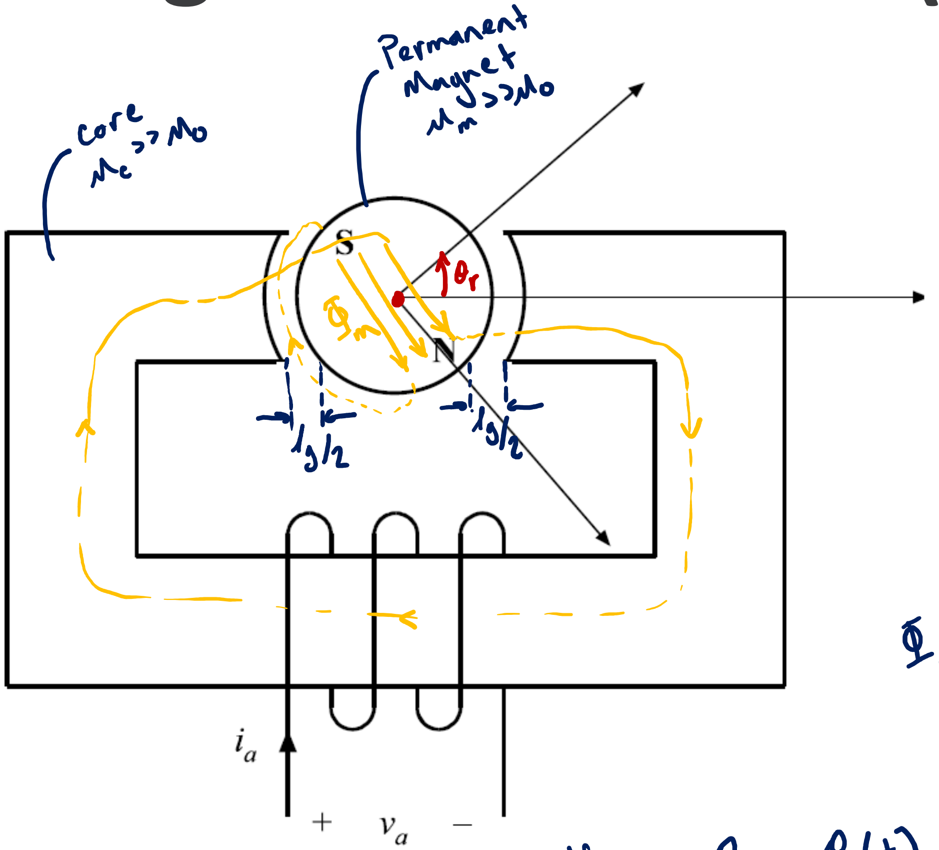
Results:

$$L \approx \frac{\mu_0 l_g n^2}{A_c}$$

$$R = \rho \frac{n \, MLT}{A_w}$$

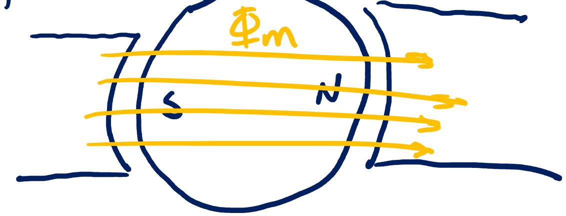
$$B(t) = \frac{1}{nA_c} \int_0^t v(t)\,dt \approx \frac{\mu_0 n}{l_g} i(t)$$

Single Phase Motor (Simplified)

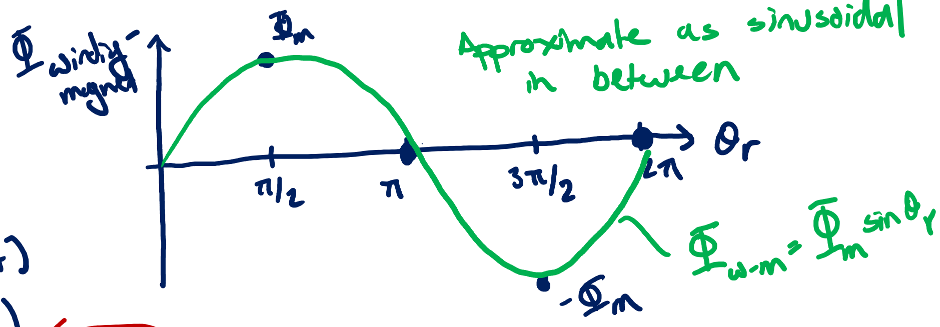


Magnet can rotate, characterized by angle θ_r

When $\theta_r = \frac{\pi}{2}$, Assume all flux Φ_m flows through the core (& winding)



At any θ_r, flux coupled into winding is



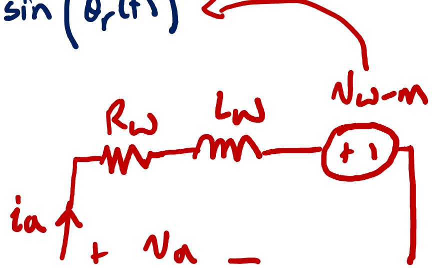
Approximate as sinusoidal in between

$\Phi_{w-m} = \Phi_m \sin\theta_r$

If magnet is rotating in time $\theta_r \rightarrow \theta_r(t)$

$$N_{w-m} = n \frac{d}{dt} \Phi_{w-m} = n \Phi_m \frac{d}{dt} \sin(\theta_r(t))$$

Circuit model of winding:



Electromechanical Conversion

$$v_{w\text{-}m} = \underbrace{n\Phi_m} \frac{d}{dt}\sin(\theta_r(t))$$

$n\Phi_m = \lambda_m \equiv$ "flux linkage"

$$v_{w\text{-}m} = \lambda_m \cos(\theta_r(t)) \frac{d\theta_r(t)}{dt}$$

$$\boxed{v_{w\text{-}m} = \lambda_m \omega_r \cos(\theta_r)}$$

If the magnet spins at a constant speed $\frac{d\theta_r(t)}{dt} = \omega_r$

Power supplied to winding $P_a = i_a \cdot v_a$

$$P_a = \underbrace{i_a^2 R_w}_{\text{conduction loss}} + \underbrace{i_a L_w \frac{di_a}{dt}}_{\text{reactive power}} + \underbrace{i_a \lambda_m \omega_r \cos(\theta_r)}_{\text{converted to mechanical power}}$$

Mechanical Power $P_m = \tau_r \omega_r$

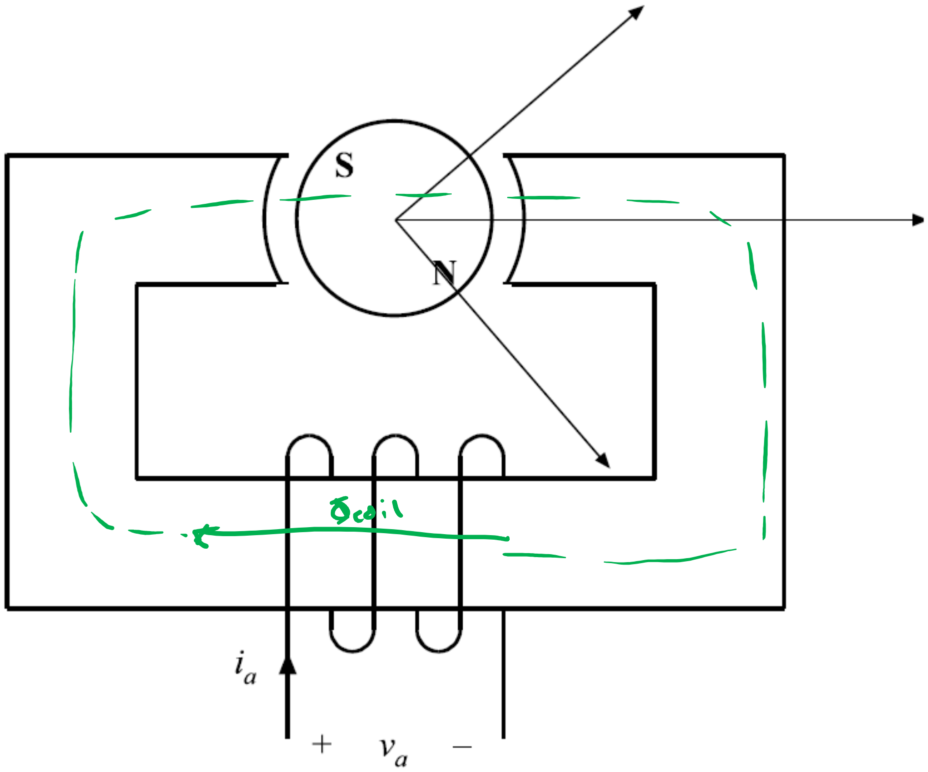
Neglecting friction or dynamics

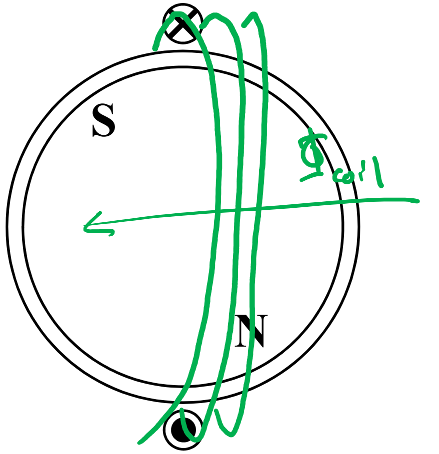
$$\tau_r \cancel{\omega_r} = i_a \lambda_m \cancel{\omega_r} \cos(\theta_r)$$

