Tellegen's Theorem

For any valid circuit (KVL $ KCL)

$$\sum_{i \in \text{element}} V_i I_i = \emptyset$$

Voltage across

current through

Passive sign convention held everywhere

For our 2-subinterval SC converter:

$$\bar{a}^{I}\bar{v}^{I} = \emptyset \quad \$ \quad \bar{a}^{II}\bar{v}^{II} = \emptyset$$

so, $\bar{a}^{I}\bar{v}^{I} + \bar{a}^{II}\bar{v}^{II} = \emptyset$

$$\underbrace{V_g\left(a_{in}^{I} + a_{in}^{II}\right)}_{\text{when testing } R_o \; = \emptyset} + \bar{a}_c^{I}\bar{v}_c^{I} + \bar{a}_c^{II}\bar{v}_c^{II} + V_{out}\underbrace{\left(a_{out}^{I} + a_{out}^{II}\right)}_{=1,\text{ by normalization}} = \emptyset$$

SSL Output Resistance

$$\bar{a}_c^{I}\bar{v}_c^{I} + \bar{a}_c^{II}\bar{v}_c^{II} = -V_{out}$$

$\bar{a}_c^{I} = -\bar{a}_c^{II}$, by charge balance in 2-subinterval SC conv.

$$\bar{a}_c^{I}\left(\bar{v}_c^{I} - \bar{v}_c^{II}\right) = -V_{out}$$

$$\bar{v}_c^{I} - \bar{v}_c^{II} = \Delta\bar{v}_c \qquad \Delta V_{ci} = \frac{q_i}{C_i} \quad \$ \quad a_{ci} = \frac{q_i}{q_{out}}$$

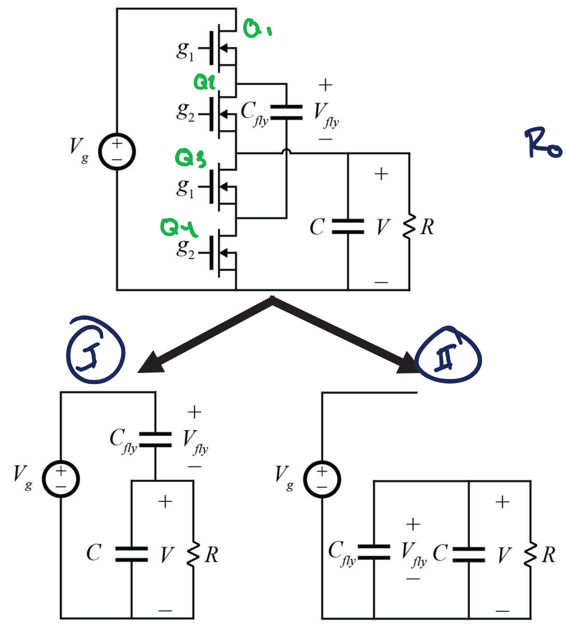
$$-V_{out} = \sum_{i \in \text{caps}} \left(\frac{q_i}{q_{out}}\right)\left(\frac{q_i}{C_i}\right) = \sum_{i \in \text{caps}} \frac{(q_i)^2}{q_{out}} \frac{1}{C_i}$$

$$\frac{-V_{out}}{q_{out} f_s} = \sum_{i \in \text{caps}} (a_{ci})^2 \frac{1}{C_i f_s} \qquad q_{out} f_s = I_{out}$$

for 2-subinterval SC converters

$$\frac{-V_{out}}{I_{out}} = R_o = \sum_{i \in \text{caps}} \frac{(a_{ci})^2}{C_i f_s} \longrightarrow \boxed{R_{o,SSL} = \sum_{i \in \text{caps}} \frac{(a_{ci})^2}{C_i f_s}}$$

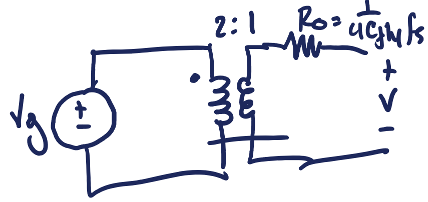
Output Resistance



$$\begin{array}{ccc} a_{in} & a_{cfly} & a_{out} \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$\bar{a}^{\mathrm{I}} = \left[-\frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2}\right] \qquad \bar{a}^{\mathrm{II}} = \left[0\ \ -\frac{1}{2}\ \ \frac{1}{2}\right]$$

$$R_{o,SSL} = \sum_{i=1}^{1} \frac{(a_{ci})^2}{C_i f_s} = \frac{(1/2)^2}{C_{fly} f_s} = \frac{1}{4 C_{fly} f_s}$$



