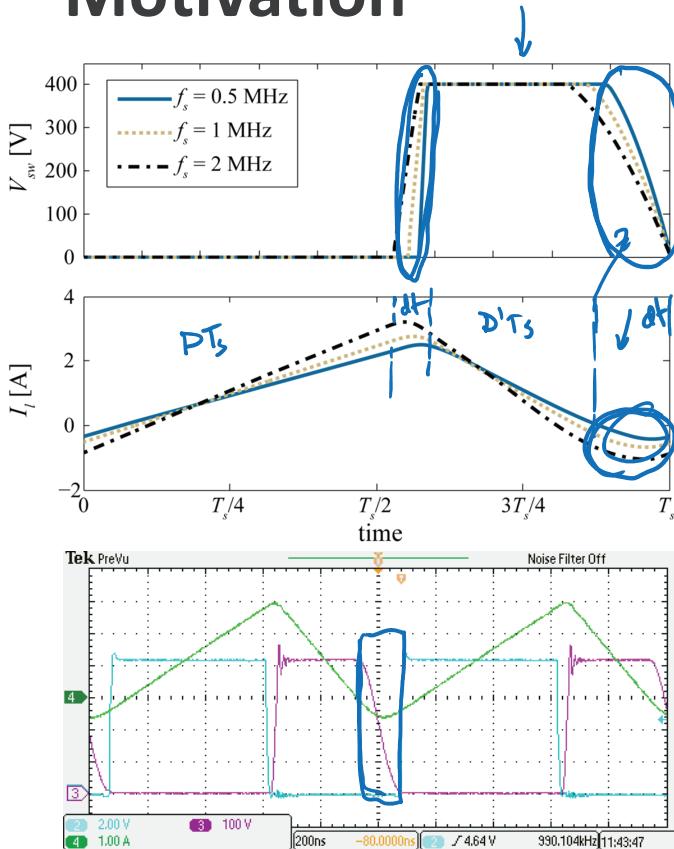


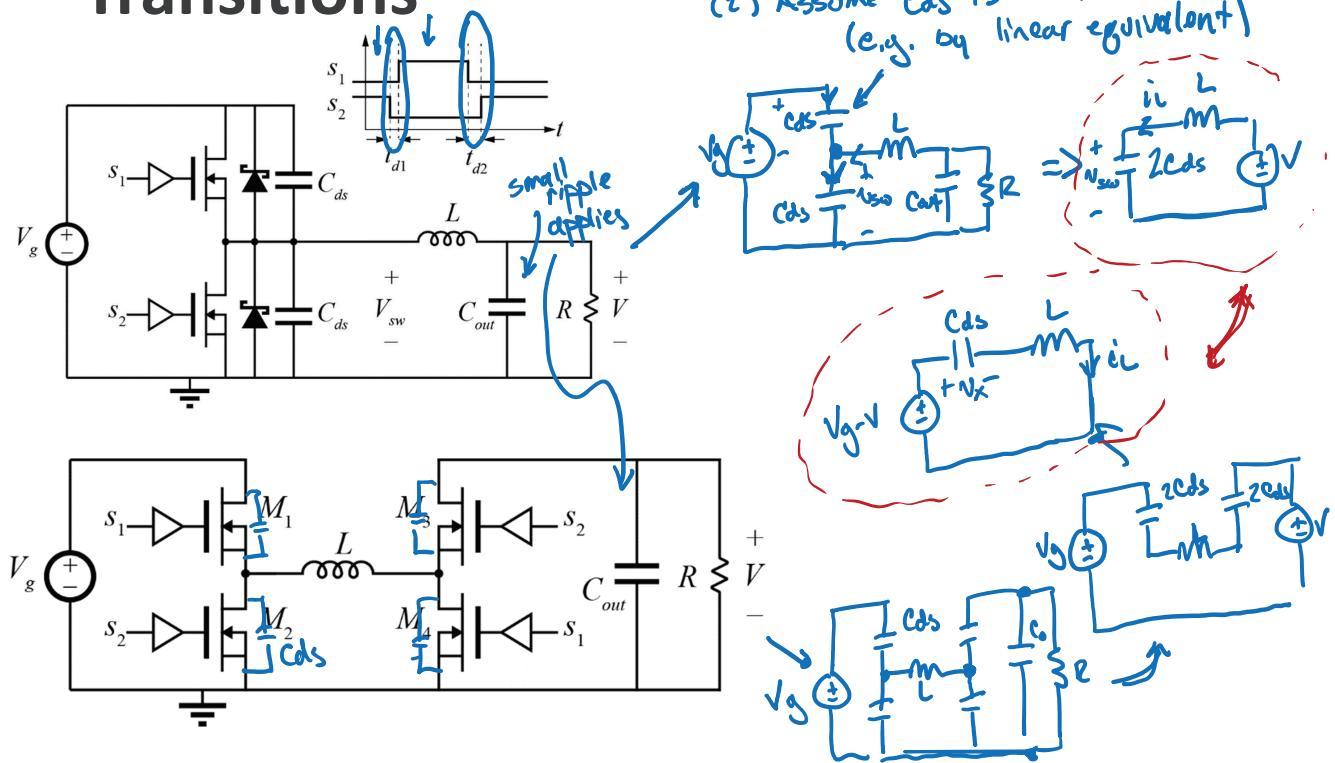
# Motivation



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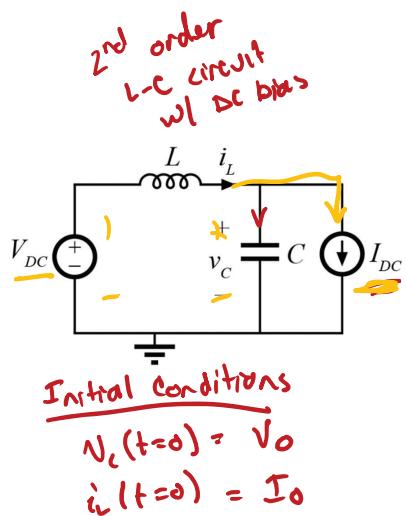
# Time-Domain Analysis of Switching Transitions

(1) Assume  $C_{out} \gg C_{ds}$   
(2) Assume  $C_{ds}$  is linear



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# Resonant Circuit Solution



$$\textcircled{1} \quad C \frac{dv_c}{dt} = i_L - I_{DC}$$

$$\textcircled{2} \quad L \frac{di_L}{dt} = V_{DC} - v_c$$

$$\downarrow$$

$$L \frac{d}{dt} \left( C \frac{dv_c}{dt} + I_{DC} \right) = V_{DC} - v_c$$

$$LC \frac{d^2 v_c}{dt^2} + v_c - V_{DC} = 0$$

$$v_c(t) = V_{DC} + (V_0 - V_{DC}) \cos\left(\frac{t}{\sqrt{LC}}\right) + (I_0 - I_{DC}) \sqrt{\frac{L}{C}} \sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$i_L(t) = I_{DC} + (I_0 - I_{DC}) \cos\left(\frac{t}{\sqrt{LC}}\right) + (V_{DC} - V_0) \sqrt{\frac{C}{L}} \sin\left(\frac{t}{\sqrt{LC}}\right)$$



## → Normalization and Notation

Notation:  $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$        $R_0 = \sqrt{L/C}$

$$\begin{cases} v_c(t) = V_{DC} + (V_0 - V_{DC}) \cos(\omega_0 t) + R_0(I_0 - I_{DC}) \sin(\omega_0 t) \\ i_L(t) = I_{DC} + (I_0 - I_{DC}) \cos(\omega_0 t) + \frac{1}{R_0}(V_{DC} - V_0) \sin(\omega_0 t) \end{cases}$$

