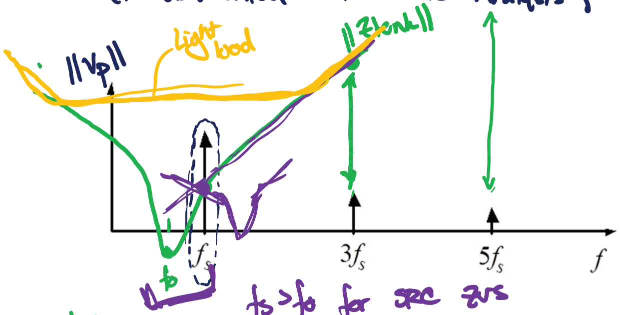
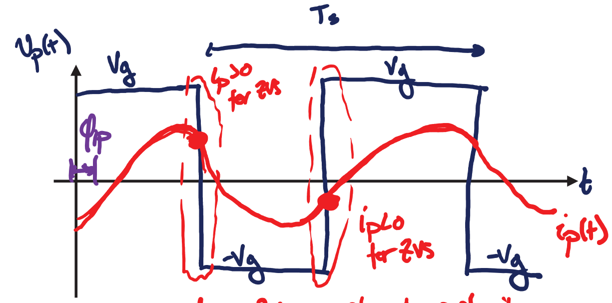
Sinusoidal Analysis (Ch 19)

Erickson & Maksimovic 2nd ed.
ch 22 in 3rd edition

(Fundamental Harmonic Analysis)



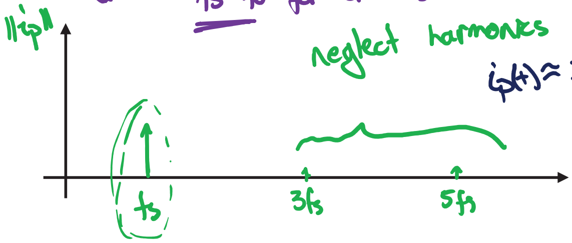
$f_s > f_0$ for SRC ZVS

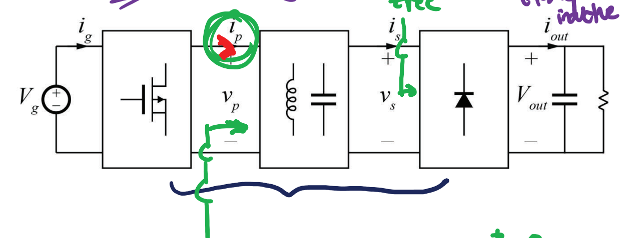


neglect harmonics

$i_p(t) \approx I_{p1} \sin(2\pi f_s t + \varphi_{ip})$

for ZVS: check polarity
i_p lag v_p for ZVS (@ f_s Z_{tank} inductive)





Good approximation if tank has good bandpass characteristics

Z_{tank} ~ tank structure & Z_{rec}

ex. for SRC $Z_{tank} = sL_r + \frac{1}{sC_r} + \frac{Z_{rec}}{n^2}$

Sinusoidal Analysis: Comments

- Generally most accurate when operating near resonance with a high Q
- Effective quality factor Q_e depends not only on resonant tank, but also on loading
- Analysis neglects switching intervals; can only predict where ZVS **cannot** be obtained

