Announcements

- PCBs ordered
  - EECS ordering will contact you directly when parts/PCBs arrive
  - FPGA / Interface boards will be made available today
    - May require some population/testing
- TNvoice available
  - https://utk.campuslabs.com/eval-home/
  - Currently, 2/6 completed
  - Closes 12/1

Remaining Tasks

- TNVoice anonymous eval due 12/1
- Track I: PCB Testing
  - Final testing report due 12/9
  - Format template available on website
    - *short* additional narrative if results do not correspond to predictions
- Track II: Final Exam
  - Posted 11/30, Due 12/9
  - Same rules/format as midterm exam
  - Comprehensive, covering all course material

Historical Perspective

Robert D Middlebrook
PhD, Standford, 1955
CalTech Professor, 1955-1998

Slobodan Cúk
PhD CalTech, 1976
CalTech Prof, 1977-1999

Dennis John Packard
PhD, CalTech 1976

Modelling, analysis, and design of switching converters

Model a switched system as an averaged, time-invariant system with

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t)$$

where

$$\boldsymbol{A} = D\boldsymbol{A_1} + D'\boldsymbol{A_2}$$
$$\boldsymbol{B} = D\boldsymbol{B_1} + D'\boldsymbol{B_2}$$

Discrete modeling and analysis of switching regulators

Model a switched system as a discrete-time system with

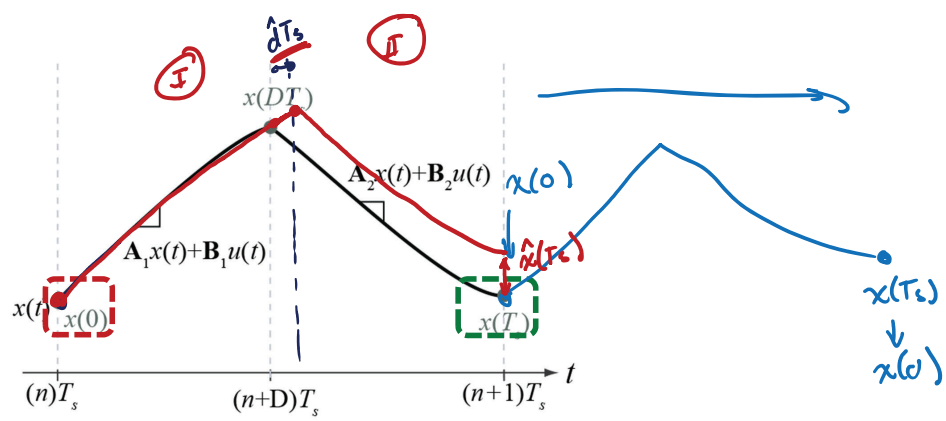
$$\boldsymbol{x}[n+1] = \boldsymbol{\Phi}\boldsymbol{x}[n] + \boldsymbol{\Psi}U[n]$$

where

$$\boldsymbol{\Phi} = \left(\prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)$$
$$\boldsymbol{\Psi} = \sum_{i=1}^{n_{sw}}\left\{\left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k}\right)\boldsymbol{A}_i^{-1}\left(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}\right)\boldsymbol{B}_i\right\}$$

A. R. Brown and R. D. Middlebrook, "Sampled-data Modeling of Switching Regulators" PESC 1981

Application to Switching Converter



$$\boldsymbol{x}(DT_s) = e^{\boldsymbol{A}_1 DT_s}\boldsymbol{x}(0) + \boldsymbol{A}_1^{-1}\left(e^{\boldsymbol{A}_1 DT_s} - \boldsymbol{I}\right)\boldsymbol{B}_1 U$$

$$\boldsymbol{x}(T_s) = e^{\boldsymbol{A}_2 D' T_s}\boldsymbol{x}(DT_s) + \boldsymbol{A}_2^{-1}\left(e^{\boldsymbol{A}_2 D' T_s} - \boldsymbol{I}\right)\boldsymbol{B}_2 U$$

$$\boxed{\boldsymbol{x}(T_s)} = e^{\boldsymbol{A}_2 D' T_s} e^{\boldsymbol{A}_1 DT_s}\boxed{\boldsymbol{x}(0)} + \boldsymbol{A}_2^{-1}\left(e^{\boldsymbol{A}_2 D' T_s} - \boldsymbol{I}\right)\boldsymbol{B}_2 U + e^{\boldsymbol{A}_2 D' T_s}\boldsymbol{A}_1^{-1}\left(e^{\boldsymbol{A}_1 DT_s} - \boldsymbol{I}\right)\boldsymbol{B}_1 U$$

General Form

Generally, for n_{sw} separate switching positions

$$\boldsymbol{x}(T_s) = \underbrace{\left(\prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)}_{\Phi}\boldsymbol{x}(0) + \underbrace{\sum_{i=1}^{n_{sw}}\left\{\left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k}\right)\boldsymbol{A}_i^{-1}(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I})\boldsymbol{B}_i\right\}}_{\Psi} U$$

Equation is in the form of a discrete-time system with

$$\boldsymbol{x}[n+1] = \boldsymbol{\Phi}\boldsymbol{x}[n] + \boldsymbol{\Psi}U[n]$$ ← LTI DT system