$$\boxed{\tau_r = i_a \lambda_m \cos(\theta_r)}$$

No matter how i_a is controlled, torque goes to 0

Alternative Diagram

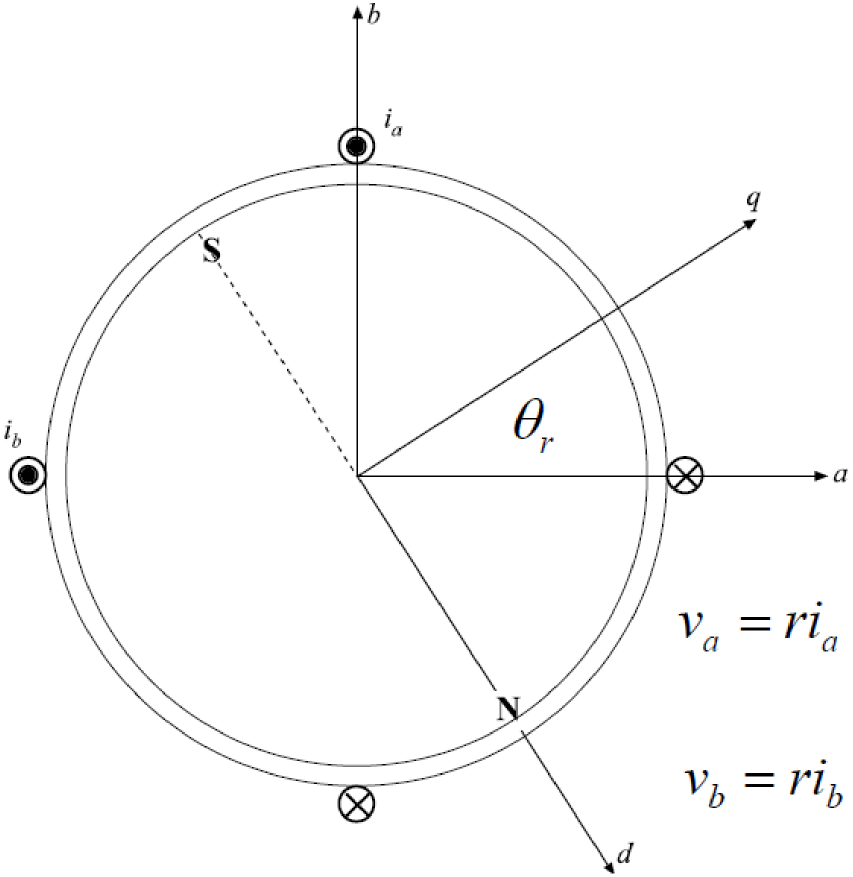




Simplified diagram

- shows how winding & magnet fluxes interact at varying θ_r

2-Pole, 2-Phase PMSM



Two-pole, two-phase PMSM terminal characteristics in stator reference frame

$$\lambda_a(\theta_r) = \lambda_M \sin(\theta_r)$$
$$\lambda_b(\theta_r) = -\lambda_M \cos(\theta_r)$$

$$v_a = ri_a + \frac{d\lambda_a}{dt} = ri_a + L\frac{di_a}{dt} + \lambda_M \omega_r \cos(\theta_r)$$
$$v_b = ri_b + \frac{d\lambda_b}{dt} = ri_b + L\frac{di_b}{dt} + \lambda_M \omega_r \sin(\theta_r)$$

$$\tau_r = \lambda_m(i_a \cos(\theta_r) + i_b \sin(\theta_r))$$

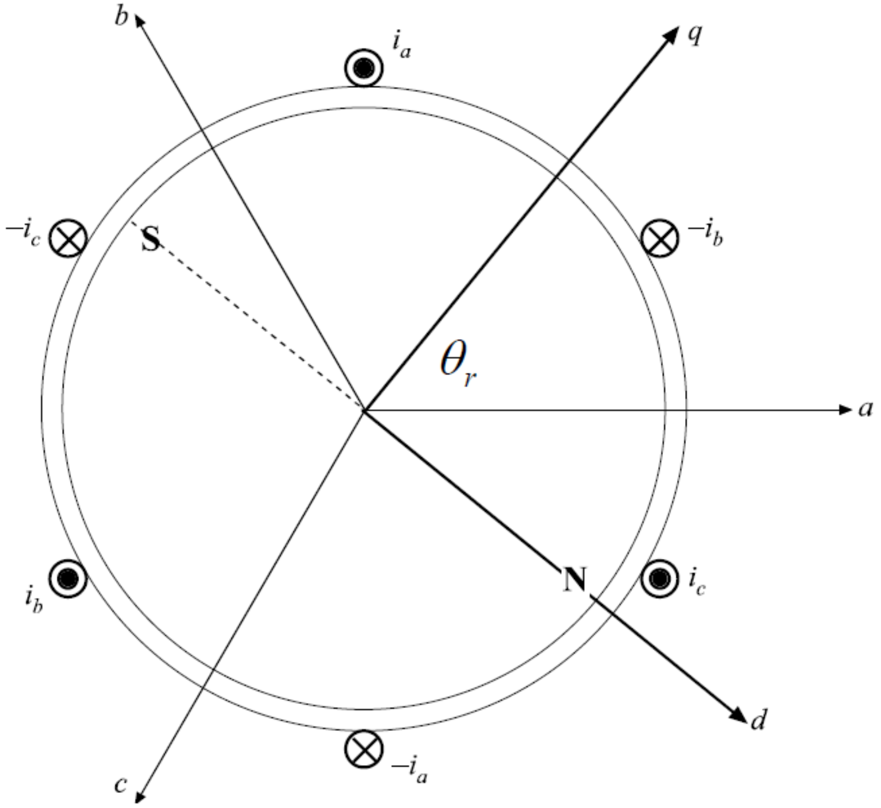
If we control the currents so that $\begin{cases} i_a = I_x \cos(\theta_r) \\ i_b = I_x \sin(\theta_r) \end{cases}$

$$\boxed{\tau_r = \lambda_m I_x}$$

No torque ripple!