Normalization:

$$m_c(t) = \frac{v_c(t)}{V_{base}}$$

$V_{base} \rightarrow$  Any constant voltage  
(you choose; some choices better than others)

$$j_L(t) = \frac{i_L(t)}{I_{base}}$$

$$I_{base} = \frac{V_{base}}{R_0}$$

$$\theta_i = \omega_0 t_i$$

$$\begin{aligned} \text{v}_{\text{base}} &= \text{v}_{DC} \\ \text{i}_{\text{base}} &= \frac{\text{v}_{DC}}{R_0} \end{aligned}$$

$$\left\{ \begin{array}{l} v_c(t) = V_{DC} + (V_0 - V_{DC}) \cos(\omega_0 t) + R_0(I_0 - I_{DC}) \sin(\omega_0 t) \\ i_L(t) = I_{DC} + (I_0 - I_{DC}) \cos(\omega_0 t) + \frac{1}{R_0}(V_{DC} - V_0) \sin(\omega_0 t) \end{array} \right.$$

$$\left\{ \begin{array}{l} m_c(\theta) = \frac{v_c}{\text{v}_{\text{base}}} = 1 + \left( \frac{V_0}{V_{DC}} - 1 \right) \cos \theta + R_0 \frac{I_0 - I_{DC}}{\text{v}_{DC}} \sin \theta \\ j_L(\theta) = \frac{i_L}{\text{i}_{\text{base}}} = R_0 \frac{I_{DC}}{V_{DC}} + R_0 \frac{I_0 - I_{DC}}{V_{DC}} \cos \theta + \left( 1 - \frac{V_0}{V_{DC}} \right) \sin \theta \end{array} \right.$$

$$A = \left( \frac{V_0}{V_{DC}} - 1 \right) \quad B = R_0 \frac{I_0 - I_{DC}}{\text{v}_{DC}}$$

$$\left\{ \begin{array}{l} m_c(\theta) = 1 + A \cos \theta + B \sin \theta \\ j_L(\theta) = R_0 \frac{I_{DC}}{V_{DC}} + B \cos \theta - A \sin \theta \end{array} \right.$$

Try looking @  $(m_c(\theta) - 1)^2 + (j_L(\theta) - R_0 \frac{I_{DC}}{V_{DC}})^2 =$

$$= A^2 \cos^2 \theta + B^2 \sin^2 \theta + 2AB \cos \theta \sin \theta + A^2 \sin^2 \theta + B^2 \cos^2 \theta - 2AB \cos \theta \sin \theta$$

$$= A^2 (\cos^2 \theta + \sin^2 \theta) + B^2 (\sin^2 \theta + \cos^2 \theta)$$

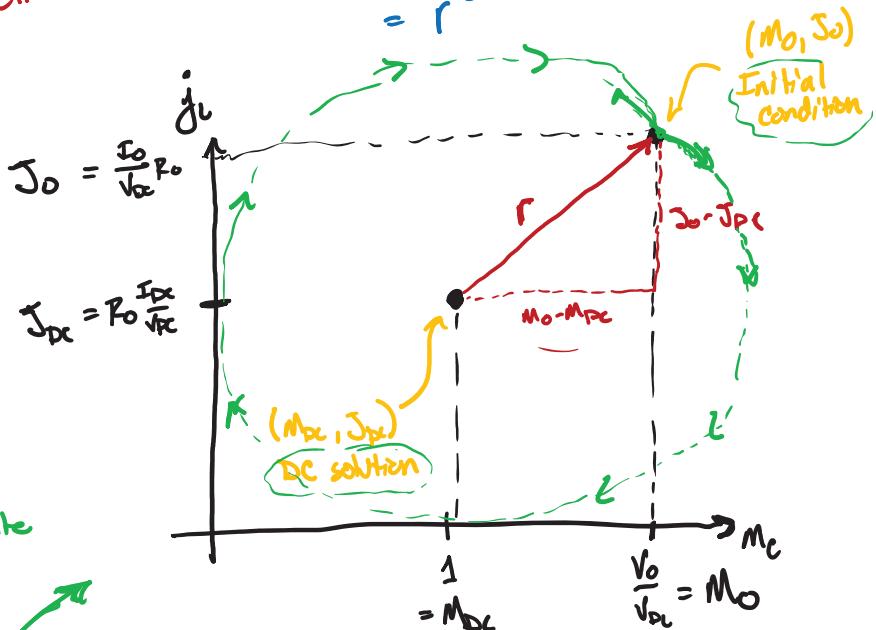
$$= A^2 + B^2$$

$$(m_c(\theta) - 1)^2 + (j_L(\theta) - R_0 \frac{I_{DC}}{V_{DC}})^2 = \left( \frac{V_0}{V_{DC}} - 1 \right)^2 + \left( R_0 \frac{I_0 - I_{DC}}{V_{DC}} \right)^2$$