Again, the effect of changing modulation (i.e. t_i) is hidden in nonlinear terms

$$\hat{\boldsymbol{x}}[n+1] = \boldsymbol{\Phi}\hat{\boldsymbol{x}}[n] + \boldsymbol{\Psi}\hat{u}[n] + \boldsymbol{\Gamma}\hat{d}[n]$$

Find $\boldsymbol{\Gamma}$ by small-signal modeling

Steady-State Large-Signal Analysis

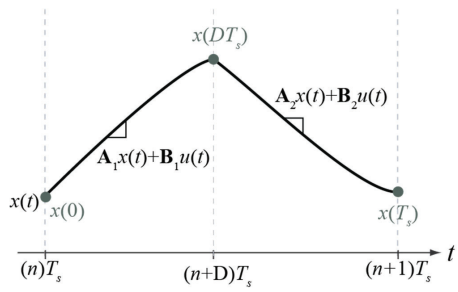
$$\boldsymbol{x}(T_s) = \left(\prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)\boldsymbol{x}(0) + \sum_{i=1}^{n_{sw}}\left\{\left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k}\right)\boldsymbol{A}_i^{-1}(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I})\boldsymbol{B}_i\right\} U$$

In steady-state, $\boldsymbol{x}(T_s) = \boldsymbol{x}(0)$

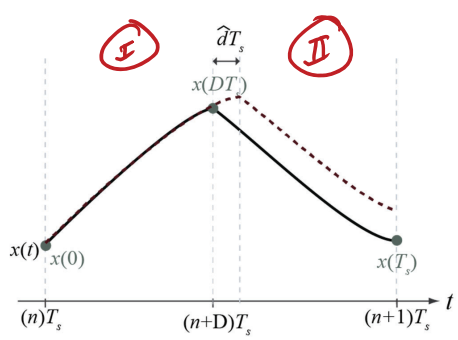
$$\boldsymbol{x}(T_s) = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)^{-1}\sum_{i=1}^{n_{sw}}\left\{\left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k}\right)\boldsymbol{A}_i^{-1}(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I})\boldsymbol{B}_i\right\} U$$

Gives explicit solution for steady-state operation of any switching circuit

Small Signal Modeling



Small Signal Modeling



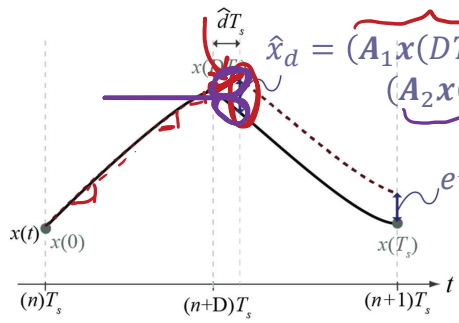
Small Signal Modeling

$\dot{x}_I = A_1 x(DT_s) + B_1 U$

$\dot{x}_I$

$$\hat{x}_d = \overbrace{(\boldsymbol{A}_1\boldsymbol{x}(DT_s) + \boldsymbol{B}_1 U)}^{\dot{x}_I}\hat{d}T_s - \underbrace{(\boldsymbol{A}_2\boldsymbol{x}(DT_s) + \boldsymbol{B}_2 U)}_{\dot{x}_{II}}\hat{d}T_s$$

$e^{\boldsymbol{A}_2 D' T_s}\hat{x}_d$



Complete Small Signal Model

This completes the small-signal model

$$\hat{\boldsymbol{x}}[n+1] = \boldsymbol{\Phi}\hat{\boldsymbol{x}}[n] + \boldsymbol{\Psi}\hat{u}[n] + \boldsymbol{\Gamma}\hat{d}[n]$$