Requires control to synchronize currents to rotation

3-Phase, 2-Pole PMSM



$$\lambda_a(\theta_r) = \lambda_m \sin(\theta_r)$$

$$\lambda_b(\theta_r) = \lambda_m \sin\left(\theta_r - \frac{2\pi}{3}\right)$$

$$\lambda_c(\theta_r) = \lambda_m \sin\left(\theta_r - \frac{4\pi}{3}\right)$$

$$\tau_r = i_a\lambda_m\cos(\theta_r) + i_b\lambda_m\cos\left(\theta_r - \frac{2\pi}{3}\right) + i_c\lambda_m\cos\left(\theta_r - \frac{4\pi}{3}\right)$$

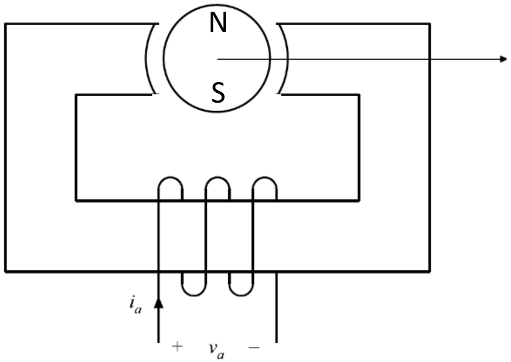
with currents controlled to sync

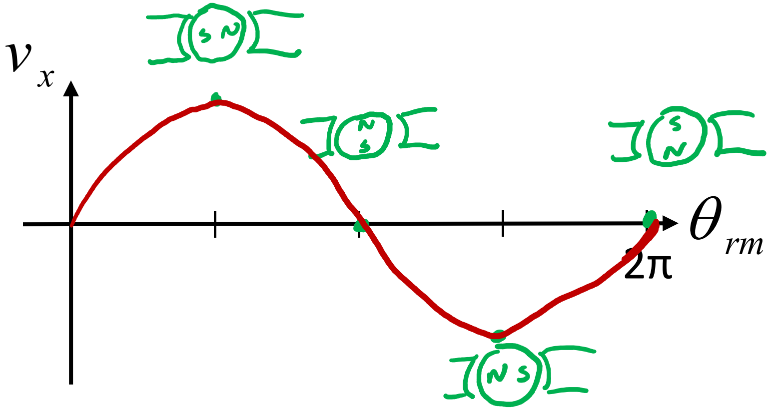
$$\boxed{\tau_r = \frac{3}{2}\lambda_m I_x}$$

