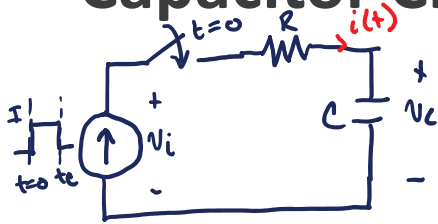
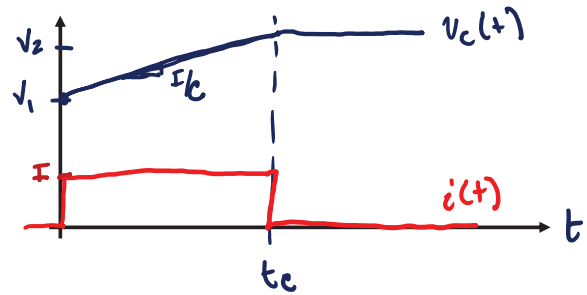


# Capacitor Charging: Current Source



$$V_c(t < 0) = V_1$$

$$I = \frac{\Delta Q_c}{t_c} = \frac{C(V_2 - V_1)}{t_c}$$



$$\Delta E_c = \frac{1}{2} C (V_2^2 - V_1^2) \quad (\text{same})$$

$$\Delta E_i = \int_0^{t_c} V_i \cdot I \, dt = I \int_0^{t_c} (V_c + IR) \, dt = I \left( \frac{V_1 + V_2}{2} \right) t_c + I^2 R t_c$$

$$= \frac{C(V_2 - V_1)}{t_c} \left( \frac{V_1 + V_2}{2} \right) t_c + \left( \frac{C(V_2 - V_1)}{t_c} \right)^2 R t_c$$

$$= \underbrace{\frac{1}{2} C (V_2^2 - V_1^2)}_{\Delta E_c} + \underbrace{\frac{1}{2} C (V_2 - V_1)^2 \left[ \frac{RC}{t_c} \right]}_{\text{Loss}}$$

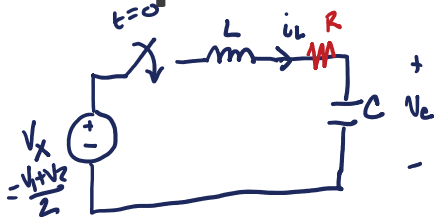
scale factor  
difference between  
V-src & I-src

$$\begin{aligned} \text{Voltage source case: } E_{\text{loss}} &= \frac{1}{2} C (V_2 - V_1)^2 \\ \text{Current source case: } E_{\text{loss}} &= \frac{1}{2} C (V_2 - V_1)^2 \left[ 2 \frac{RC}{t_c} \right] \end{aligned}$$

if  $t_c \gg RC \rightarrow$  current source has less loss

if  $t_c \ll RC \rightarrow$  Don't know  
V-src charging analysis assumes  
 $t_c \gg RC$

# Capacitor Charging: Resonant



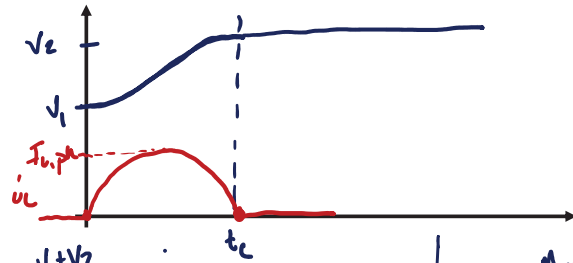
High-efficiency approx: assume  $R=0$  when solving waveforms

$$\Delta E_c = \frac{1}{2} C (V_2^2 - V_1^2)$$

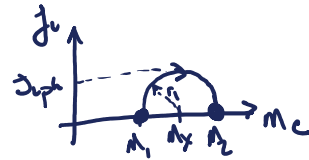
$$E_{loss} = I_{rms}^2 R t_c = \frac{I_{Lpk}^2}{2} R t_c$$

$$= \left( \frac{V_2 - V_1}{2 R_0} \right)^2 \frac{R t_c}{2} = \frac{(V_2 - V_1)^2}{4 R_0^2} \frac{R \pi}{2 \omega_0}$$

$$E_{loss} = \frac{(V_2 - V_1)^2}{4 R_0} \frac{R C \pi}{2}$$



$$V_x = \frac{V_1 + V_2}{2}$$



$$I_{Lpk} = M_2 - M_1 = \frac{M_2 - M_1}{2}$$

$$I_{Lpk} = I_{Lpk} I_{avg} = \frac{V_2 - V_1}{2 R_0}$$

$$t_c = \frac{\pi}{\omega_0}$$

$$\frac{1}{R_0 \omega_0} = \frac{\sqrt{LC}}{\sqrt{4C}} = C$$

$$E_{loss} = \frac{1}{2} C (V_2 - V_1)^2 \boxed{\frac{R \pi}{4 R_0}}$$

Loss in V-src case

$\frac{R}{R_0} \rightarrow$  small for high-Q resonance  
 $R_0 \gg R$

$$\frac{R \pi}{4 R_0} = \frac{R \pi \omega_0 C}{4} = \boxed{\frac{\pi^2 R C}{4 t_c}} \rightarrow t_c \gg R C$$