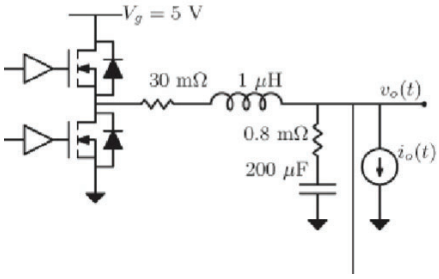
where

$x(DT_s)$ in steady-state

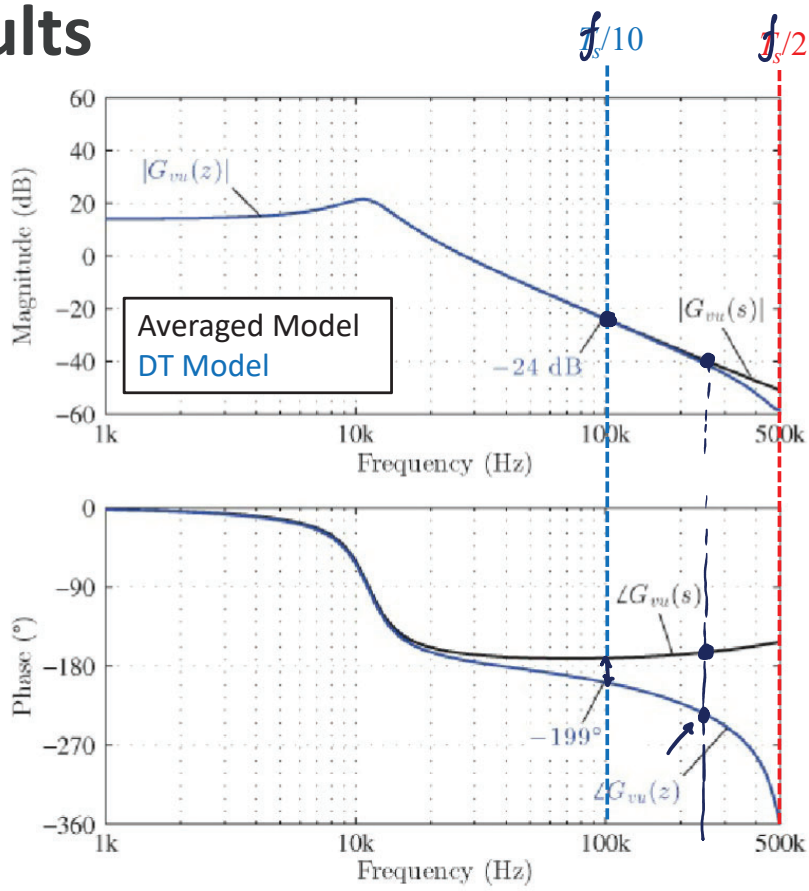
$$\boldsymbol{\Gamma} = e^{\boldsymbol{A}_2 D' T_s}\big((\boldsymbol{A}_1 - \boldsymbol{A}_2)\boldsymbol{X}_D + (\boldsymbol{B}_1 - \boldsymbol{B}_2)U\big)T_s$$