& startup direction is guaranteed

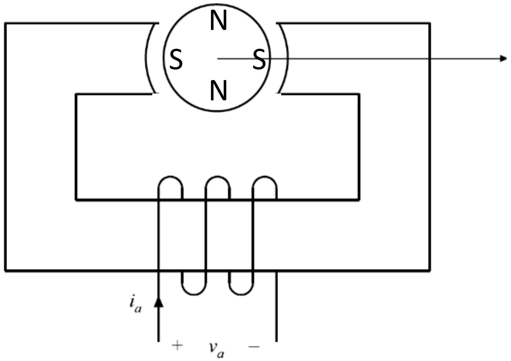
Different Number of Poles

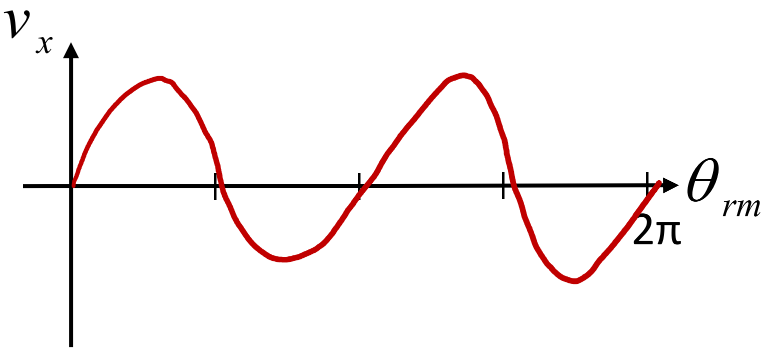
2 pole



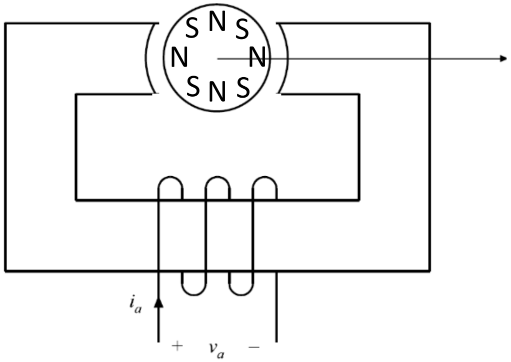


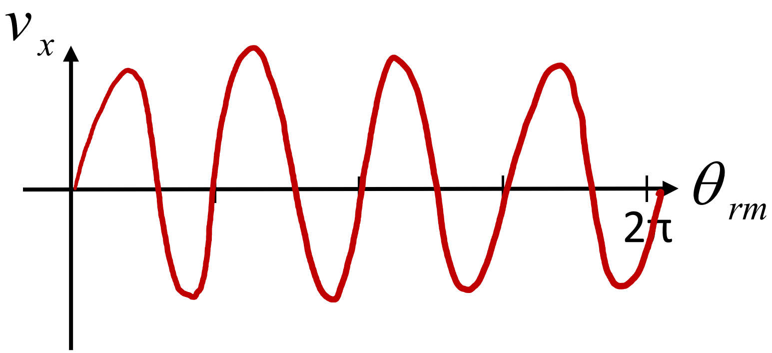
4 pole





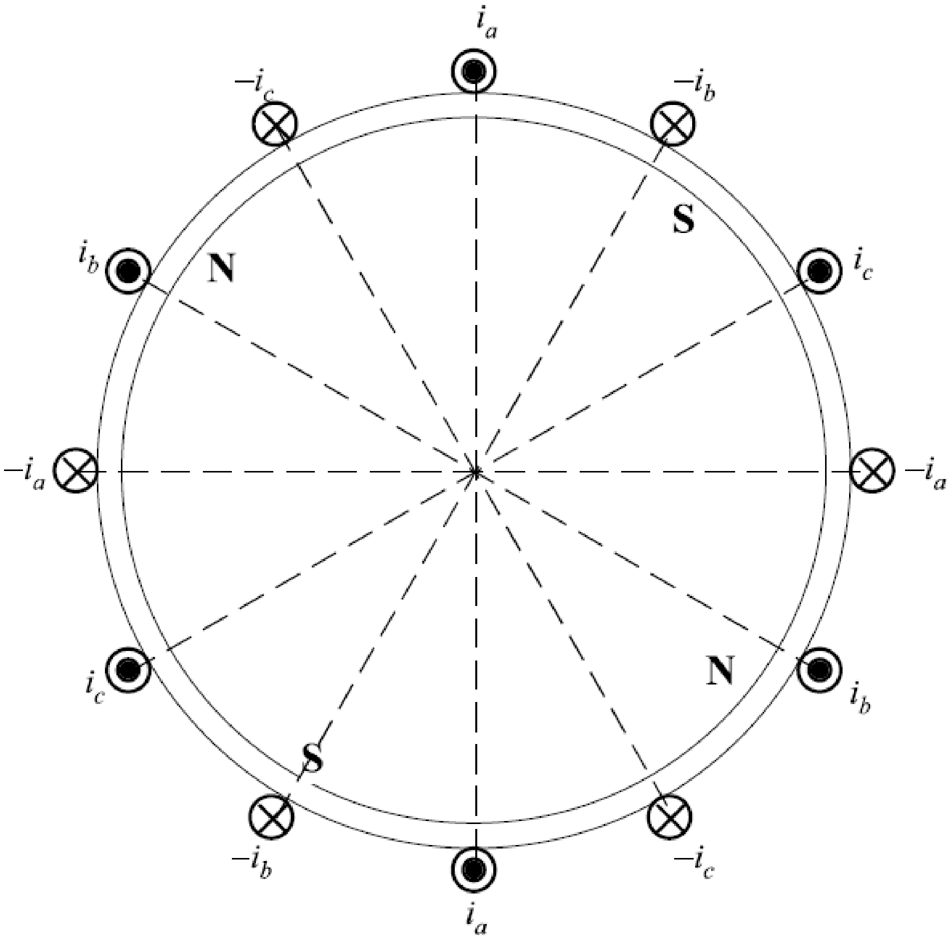
8 pole





3-Phase, P-Pole PMSM

P = 4 example



Poles always occur in pairs

Electrical and mechanical angle

$$\theta_r = \frac{P}{2}\theta_{rm}$$

Electrical and mechanical speed

$$\omega_r = \frac{P}{2}\omega_{rm}$$

Max torque per amp

$$T_m \le \lambda_m \frac{P}{2}\frac{3}{2} I$$

Outer- vs. Inner-Rotor

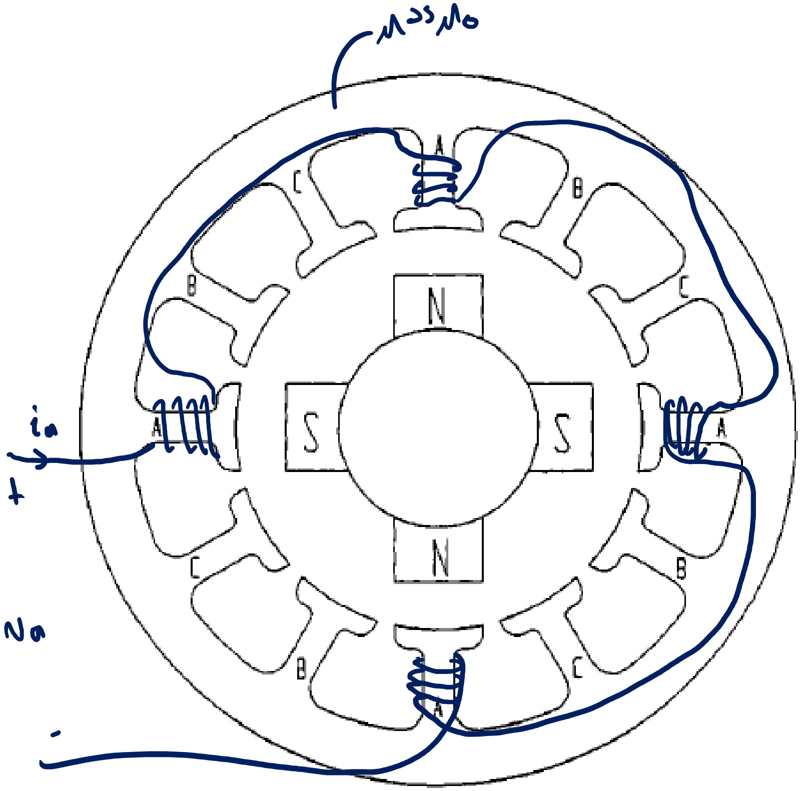


FIGURE 5.15 Multiphase inner-rotor motor.