$$\frac{\pi^2}{4} \approx 2.5$$

# Comparison of Capacitor Charging

for capacitor charged from  $V_1$  to  $V_2$  in time  $t_c$

Voltage - source

$$\frac{E_{loss}}{\frac{1}{2} C (V_2 - V_1)^2}$$

Assumptions  
 $t_c \gg RC = \tau$

Current - source

$$\frac{1}{2} C (V_2 - V_1)^2 \left[ \frac{2RC}{t_c} \right]$$

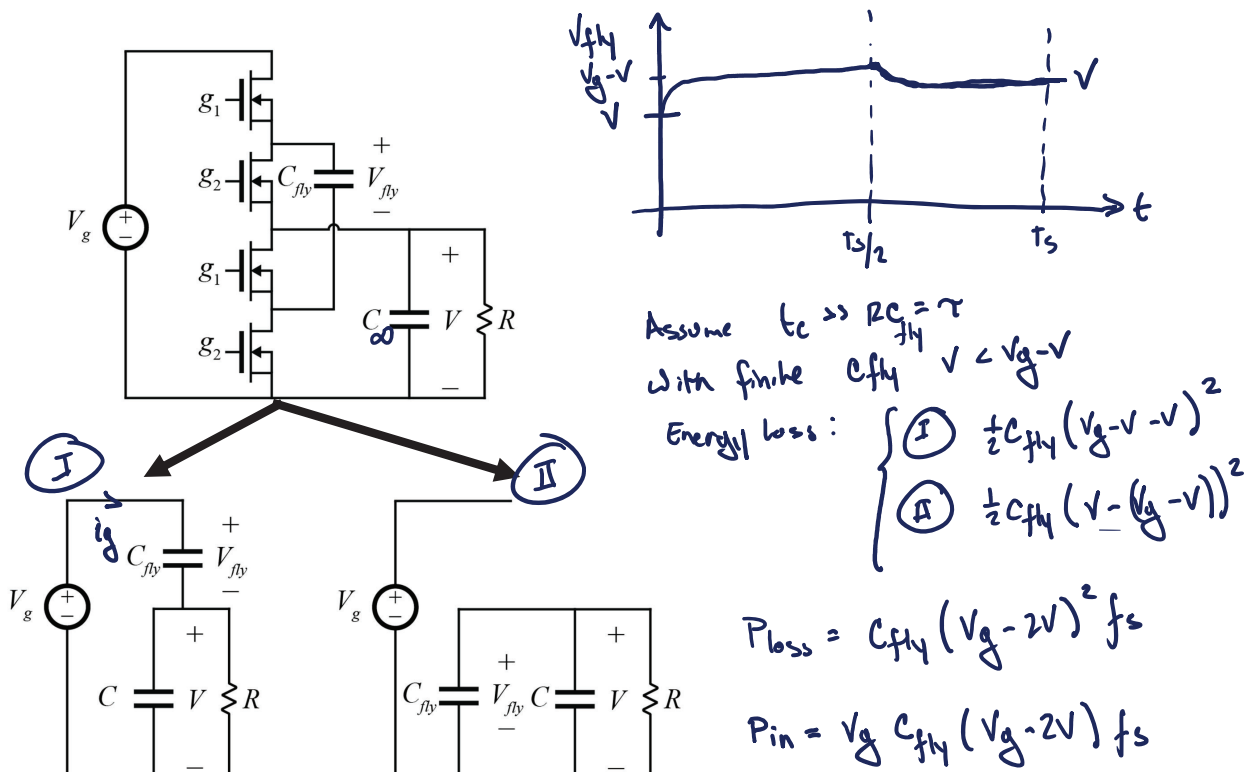
X

Resonant

$$\frac{1}{2} C (V_2 - V_1)^2 \left[ \frac{\pi^2 RC}{4 t_c} \right]$$

High-Q resonance  
 $R_0 \gg R$

## 2:1 SC Revisited

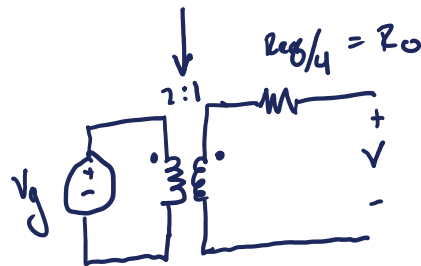
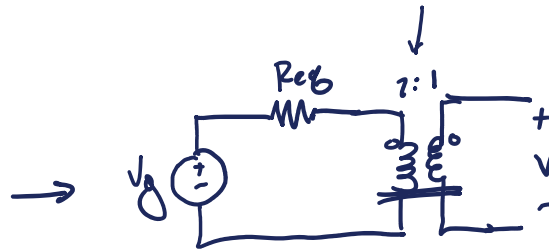
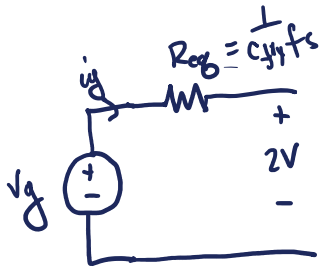


# Equivalent Circuit Model

$$P_{\text{loss}} = \underline{C_{fh} f_s} (V_g - 2V)^2$$

$$R_{eg} = \frac{1}{C_{fh} f_s}$$

$$P_{\text{in}} = V_g \underline{C_{fh} f_s} (V_g - 2V)$$



for high  $\eta$ , want small  $R_{eg}$   
 $\frac{1}{C_{fh} f_s} \rightarrow$  large  $C_{fh}$   
 large  $f_s$