with $\boldsymbol{X}_D = \boldsymbol{x}(DT_s)$ in steady-state

Example Results

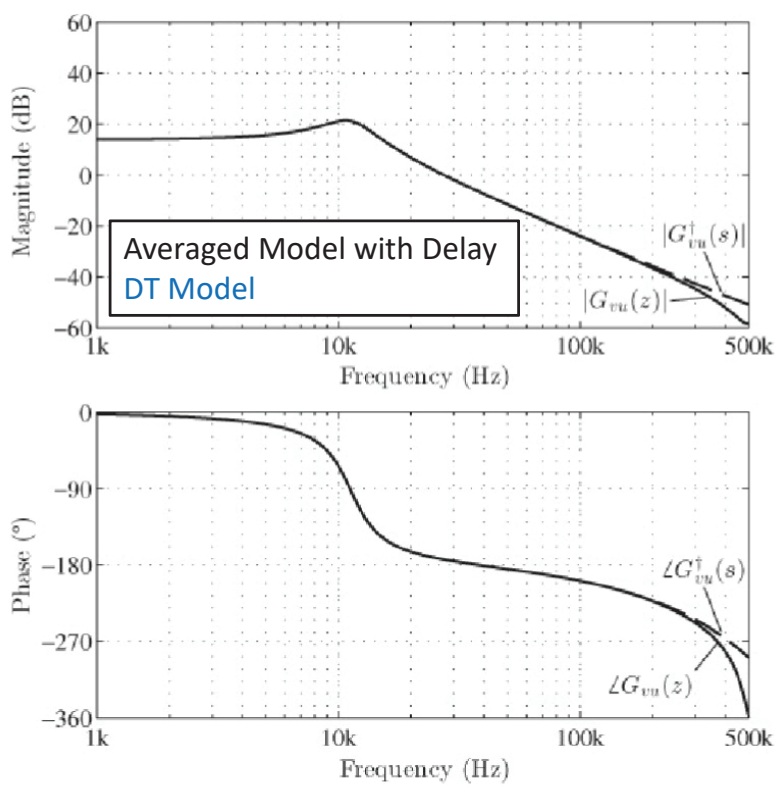


* Includes t_d=760ns of delay in feedback loop

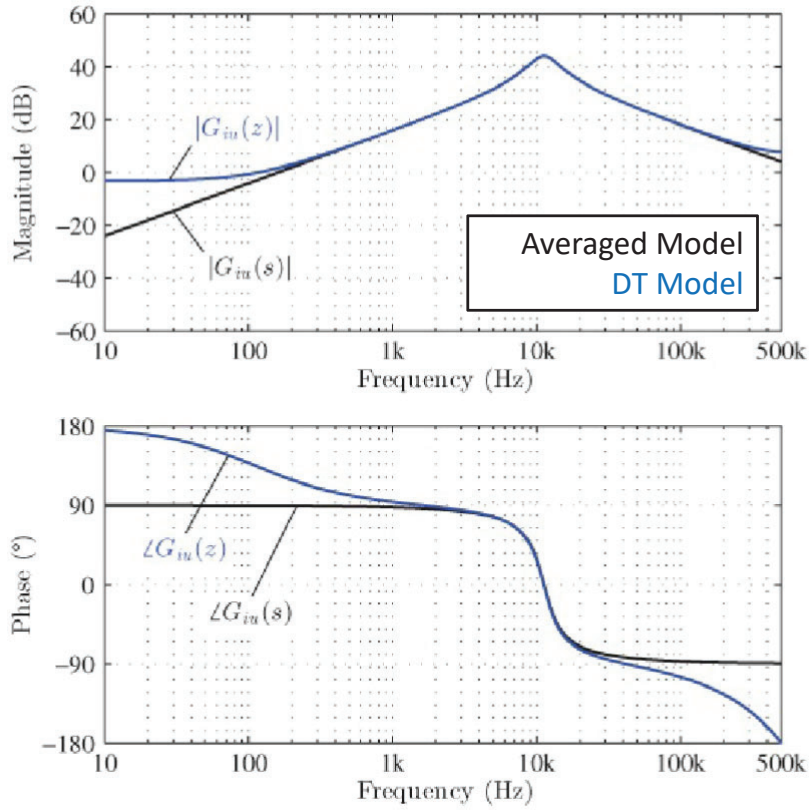


Inclusion of Delay

$$G_{vu}^{\dagger}(s) = G_{vu}(s)e^{-st_d}$$



Current Control



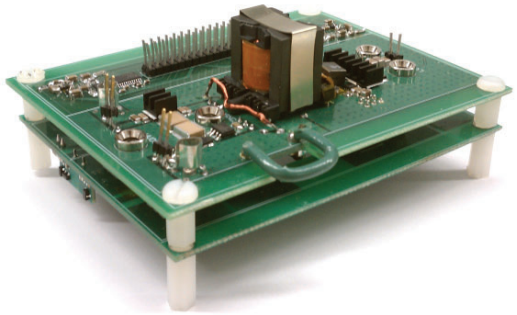
L. Corradini et. al. *Digital Control of High Frequency Switched-Mode Power Converters,* Section 3.2

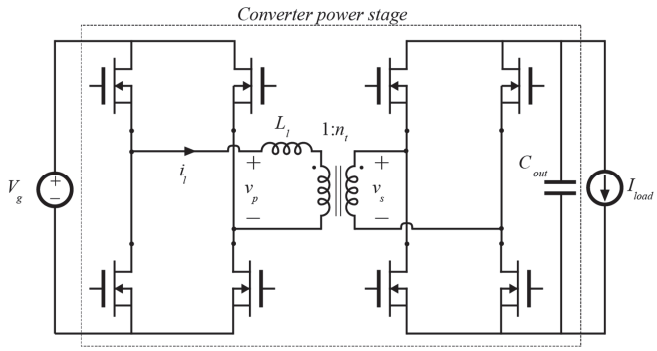
Discrete Time Analysis: Results

$$\boldsymbol{X_{ss}} = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i} \right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k} \right) \boldsymbol{A}_i^{-1} (e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}) \boldsymbol{B}_i \right\} U_i$$

- Valid for any switched circuit, as long as
  1. Inputs, U, are constant or slowly varying
  2. All times t_i are known
  3. Every subinterval can be described by a linear circuit
- Requires no dedicated analysis other than finding $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$
- Decisively **not** a design-oriented equation

Example: DAB Design Using Dedicated Analysis

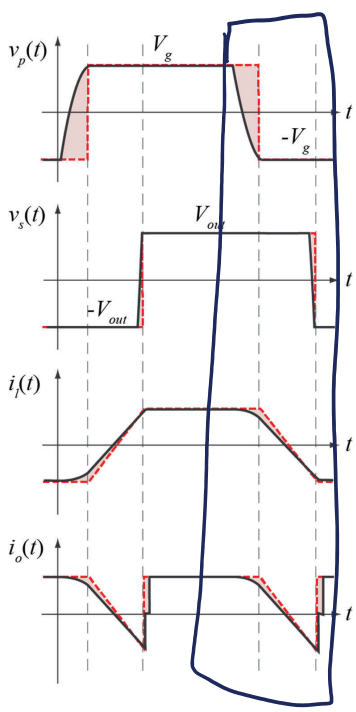


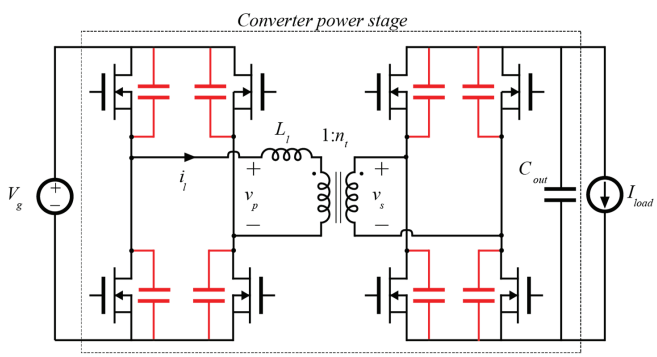


- Design of a high step-down DAB for Data Centers
- 150-to-12V, 120 W, 1MHz, design
- Prototype achieved 98.4% peak efficiency

D Costinett, D. Maksimovic, and R Zane, "Design and Control for High fficiency in High Step-Down Dual Active Bridge Converters Operating at High Switching Frequency," *IEEE Trans. On Pwr. Elec.*, 2013