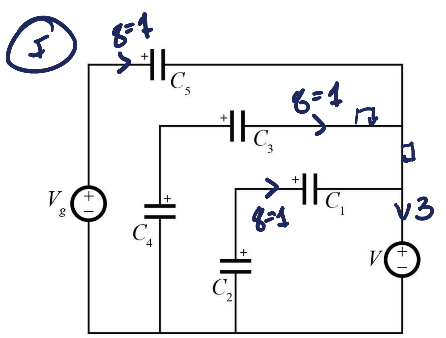
$$\begin{array}{ccccc} & g_1 & g_2 & g_3 & g_4 \\ \bar{a}_r = & [1/2 & 1/2 & 1/2 & 1/2] \end{array}$$

$$R_{o,FSL} = \sum_{i=trans.} (a_{ri})^2 \cdot 2 R_{on,i}$$

if all $R_{on,i}$ are the same

$$R_{o,FSL} = 2 R_{on}$$

Dickson Charge Vector Analysis

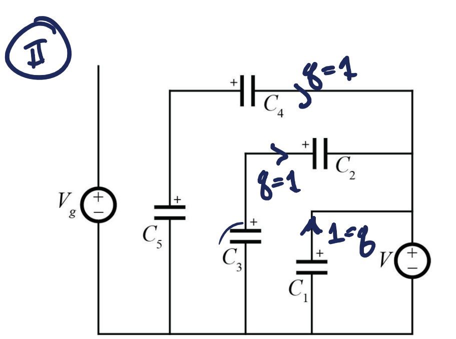


$$\begin{array}{cccccccc} & in & c_1 & c_2 & c_3 & c_4 & c_5 & out \\ \bar{a}^{I} = & [-1 & 1 & -1 & 1 & -1 & 1 & 3]/q_{out} \end{array}$$

$$\bar{a}^{I} = [-1/6\ \ 1/6\ \ -1/6\ \ 1/6\ \ -1/6\ \ 1/6\ \ 1/2]$$

$$\bar{a}^{II} = [0\ \ -1\ \ 1\ \ -1\ \ 1\ \ -1\ \ 3]/q_{out}$$

$$\bar{a}^{II} = [0\ \ -1/6\ \ 1/6\ \ -1/6\ \ 1/6\ \ -1/6\ \ 1/2]$$



$$q_{out} = q_{out}^{I} + q_{out}^{II} = 6$$

$$q_{in} = -1/6 \qquad q_{out} = 1$$

$$M = \frac{1}{6} = \frac{q_{in}}{q_{out}} = \frac{V_{out}}{V_g} \quad \checkmark$$

Dickson Output Resistance

$$R_{o,SSL} = \sum_{i \in caps} \frac{(a_{ci})^2}{C_i f_s}$$

Assume all $C_i = C_{fly}$

$$R_{o,SSL} = 5 \frac{(1/6)^2}{C_{fly} f_s} = \boxed{\frac{5}{36} \frac{1}{C_{fly} f_s} = R_{o,SSL}}$$

Charge Vector Analysis in FSL

$\bar{a}_r = \frac{\bar{q}_r}{q_{out}}$ where q_{ri} is the charge flowing through transistor i when it is conducting

No supersript because each transistor only conducts in one interval

Find $\bar{a}_r$ by inspection as some linear combination $\bar{a}_c^{\,I}$ & $\bar{a}_c^{\,II}$

i_{ri} = transistor current when conducting

$i_{ri} = q_{ri} \cdot \frac{2}{T_s}$ assume 2-subinterval SC converter @ D=50%

$i_{ri} = a_{ri}\, q_{out} \frac{2}{T_s}$

$P_{ri} = (i_{ri})^2 R_{on i} \cdot \left(\frac{1}{2}\right)$

$P_{ri} = \left(a_{ri}\, q_{out} \frac{2}{T_s}\right)^2 R_{on,i} \left(\frac{1}{2}\right) = (a_{ri})^2 q_{out}^2 f_s^2 \cdot 2 \cdot R_{on,i}$

$P_{ri} = I_{out}^2 (a_{ri})^2 \cdot 2 R_{on,i}$

Doesn't include ESR or other resistances

$$\boxed{R_{o,FSL} = \sum_{i \in transistors} (a_{ri})^2 \cdot 2 R_{on,i}}$$