$$= (M_0 - M_{DC})^2 + (J_0 - J_{DC})^2$$

equation for a circle in the  
m-j plane

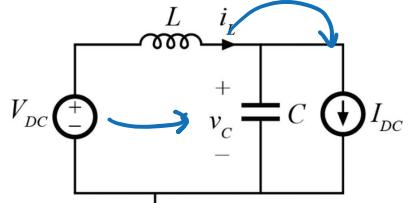
Always a direction  
associated w/ state  
plane



### State Plane:

Transforms Diff EQ's into  
Geometry → try  
- For L-C, I-L circuits

# State Plane Analysis



$$V_{base} = V_{DC}$$

$$I_{base} = \frac{V_{DC}}{R_0}$$

DC solution:

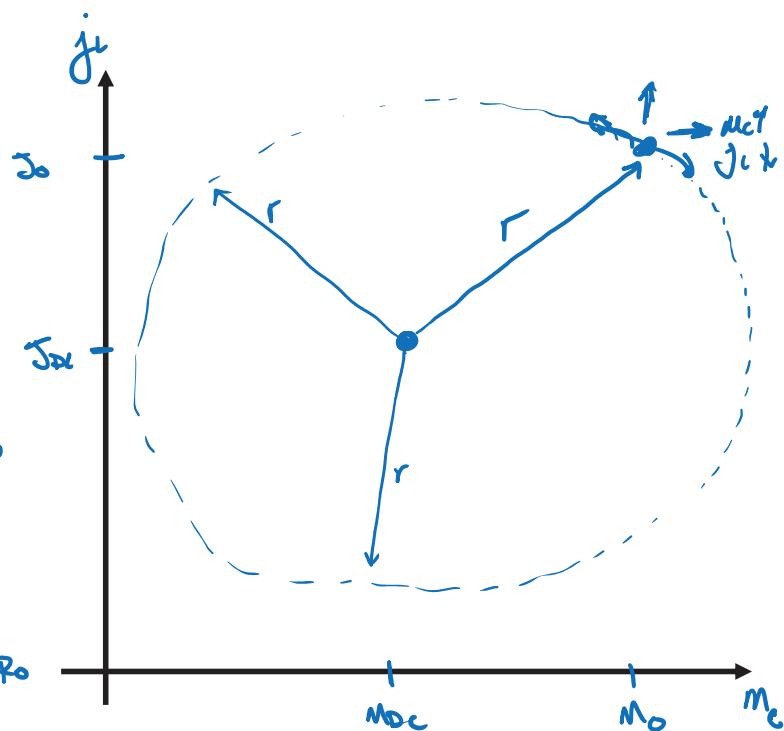
$$\begin{aligned} V_C &= V_{DC} \rightarrow M_C = M_{PC} = 1 \\ i_L &= I_{DC} \rightarrow J_L = J_{DC} = \frac{I_{DC}}{R_0} \end{aligned}$$

Initial Conditions:

$$\begin{aligned} V_C(0) &= V_0 \rightarrow M_C = M_0 = \frac{V_0}{V_{base}} \\ i_L(+) &= J_0 \rightarrow J_L = J_0 = \frac{I_0}{R_0} \end{aligned}$$

Direction?

find  $\frac{dV_C}{dt}$  at  $t=0$   
and/or  $\frac{di_L}{dt}$  at  $t=0$



- [1] R. Oruganti and F. C. Lee, "Resonant Power Processors, Part I – State Plane Analysis", Industry Applications, IEEE Tran. on, vol. 21, no. 6, nov 1985.

- [2] D. P. Atherton, Nonlinear Control Engineering. London: Van Nostrand Reinhold, 1982, Ch. 2.