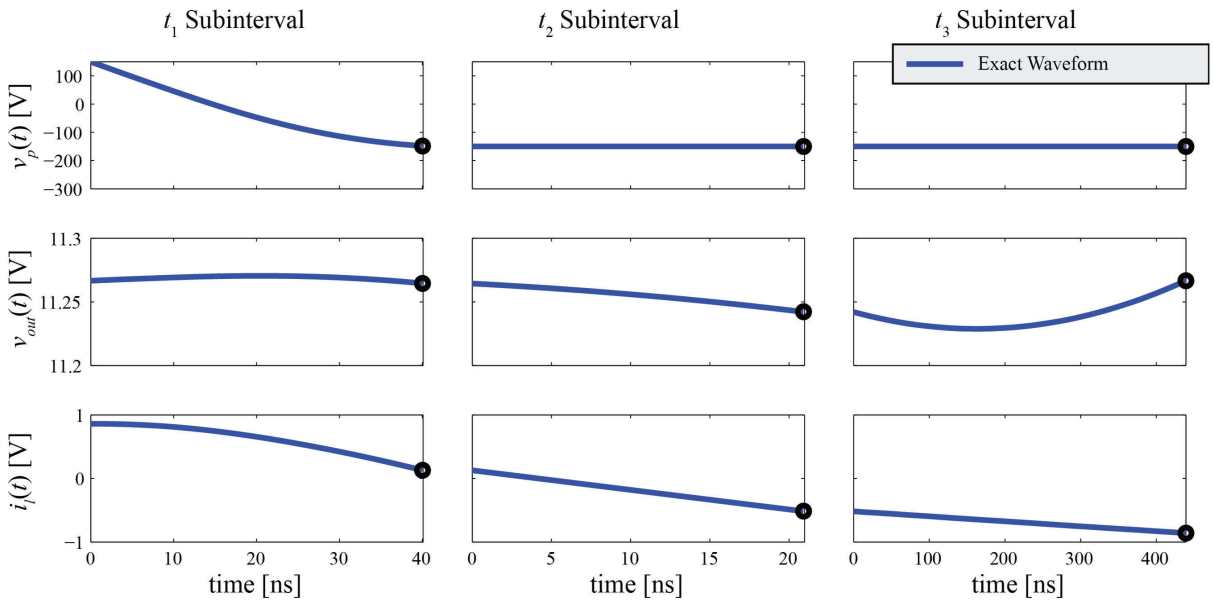
DAB Operated at High Frequency





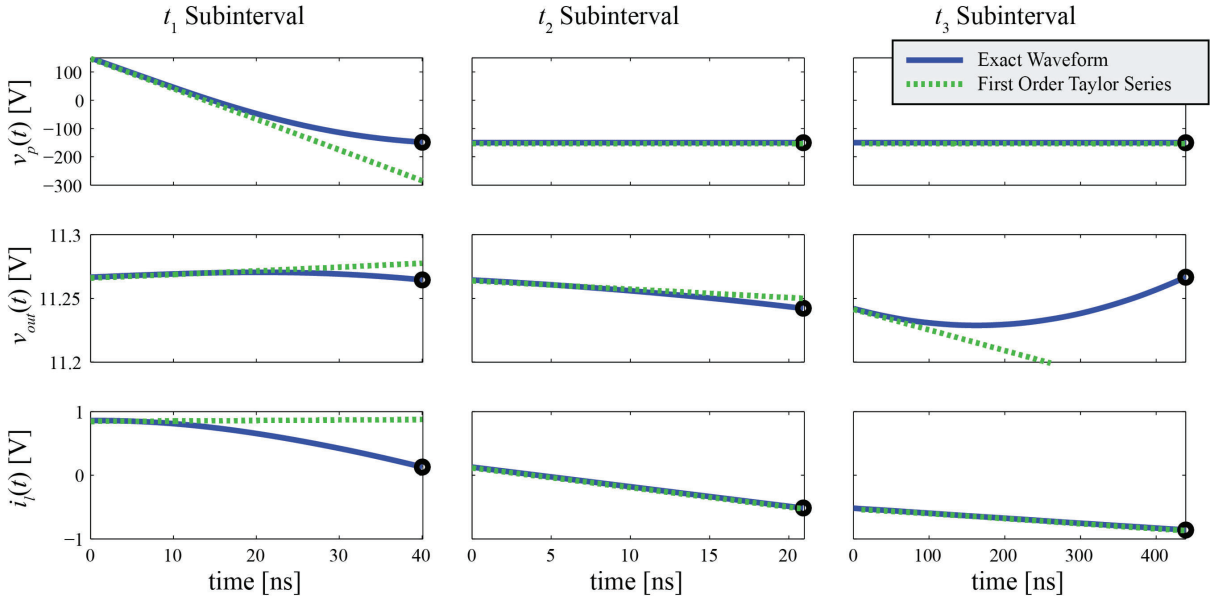
- Resonance between L_l and transistor capacitance distorts waveforms
- Resonance *may* need to be modeled when operating at high frequency

