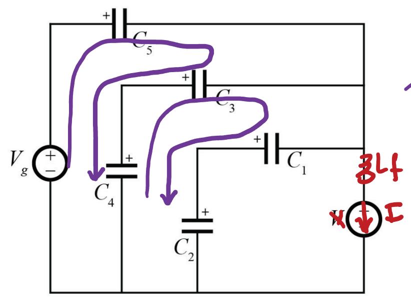
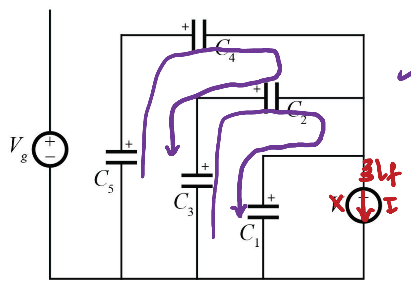
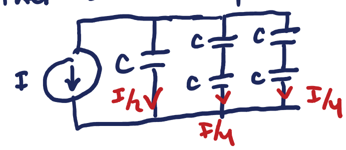
Hybrid Dickson converter





In either subinterval, DC solution is



However, there are also pure V-C loops. If any current flows in the V-C loops, there will still be charge-sharing loss.

Assume current is only I_{out} DC solution: i.e. zero current in V-C paths

$$\begin{array}{cccccccc} & & \text{in} & C_1 & C_2 & C_3 & C_4 & C_5 & \text{out} \\ \bar{a}^{I} \neq & [& -\frac{1}{4} & \frac{1}{8} & -\frac{1}{8} & \frac{1}{8} & -\frac{1}{8} & \frac{1}{4} & \frac{1}{2} &] \\ \bar{a}^{II} \neq & [& \emptyset & -\frac{1}{4} & \frac{1}{8} & -\frac{1}{8} & \frac{1}{8} & -\frac{1}{8} & \frac{1}{2} &] \end{array}$$

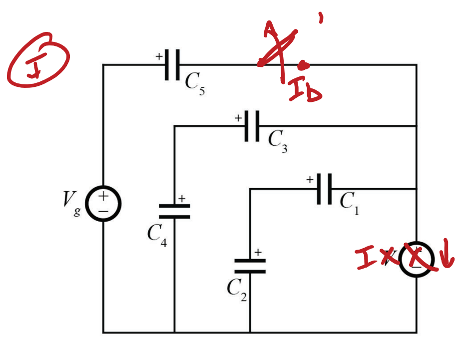
~~Iout Ts~~
~~Iout Ts~~

Because C_1 & C_5 don't have zero charge over T_s, this operation will have some hard charging

Split-Phase Control

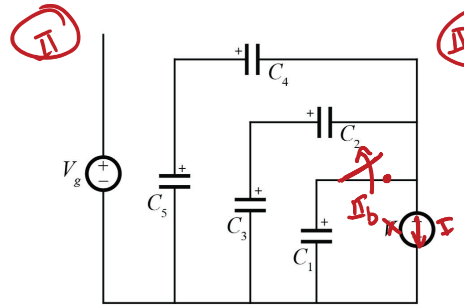
I

Ib :



II

IIb



$$\bar{a}^{I} = [-\tfrac{1}{4}\ \ \tfrac{1}{8}\ \ -\tfrac{1}{8}\ \ \tfrac{1}{8}\ \ -\tfrac{1}{8}\ \ \tfrac{1}{4}\ \ \tfrac{1}{2}]$$

$$\begin{array}{ccccccccc} \bar{a}^{Ib} = & [& \emptyset & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \emptyset & \frac{1}{2} &] \\ & & & C_1 & C_2 & C_3 & C_4 & C_5 & & \end{array}$$

$$\bar{a}^{II} = [\emptyset\ \ -\tfrac{1}{4}\ \ \tfrac{1}{8}\ \ -\tfrac{1}{8}\ \ \tfrac{1}{8}\ \ -\tfrac{1}{8}\ \ \tfrac{1}{2}]$$

$$\bar{a}^{IIb} = [\emptyset\ \ \emptyset\ \ \tfrac{1}{4}\ \ -\tfrac{1}{4}\ \ \tfrac{1}{4}\ \ -\tfrac{1}{4}\ \ \tfrac{1}{2}]$$