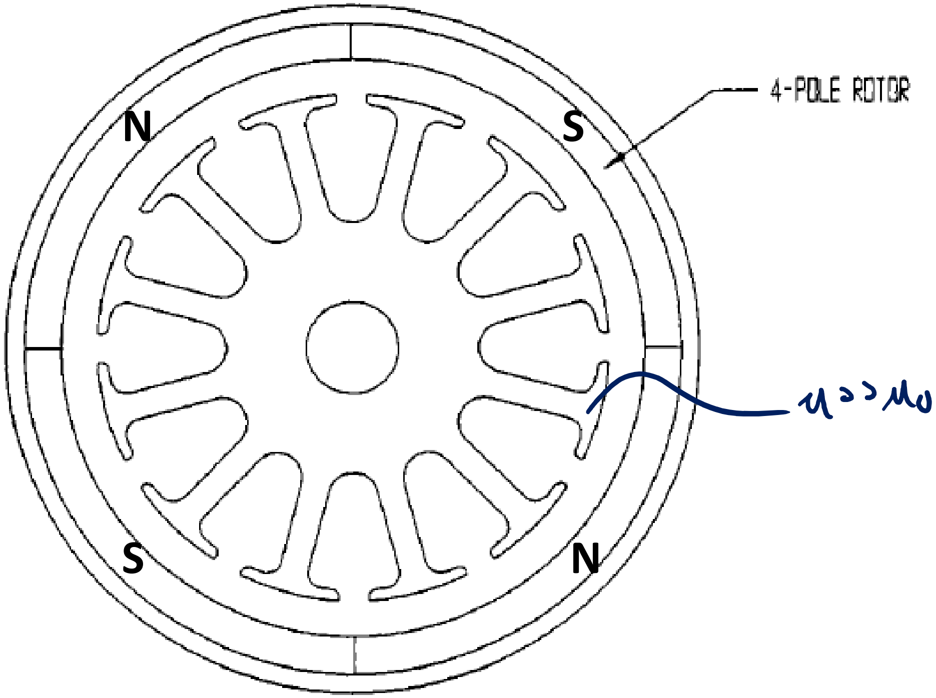
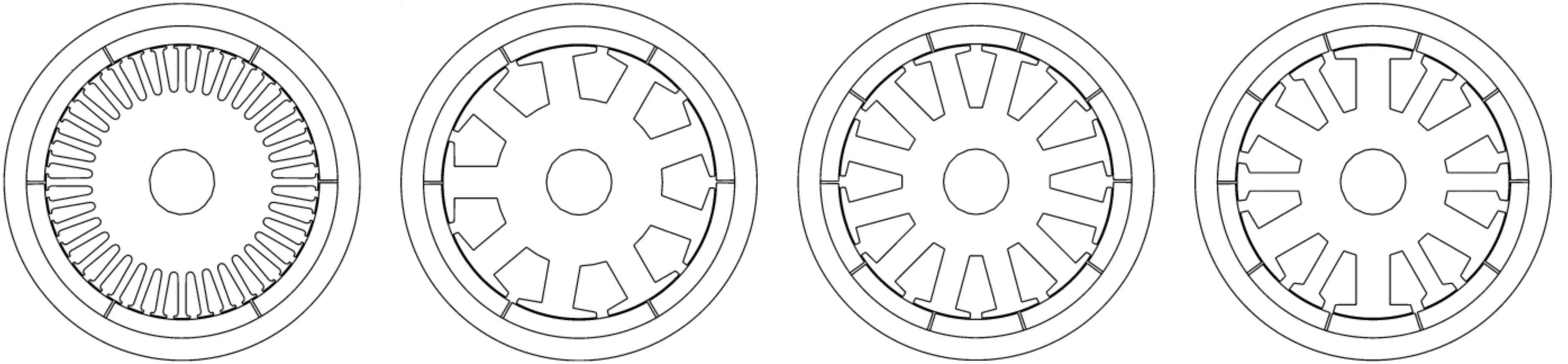


FIGURE 5.13 Multiphase outer-rotor motor.

- Traditional motors are inner-rotor
- On e-bike, need hub to remain stationary and outer wheel to spin

Motor Teeth/Poles Example



(a)
36-slot/6-pole

(b)
9-slot/6-pole

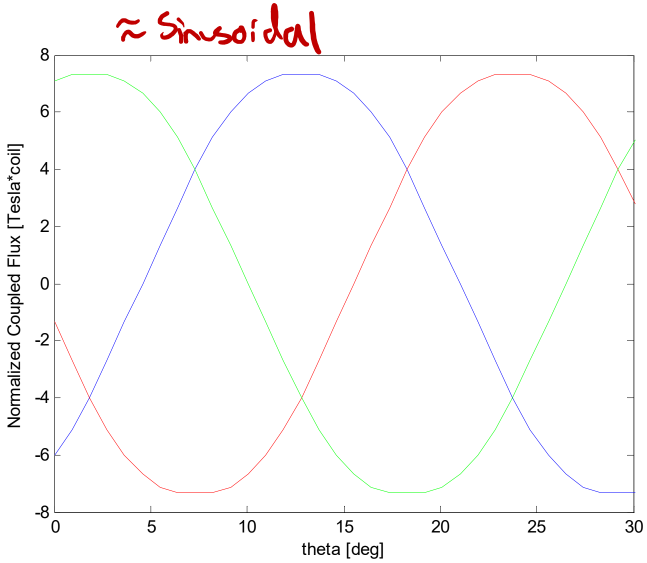
(c)
12–slot/10-pole
(all teeth wound)

