Analysis: Intervals 1-3

$V_{base} = V_g + \frac{V}{n}$

① Nothing yet. $\rightarrow$ $T_s = \sum_{i=1}^{6} t_i$ $\rightarrow$ $\boxed{\frac{2\pi}{F} = \theta_1 + \theta_2 + \gamma + \theta_4 + \varepsilon + \theta_6}$

② $\left(\frac{1}{I_{base}}\right)\frac{I_m}{C_{ds}} t_2 = V_g + \frac{V}{n}\left(\frac{1}{I_{base}}\right)$

$\frac{1}{R_0}\frac{1}{C_{ds}} = \sqrt{\frac{C_{ds}}{L_\ell}}\frac{1}{C_{ds}} = \frac{1}{\sqrt{L_\ell C_{ds}}} = \omega_0$

$J_m \frac{1}{R_0}\frac{1}{C_{ds}} t_2 = \frac{V_g + V/n}{V_{base}} = 1$

$$\boxed{J_m \theta_2 = 1}$$

③ $r_3^2 = J_m^2$

$r_3^2 = J_3^2 + (M_s - 1)^2$

$$\boxed{J_m^2 = J_3^2 + (M_s - 1)^2}$$

$$\boxed{\gamma = \tan^{-1}\left(\frac{M_s - 1}{J_3}\right)}$$

Analysis: Intervals 4-6

④ $-\left(\frac{V_g + \frac{V}{n} - V_s}{L_\ell}\right) t_4 = I_3 + I_4$

$$\boxed{-(1 - M_s)\theta_4 = J_3 + J_4}$$

from $r_5^2 = r_5^2$

⑤ $\boxed{(M_s - 1)^2 + J_4^2 = 1 + J_5^2}$

$$\boxed{\varepsilon = \pi - \tan^{-1}\left(\frac{J_5}{1}\right) - \tan^{-1}\left(\frac{J_4}{M_s - 1}\right)}$$

⑥ $\frac{V_g + \frac{V}{n}}{L_\ell} t_6 = I_5 + I_m$

$$\boxed{(1)\theta_6 = J_5 + J_m}$$

Analysis: Averaging

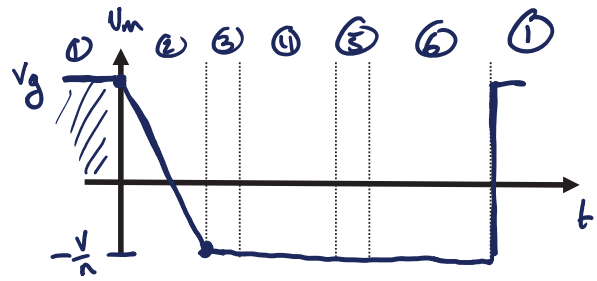
Get one additional equation from applying averaging to every filter element.

$C_s \rightarrow$ Cap-Q balance

$$\langle i_{cs} \rangle |_{T_s} = \phi$$

$$I_3 = I_4$$

$$\boxed{J_3 = J_4}$$



$L_m \rightarrow$ volt-sec balance

$$\langle v_m \rangle |_{T_s} = \phi$$

$$\langle v_m \rangle |_{T_s} = \frac{1}{T_s}\int_0^{T_s} v_m(t)\,dt$$

$$\phi = \cancel{\frac{1}{T_s}}\left[V_g t_1 + t_2 \frac{V_g - \frac{V}{n}}{2} + -\frac{V}{n}\left(t_3 + t_4 + t_5 + t_6\right)\right]$$

$$V_g\left(t_1 + \frac{t_2}{2}\right) = \frac{V}{n}\left(\frac{t_2}{2} + t_3 + t_4 + t_5 + t_6\right) \rightarrow \boxed{\frac{V}{V_g} = M = n\,\frac{t_1 + \frac{t_2}{2}}{\frac{t_2}{2} + t_3 + t_4 + t_5 + t_6}} \approx n\frac{D}{D'}$$

Average Output Current

Output cap $C \rightarrow$ cap-Q balance

$$\langle i_c \rangle |_{T_s} = \phi = \langle i_{D1} - i_{out} \rangle |_{T_s}$$

$$\langle i_{out} \rangle |_{T_s} = \langle i_{D1} \rangle |_{T_s} = \frac{1}{T_s}\int_0^{T_s} (I_m - i_L)\frac{1}{n}\,dt$$

$$n I_{out} = \frac{1}{T_s}\int_0^{T_s} I_m - i_L\,dt$$

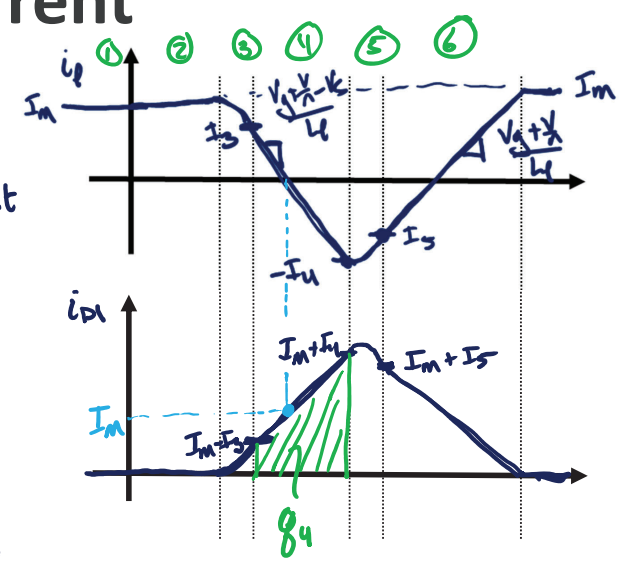
$$= \frac{1}{T_s}\left[\cancel{q_1} + \cancel{q_2} + q_3 + q_4 + q_5 + q_6\right]$$

$$q_3 = I_m t_3 - C_{ds}\left(\cancel{V_s} - V_g - \frac{V}{n}\right)$$

$$q_4 = t_4\left(\frac{(I_m - \cancel{I_3}) + (I_m + \cancel{I_4})}{2}\right) = I_m t_4$$

$$q_5 = I_m t_5 + \cancel{C_{ds} V_s}$$

$$q_6 = t_6 \frac{I_m + I_5}{2}$$



$$n\frac{I_{out}}{I_{base}} = n J_{out} = \frac{1}{T_s}\left[J_m t_3 + 1 C_{ds} R_b + J_m t_4 + J_m t_5 + \frac{J_m + J_5}{2} t_6\right]$$

$$\boxed{J_{out} = \frac{F}{2\pi}\left[J_m\left(\gamma + \theta_4 + \epsilon + \frac{\theta_6}{2}\right) + J_5 \frac{\theta_6}{2} + 1\right]}$$