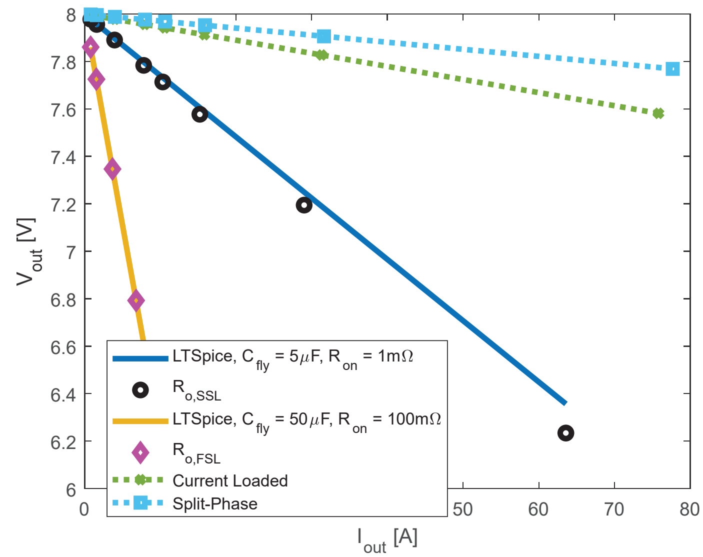
$$[D_I\ \ D_{Ib}\ \ D_{II}\ \ D_{IIb}] \begin{bmatrix} \bar{a}^{I} \\ \bar{a}^{Ib} \\ \bar{a}^{II} \\ \bar{a}^{IIb} \end{bmatrix} = [-\tfrac{1}{6}\ \ \underbrace{0\ 0\ 0\ 0\ 0}\ \ 1]$$

$D_I + D_{Ib} = \frac{1}{2}$ $D_{II} + D_{IIb} = 1/2$

$$\begin{cases} D_I = 2/3 = D_{II} \\ D_{Ib} = 1/3 = D_{IIb} \end{cases}$$

Y. Lei, R. May, and R. Pilawa-Podgurski, "Split-Phase Control: Achieving Complete Soft-Charging Operation of a Dickson Switched- Capacitor Converter," 2016

LTSpice Simulation



Switching Losses in SC Converters

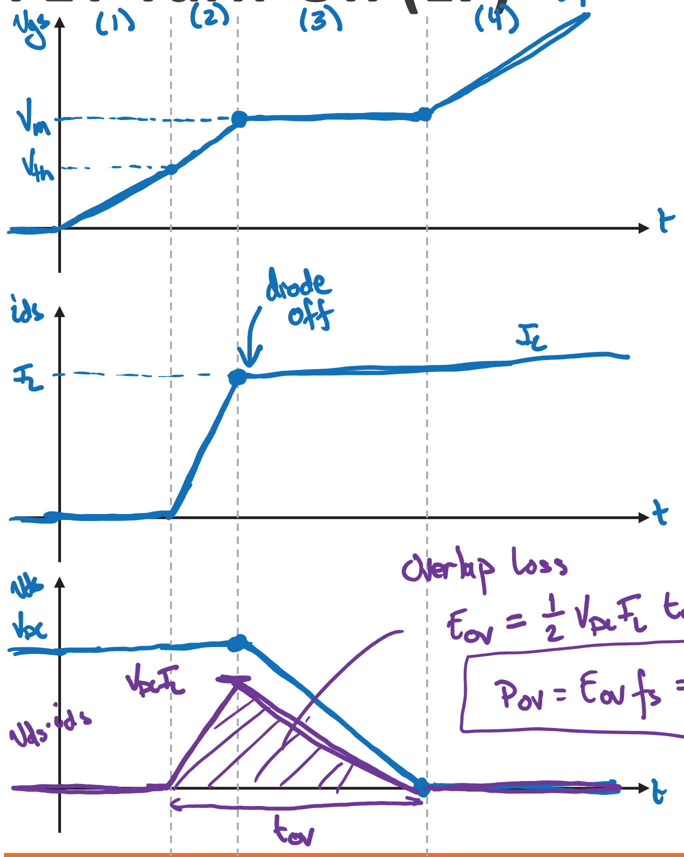
Pure SC converter in SSL:

Full Coss losses at turn-on

zero-current turn-off (≈ lossless)

some overlap losses may occur, but with resistive loading rather than inductively-clamped switching

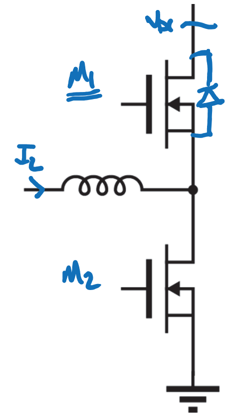
FET Turn-On (L7) $(M_2,\ I_L > 0)$

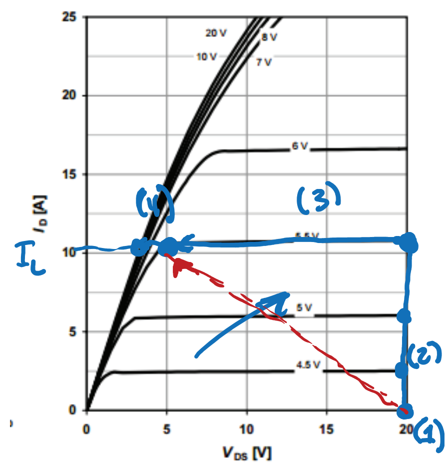


Overlap loss

$$E_{ov} = \frac{1}{2} V_{bc} I_L t_{ov}$$

$$\boxed{P_{ov} = E_{ov} f_s = \frac{1}{2} V_{bc} I_L \frac{t_{ov}}{T_s}}$$

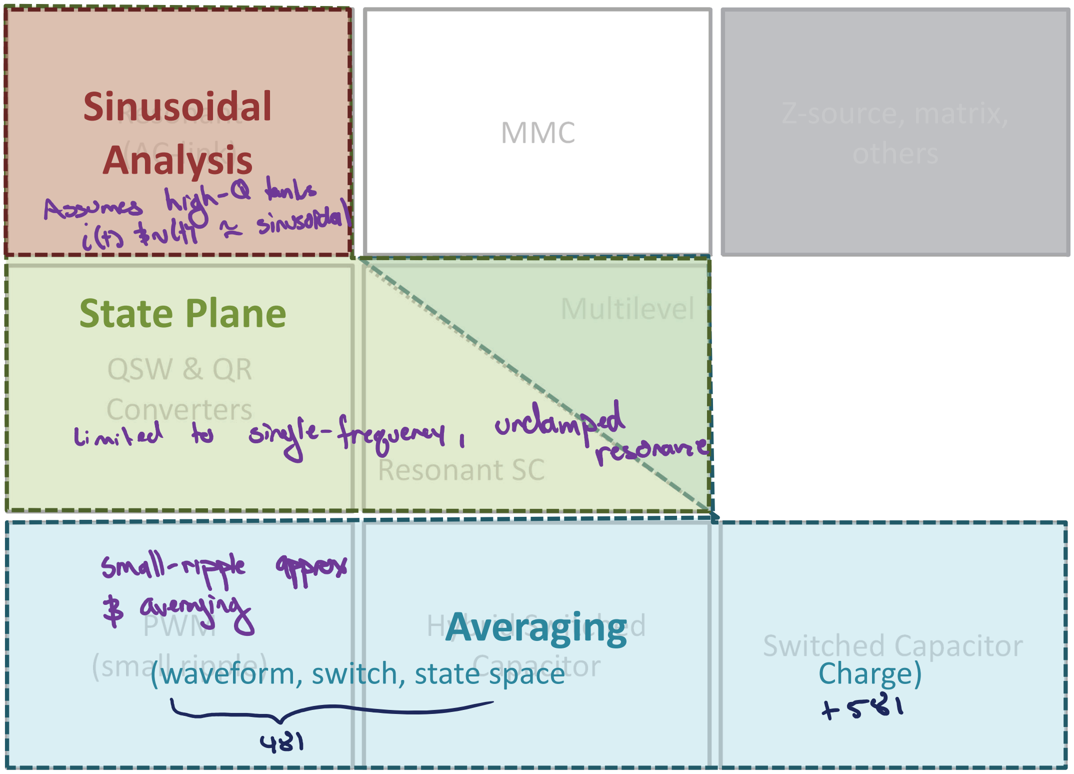




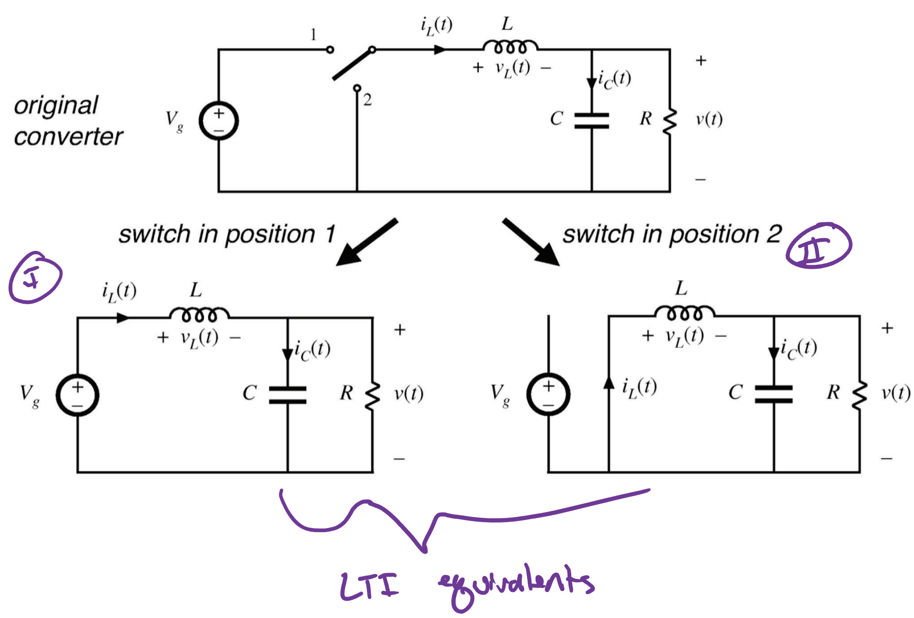
DISCRETE TIME MODELING

– Covered in detail in ECE 692

Converter Analysis



Switched Circuits



Historical Perspective

Robert D Middlebrook
PhD, Standford, 1955
CalTech Professor, 1955-1998

Slobodan Cúk
PhD CalTech, 1976
CalTech Prof, 1977-1999

Modelling, analysis, and design of switching converters

Model a switched system as an averaged, time-invariant system with

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t)$$

where

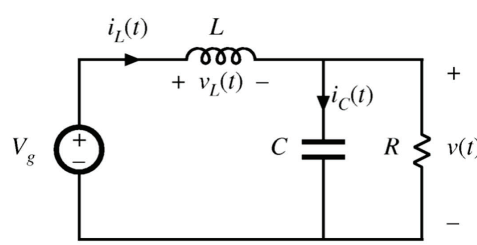
$$\boldsymbol{A} = D\boldsymbol{A}_1 + D'\boldsymbol{A}_2$$
$$\boldsymbol{B} = D\boldsymbol{B}_1 + D'\boldsymbol{B}_2$$