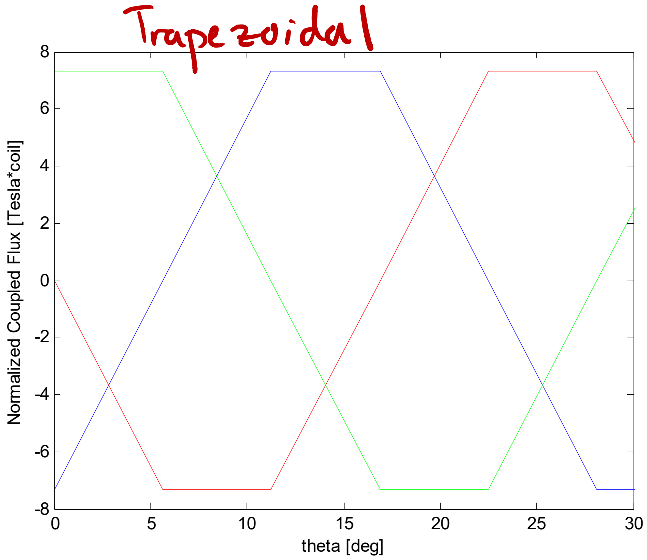
(d)
12-pole/10-pole
(alternate teeth wound)

Shaping Back-EMF

- Earlier, assumed $f(\theta_r) = \sin(\theta_r)$ resulting in sinusoidal back-EMF
- Ways to achieve:
  1. Sinusoidal distribution of windings
  2. Altering slot/pole/phase
- #2 is used in our motor

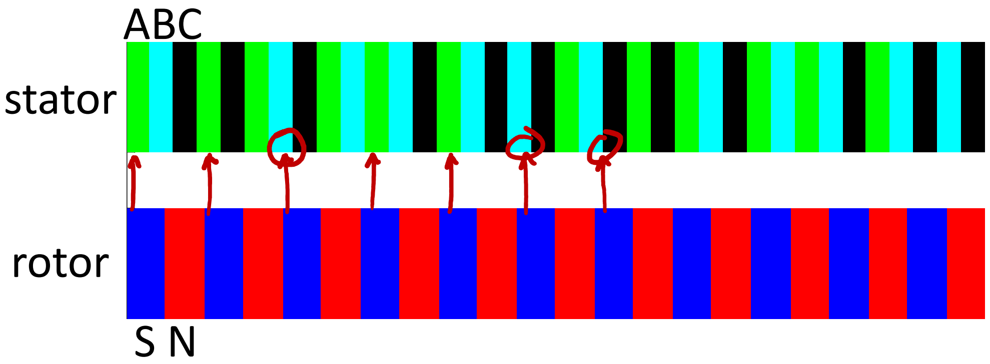
Shape of Back EMF

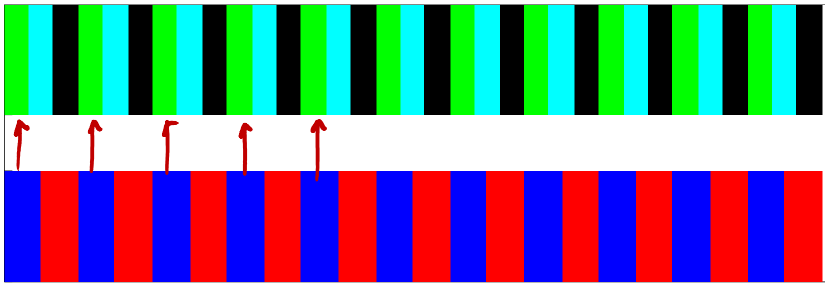

≈ Sinusoidal


Trapezoidal

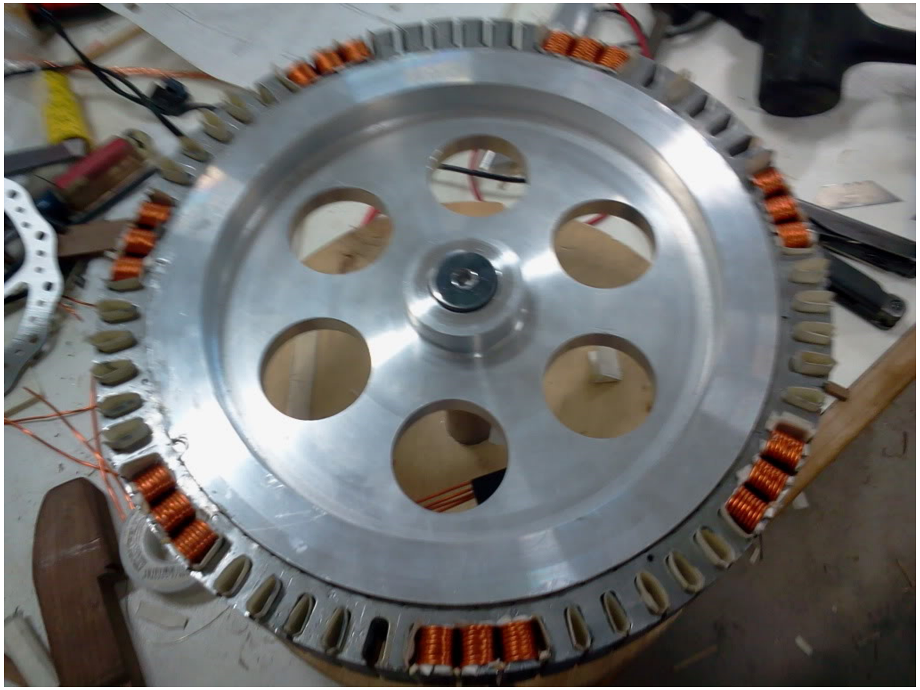
- 36 Teeth, 22 Poles
- Teeth/Pole/Phase = 0.5455