DAB Waveforms



$e^{A_1 t_1}$ $e^{A_2 t_2}$ $e^{A_3 t_3}$

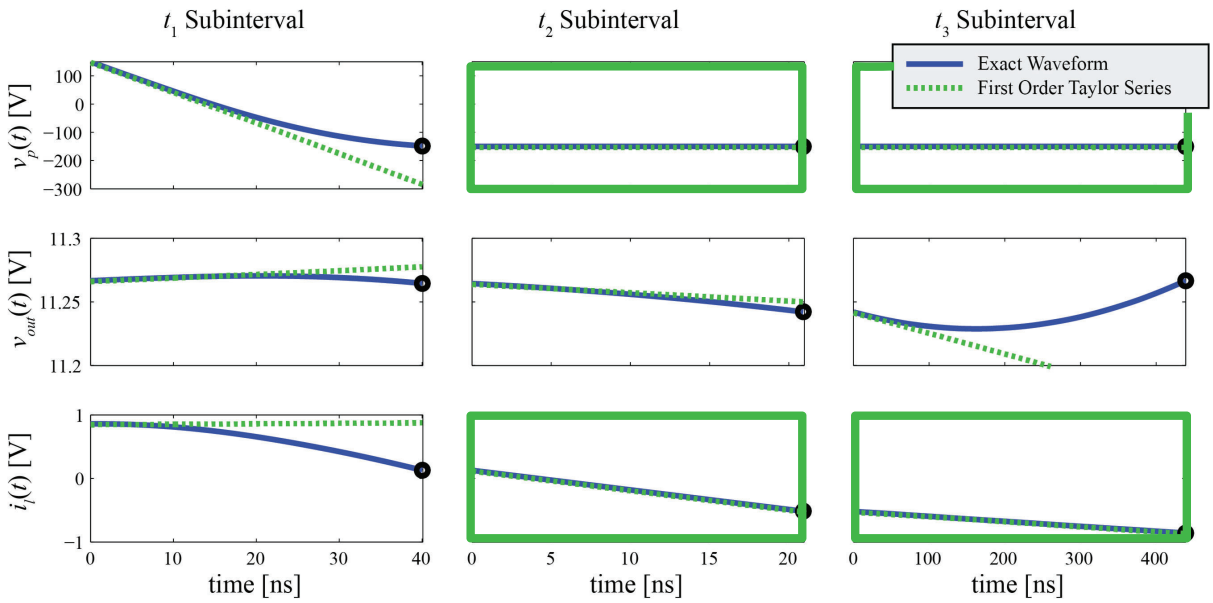
Linear Waveform Approximation



$e^{A_1 t_1}$ $e^{A_2 t_2}$ $e^{A_3 t_3}$

$I + A_1 t_1$ $I + A_2 t_2$ $I + A_3 t_3$

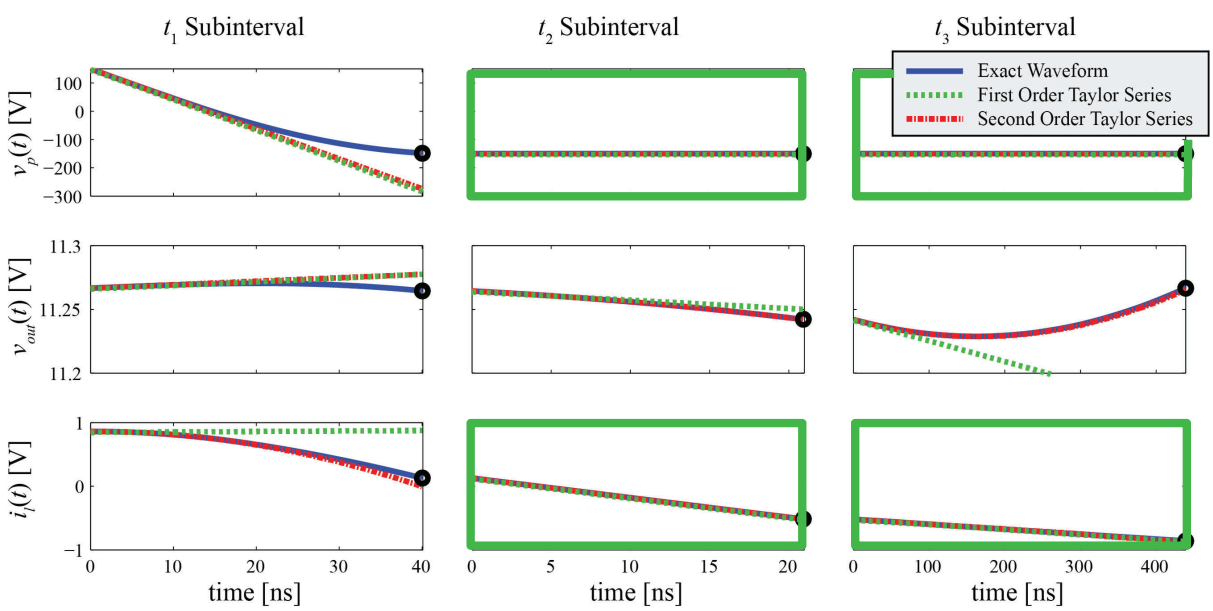
Linear Waveform Approximation



$e^{A_1 t_1}$ $e^{A_2 t_2}$ $e^{A_3 t_3}$

$I + A_1 t_1$ $I + A_2 t_2$ $I + A_3 t_3$

Second Order Approximation

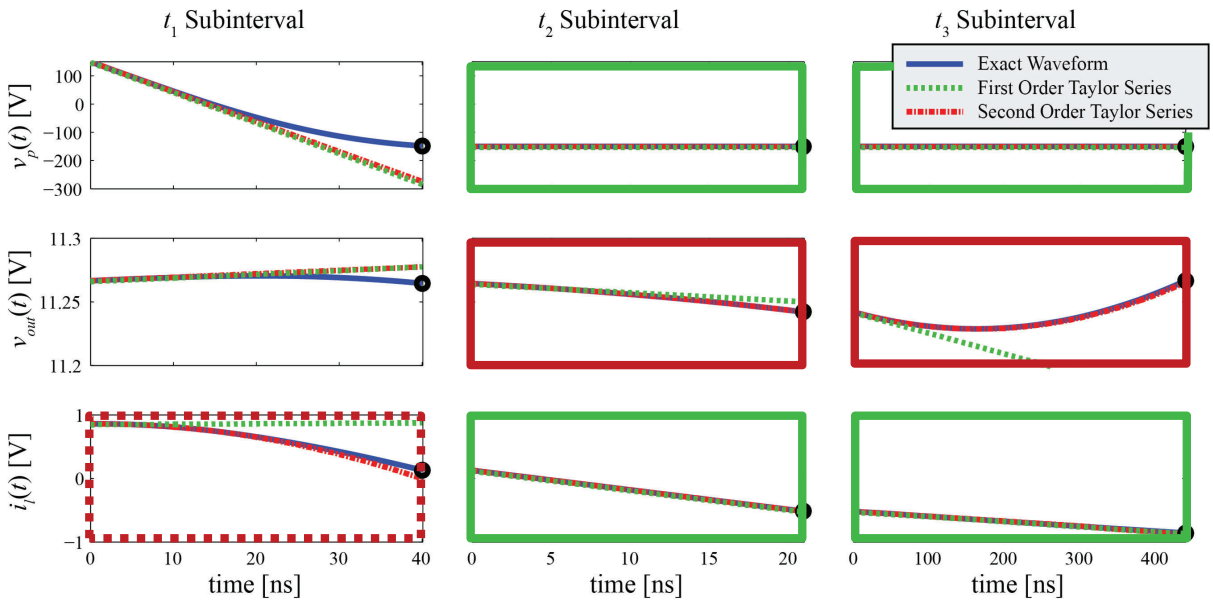


$e^{A_1 t_1}$ $e^{A_2 t_2}$ $e^{A_3 t_3}$

$I + A_1 t_1$ $I + A_2 t_2$ $I + A_3 t_3$

$I + A_1 t_1 + \frac{1}{2}(A_1 t_1)^2$ $I + A_2 t_2 + \frac{1}{2}(A_2 t_2)^2$ $I + A_3 t_3 + \frac{1}{2}(A_3 t_3)^2$

Second Order Approximation



$e^{A_1 t_1}$	$e^{A_2 t_2}$	$e^{A_3 t_3}$
$I + A_1 t_1$	$I + A_2 t_2$	$I + A_3 t_3$