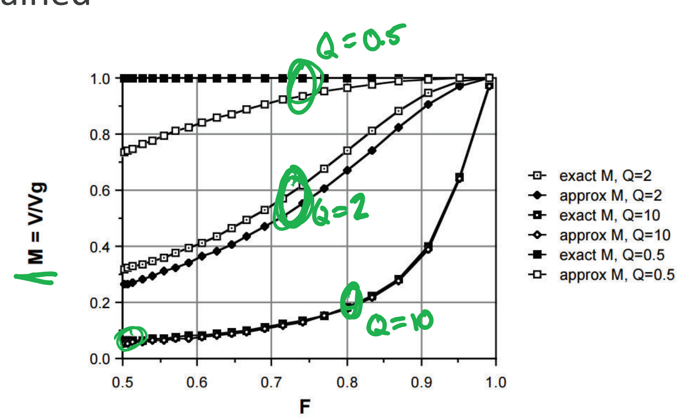
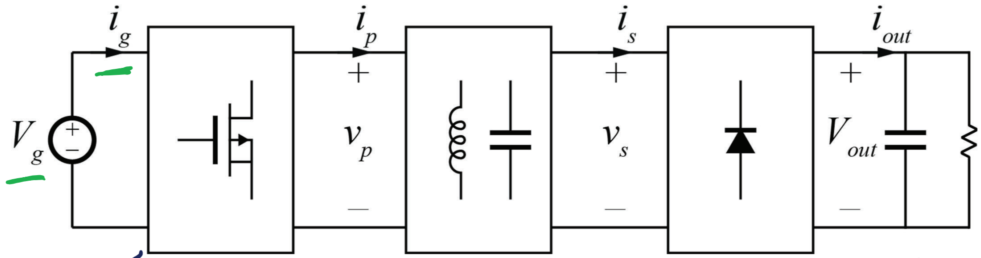


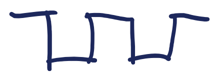
Fig. 2.14. Comparison of exact and approximate series resonant converter characteristics, below resonance.

"exact" = state plane
"approx" = sinusoidal analysis

AC Link Waveforms

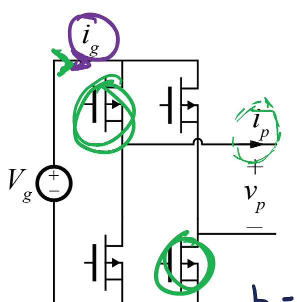


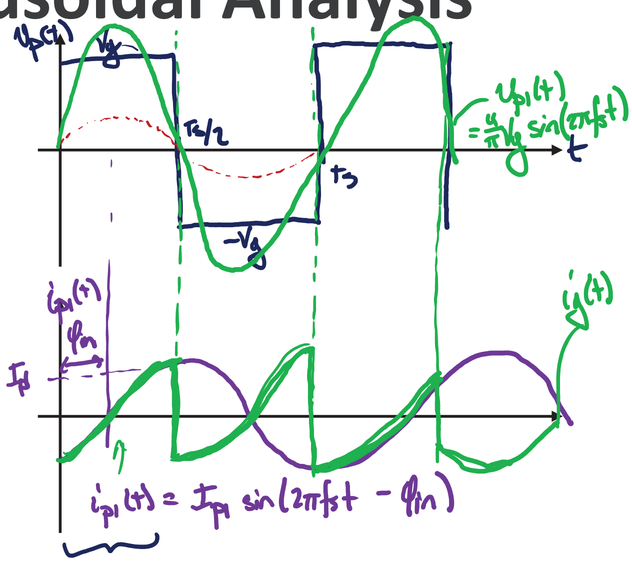
DC Average | AC sinusoidal | DC Average

$v_p(t) \rightarrow$ full signal 

$v_{p1}(t) \rightarrow$ fundamental component 

Switch Network Sinusoidal Analysis





Fourier Series

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(x)\,dx$$

For $v_p(t)$:

$$b_1 = \frac{2}{T_s}\int_0^{T_s} v_p(t)\sin(2\pi f_s t)\,dt$$

$$= \frac{4}{T_s}\int_0^{T_s/2} V_g \sin(2\pi f_s t)\,dt$$

$\theta = 2\pi f_s t$

$d\theta = 2\pi f_s\,dt$

$$= \frac{4}{T_s}V_g\int_0^{\pi}\sin(\theta)\,\frac{1}{2\pi f_s}\,d\theta$$

$$= \frac{4}{T_s}V_g\,\frac{1}{2\pi f_s}\Big[-\cos(\theta)\Big]_0^{\pi}$$

$$= \frac{4}{\cancel{T_s}}V_g\,\frac{1}{\cancel{2\pi f_s}}\,(\cancel{2}) = \boxed{\frac{4}{\pi}V_g}$$

