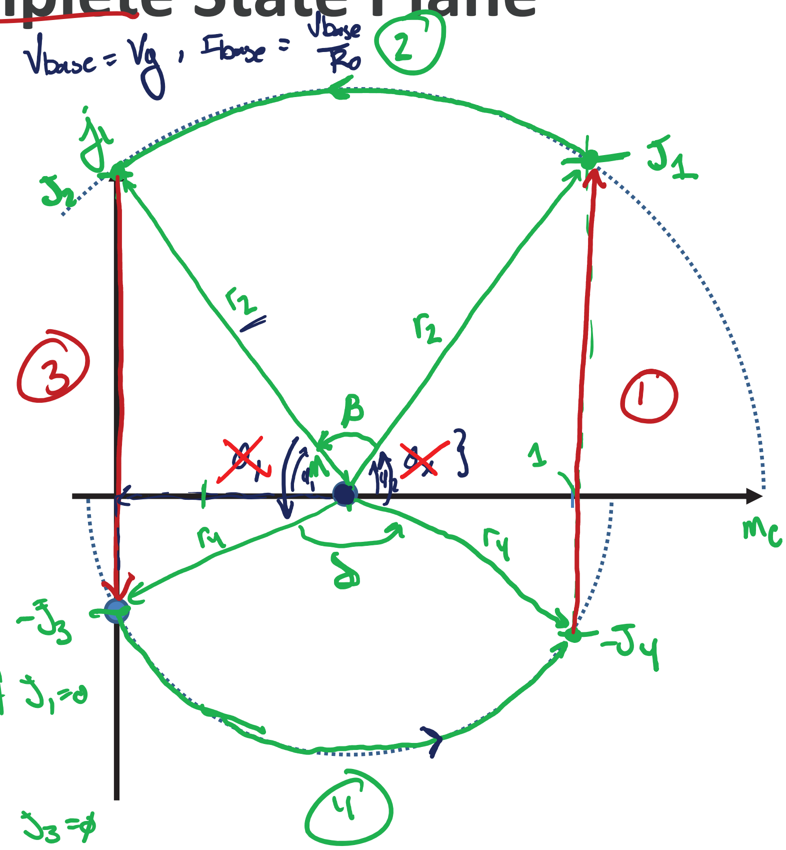
Sync-Buck Complete State Plane

(2): I_c: $i_L = I_1$ $v_{sw} = V_g$

$j_L = J_1$ $m_c = 1$



ZVS Condition:

(2) $M - \sqrt{J_1^2 + (1-M)^2} \leq 0$

$M < 1/2$, obtained even if $J_1 = 0$

(4) $M + \sqrt{J_3^2 + M^2} \geq 1$

$M > 1/2$, obtained even if $J_3 = \phi$

State Plane Solution: Intervals 1 & 2

(1) Non-resonant interval

$\left(\frac{1}{I_{base}}\right)\frac{V_g - V}{L} t_1 = I_1 + I_4 \left(\frac{1}{I_{base}}\right)$

$(1-M)\frac{R_0}{L} t_1 = J_1 + J_4$

$$\boxed{(1-M)\,\theta_1 = J_1 + J_4}$$

$\frac{1}{I_{base}} = \frac{1}{V_{base}} R_0$, $V_{base} = V_g$

$\frac{R_0}{L} = \frac{\sqrt{L/C}}{L} = \frac{1}{\sqrt{LC}} = \omega_0$

(2) Resonant interval

$\left\{ \begin{array}{l} r_2^2 = J_1^2 + (1-M)^2 \\ r_2^2 = J_2^2 + M^2 \end{array} \right\}$

$$\boxed{J_1^2 + (1-M)^2 = J_2^2 + M^2}$$

$$\boxed{\beta = \pi - \tan^{-1}\left(\frac{J_2}{M}\right) - \tan^{-1}\left(\frac{J_1}{1-M}\right)}$$

State Plane Solution: Intervals 3 & 4

(3) Non-resonant interval

$$\frac{V}{L} t_3 = I_2 + I_3$$

$$\boxed{M \theta_3 = J_2 + J_3}$$

(4) Resonant

$$\boxed{J_3^2 + M^2 = J_4^2 + (1-M)^2}$$

$$\boxed{\delta = \pi - \tan^{-1}\left(\frac{J_3}{M}\right) - \tan^{-1}\left(\frac{J_4}{1-M}\right)}$$

State Plane Solution: Averaging Step

$$I_{out} = \frac{1}{T_S}\int_0^{T_S} i_{out}(t)\,dt = \frac{1}{T_S}\int_0^{T_S} i_L(t)\,dt$$

$$I_{out} = \frac{1}{T_S}\left[\int_0^{t_1} i_L(t)\,dt + \int_{t_1}^{t_1+t_2} i_L(t)\,dt + \int_{t_1+t_2}^{t_1+t_2+t_3} i_L(t)\,dt + \int_{\sum_{i=1}^{3} t_i}^{\sum_{i=1}^{4} t_i} i_L(t)\,dt\right]$$

$$I_{out} = \frac{1}{T_S}\left[\frac{I_1 - I_4}{2} t_1 + \cancel{q_2} + \frac{I_2 - I_3}{2} t_3 + \cancel{q_4}\right]$$

$$q_2 = \int_{(2)} i_L = \int_{(2)} i_{csw} = \int_{(2)} C \frac{dv_{sw}}{dt} dt = C_{sw} \Delta V_{sw}$$

$$q_2 = C_{sw}(-V_g) \qquad q_4 = C_{sw}(V_g)$$

$$\left(\frac{1}{I_{base}}\right) I_{out} = \frac{1}{T_S}\left[\frac{I_1 - I_4}{2} t_1 + \frac{I_2 - I_3}{2} t_3\right]\left(\frac{1}{I_{base}}\right)$$

$$J_{out} = \frac{1}{T_S}\left[\frac{J_1 - J_4}{2} t_1 + \frac{J_2 - J_3}{2} t_3\right]\left(\frac{\omega_0}{\omega_0}\right)$$

$$\boxed{J_{out} = \frac{F}{2\pi}\left[\frac{J_1 - J_4}{2}\theta_1 + \frac{J_2 - J_3}{2}\theta_3\right]}$$

Define

$$F = \frac{f_s}{f_0} = \frac{\omega_s}{\omega_0}$$

Normalized Period

$t_1 + t_2 + t_3 + t_4 = T_s$

$$\theta_1 + \beta + \theta_3 + \delta = \omega_0 T_s = \frac{\omega_0}{f_s} = \frac{2\pi}{F}$$

8 equations

Unknowns:

J_1, J_2, J_3, J_4

$\theta_1, \beta, \theta_3, \delta$

Operating point & converter:

M, J_{out}, F

L & C_{sw}