$I + A_1 t_1 + \frac{1}{2}(A_1 t_1)^2$	$I + A_2 t_2 + \frac{1}{2}(A_2 t_2)^2$	$I + A_3 t_3 + \frac{1}{2}(A_3 t_3)^2$

State Plane Solution

Solution "Read off" state plane

$$\alpha = \sin^{-1}\frac{2}{J_p}$$ Primary Dead Time

$$\beta = \frac{1}{2}(J_2 + J_1)$$ Phase Shift

$$\delta = n_t \sin^{-1}\frac{2n_t^2}{\frac{R_0'}{R_0}J_p}\sqrt{\frac{C_s}{C_p}}$$ Secondary Dead Time

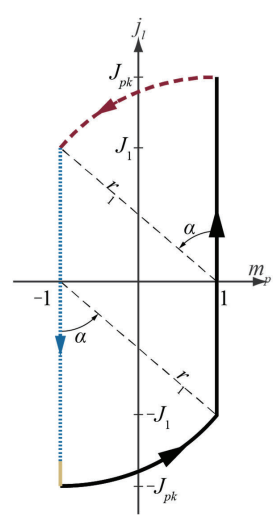
$$\zeta = \frac{F}{\pi} - \alpha - \beta - \delta$$

$$J_1 = \sqrt{J_p^2 - 4}$$

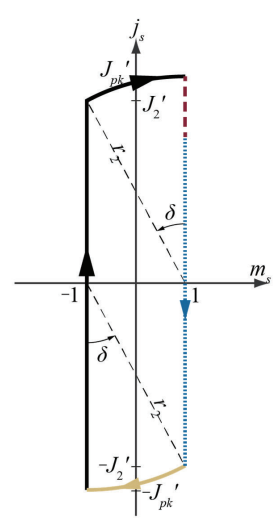
$$J_2 = \frac{R_0}{R_0'}\sqrt{\left(J_p\frac{R_0'}{R_0}\right)^2 - (2n_t)^2}$$

$$J = \frac{F}{\pi}\left[2 + \frac{1}{4}\left(J_1^2 - J_2^2\right) + J_p\left(\frac{\pi}{F} - \alpha - \beta - \delta\right)\right]$$