$\approx 1.27\,V_g$

Input Current

$i_{p1}(t) = I_{p1}\sin(2\pi f_s t - \phi_{in})$ ← Generic definition at the moment

$$\langle i_g\rangle|_{T_s} = \frac{2}{T_s}\int_0^{T_s/2} i_{p1}(t)\,dt$$

$$\theta = 2\pi f_s t - \phi_{in}$$
$$d\theta = 2\pi f_s\,dt$$

$$= \frac{2}{T_s}\int_0^{T_s/2} I_{p1}\sin(2\pi f_s t - \phi_{in})\,dt$$

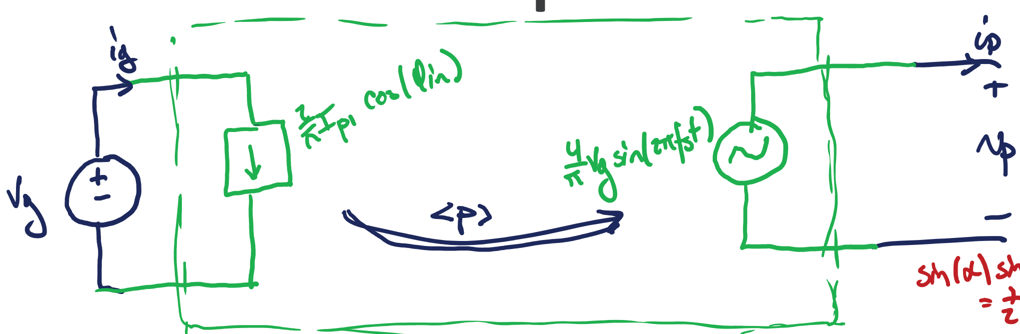
$$= \frac{2}{T_s}I_{p1}\int_{-\phi_{in}}^{\pi-\phi_{in}} \sin\theta\,\frac{1}{2\pi f_s}\,d\theta$$

$$= \frac{2}{T_s}I_{p1}\frac{1}{2\pi f_s}\Big[-\cos\theta\Big]_{-\phi_{in}}^{\pi-\phi_{in}}$$

$$= \frac{2}{\cancel{T_s}}I_{p1}\frac{1}{2\pi \cancel{f_s}}\left[\cancel{2}\cos\phi_{in}\right]$$

$$\langle i_g\rangle|_{T_s}\ \boxed{= \frac{2}{\pi}I_{p1}\cos(\phi_{in})}$$

Switch Network Equivalent Circuit



$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$

dt

$$P_g = V_g\langle i_g\rangle_{T_s} = V_g I_{p1}\frac{2}{\pi}\cos\phi_{in}\ \checkmark$$

$$P_p(t) = v_p(t)\cdot i_p(t)$$

$$\langle P_p\rangle|_{T_s} = \frac{1}{T_s}\int_0^{T_s}\frac{4}{\pi}V_g\sin(\omega_s t)\cdot I_{p1}\sin(\omega_s t - \phi_{in})$$

$$\langle P_p\rangle|_{T_s} = \frac{1}{T_s}\frac{4}{\pi}V_g I_{p1}\frac{1}{2}\int_0^{T_s}\left(\cos(\cancel{\omega_s t} - \cancel{\omega_s t} + \phi_{in}) - \cos(\omega_s t + \omega_s t - \phi_{in})\right)dt$$

$$= \frac{1}{\cancel{T_s}}\frac{4}{\pi}V_g I_{p1}\frac{1}{2}\left[\cancel{T_s}\cos(\phi_{in}) - \phi\right]$$

$$= \frac{2}{\pi}V_g I_{p1}\cos\phi_{in}\ \checkmark$$