A. R. Brown and R. D. Middlebrook, "Sampled-data Modeling of Switching Regulators" PESC 1981

Linear Circuit Modeling Using State Space

In switch position 1

$$\begin{cases} v_g(t) - v_c(t) = L\dfrac{di_L(t)}{dt} \\ i_L(t) - \dfrac{v_c(t)}{R} = C\dfrac{dv_c(t)}{dt} \end{cases}$$



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Linear Circuit Modeling Using State Space

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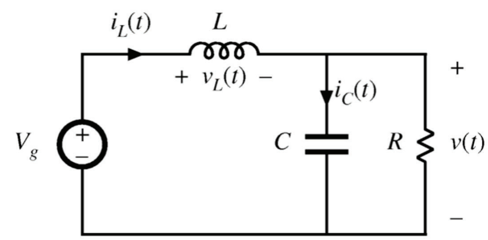


Which can be written, in state space, form as

$$\frac{d}{dt}\begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix}}_{A_1} \cdot \begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 1/L \\ 0 \end{bmatrix}}_{B_1} v_g(t)$$

Or, generally,

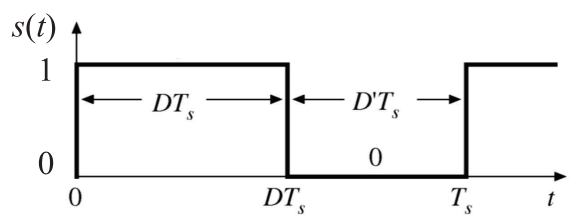
$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{B}_1 u(t)$$

In the second switch position, we will have a new (linear) circuit with

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_2\boldsymbol{x}(t) + \boldsymbol{B}_2 u(t)$$

Switching Signal

In a PWM converter with two switch positions, the two linear circuits combine according to a switching function $s(t)$



$$\dot{\boldsymbol{x}}(t) = [\boldsymbol{A}_1 s(t) + \boldsymbol{A}_2 s'(t)]\boldsymbol{x}(t) + [\boldsymbol{B}_1 s(t) + \boldsymbol{B}_2 s'(t)]u(t)$$

where

$$s(t) = \begin{cases} 1, & \text{if } \ nT_s < t < (n+D)T_s \\ 0, & \text{if } \ (n+D)T_s < t < (n+1)T_s \end{cases}$$

$$s'(t) = 1 - s(t)$$

SMPS State Space

In traditional state space modeling of linear systems

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t)$$

with $u(t)$ containing a control input. When $\boldsymbol{A}$ and $\boldsymbol{B}$ are constant, this is a linear system. However, we have

$$\dot{\boldsymbol{x}}(t) = \underbrace{[\boldsymbol{A}_1 s(t) + \boldsymbol{A}_2 s'(t)]}_{A_{eq}}\boldsymbol{x}(t) + \underbrace{[\boldsymbol{B}_1 s(t) + \boldsymbol{B}_2 s'(t)]}_{B_{eq}}u(t)$$

or, equivalently

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{eq}(t)\boldsymbol{x}(t) + \boldsymbol{B}_{eq}(t)u(t)$$

which is nonlinear: how do we deal with it?

Converting to Linear System

Assume that our system model

$$\dot{\boldsymbol{x}}(t) = [\boldsymbol{A}_1 s(t) + \boldsymbol{A}_2 s'(t)]\boldsymbol{x}(t) + [\boldsymbol{B}_1 s(t) + \boldsymbol{B}_2 s'(t)]u(t)$$

can be approximated by some linear system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t)$$