- 33 Teeth, 22 Poles
- Teeth/Pole/Phase = 0.5

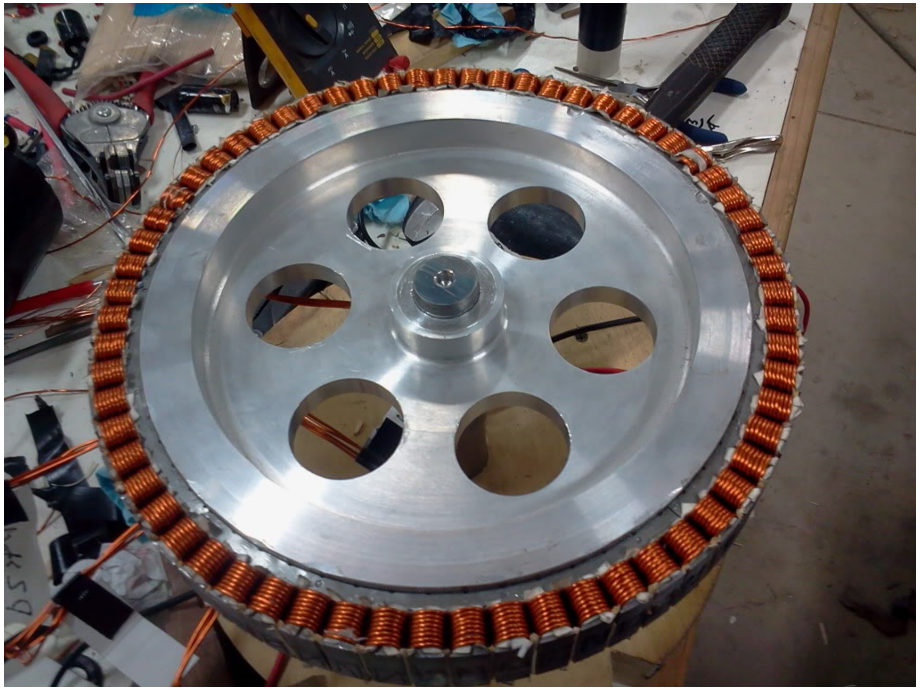




Stator Winding



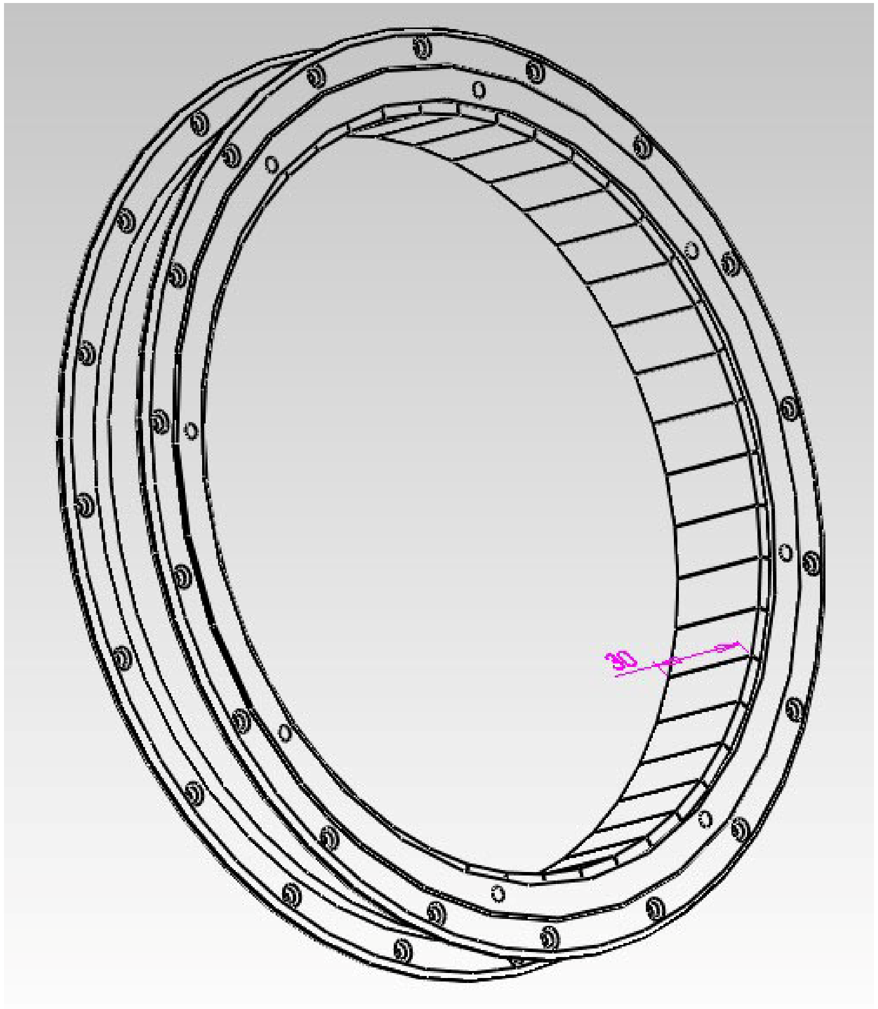
Complete winding of Phase A



Complete winding of all phases

56 pole
63 teeth

Rotor and Poles



- Outer rotor (to which spokes/wheel are attached)
- Magnets alternate N-S