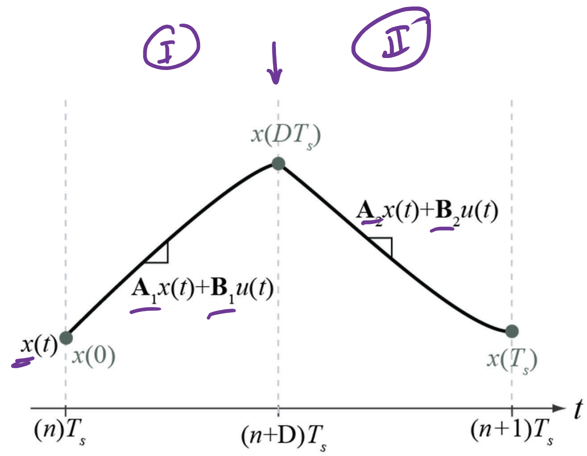
which removes the nonlinearity of the system

– Nonlinearities came from switching

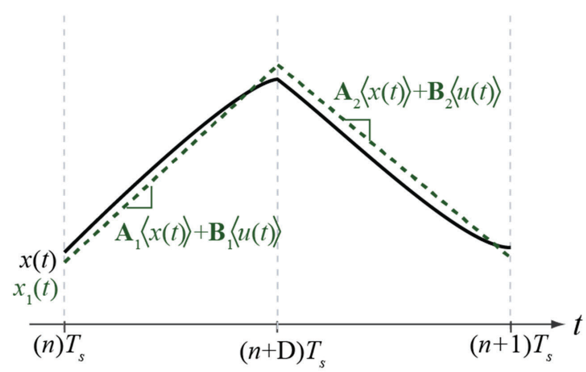
– Expect that switching dynamics will be lost

Note: This system is now linear in $\boldsymbol{x}(t)$ and $u(t)$, but not in our control signal, $s(t)$

Approximate Steady State Waveforms

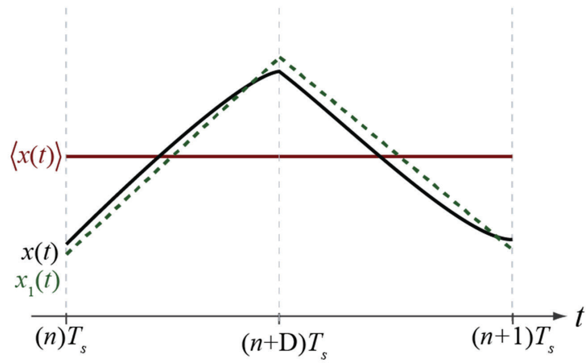


Approximate Steady State Waveforms



$$\langle x(t)\rangle = \frac{1}{T_s}\int_0^{T_s} x(t)dt$$

Approximate Steady State Waveforms



The Averaging Approximation

If waveforms can be approximated as linear

$$\dot{\boldsymbol{x}}(t) = \begin{cases} \boldsymbol{A_1}\langle \boldsymbol{x}(t)\rangle + \boldsymbol{B_1}\langle u(t)\rangle, & \text{if } \; nT_s < t < (n+D)T_s \\ \boldsymbol{A_2}\langle \boldsymbol{x}(t)\rangle + \boldsymbol{B_2}\langle u(t)\rangle,, & \text{if } \; (n+D)T_s < t < (n+1)T_s \end{cases}$$

so the average slope is

$$\langle \dot{\boldsymbol{x}}(t)\rangle$$
$$= \frac{1}{T_s}(\boldsymbol{A_1}\langle \boldsymbol{x}(t)\rangle + \boldsymbol{B_1}\langle u(t)\rangle)DT_s + (\boldsymbol{A_2}\langle \boldsymbol{x}(t)\rangle + \boldsymbol{B_2}\langle u(t)\rangle)(1-D)T_s$$

or, rearranging

Averge model

$$\langle \dot{\boldsymbol{x}}(t)\rangle = (D\boldsymbol{A_1} + D'\boldsymbol{A_2})\langle \boldsymbol{x}(t)\rangle + (D\boldsymbol{B_1} + D'\boldsymbol{B_2})\langle u(t)\rangle$$

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