Primary

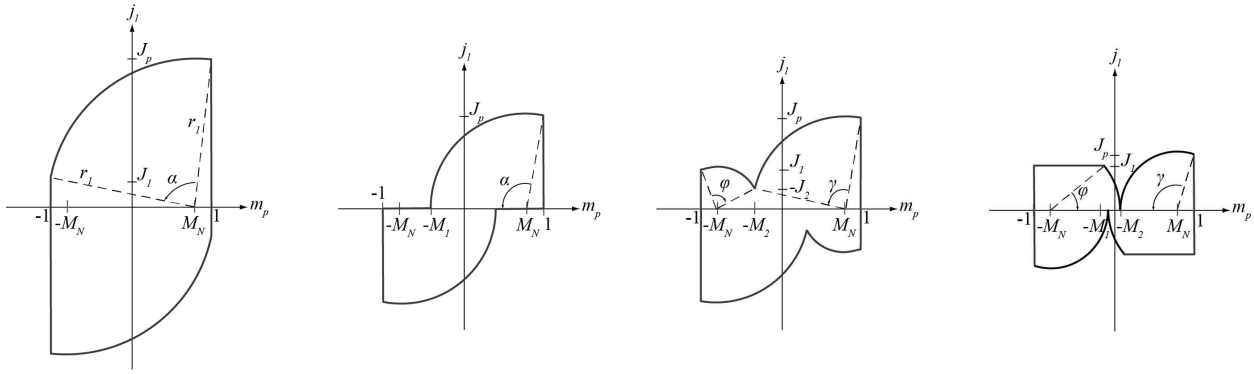


Secondary



Different Operating Modes

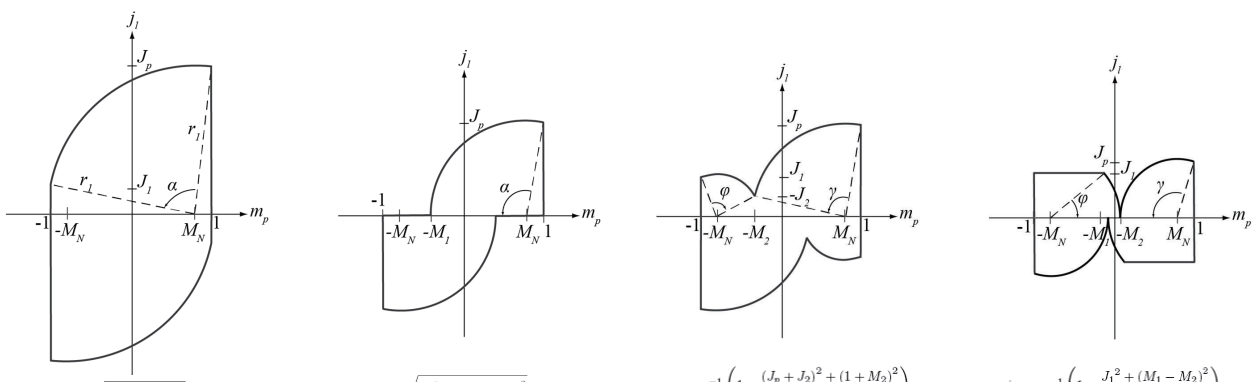
Mode I | Mode II | Mode III | Mode IV



- As control, input and load vary, operating mode changes
- In each mode, solution is a set of transcendental equations

Different Operating Modes

Mode I | Mode II | Mode III | Mode IV



$$J_1 = \sqrt{{J_p}^2 - 4M_N}$$

$$\alpha = \cos^{-1}\left(1 - \frac{(J_p - J_1)^2 + 4}{2{J_p}^2 + 2(1 - M_n)^2}\right)$$

$$\beta = \frac{J_1 + J_2}{1 + M_N}$$

$$\zeta = \frac{J_p - J_2}{1 - M_N}$$

$$\frac{\pi}{F} = \alpha + \beta + \delta$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2 + \frac{J_p + J_2}{2}\zeta + \frac{J_1 - J_2}{2}\beta\ .\right)$$

$$M_1 = \sqrt{{J_p}^2 + (1 - M_N)^2} - M_N$$

$$\alpha = \cos^{-1}\left(1 - \frac{{J_p}^2 + (1 + M_1)^2}{2(M_1 - M_n)^2}\right)$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(1 + M_1 + \frac{J_p + J_2}{2}\zeta - \frac{J_2}{2}\beta\right)$$

$$\beta = \frac{J_1 + J_2}{1 + M_N}$$

$$\zeta = \frac{J_p - J_2}{1 - M_N}$$

$$\frac{\pi}{F} = \alpha + \beta + \delta$$

$$\gamma = \cos^{-1}\left(1 - \frac{(J_p + J_2)^2 + (1 + M_2)^2}{2(1 - M_n)^2 + 2{J_p}^2}\right)$$

$$\phi = \cos^{-1}\left(1 - \frac{(J_1 + J_2)^2 + (1 - M_2)^2}{2(M_n - M_2)^2 + 2{J_2}^2}\right)$$

$${J_2}^2 = (1 - M_N)^2 + {J_p}^2 - (M_2 + M_N)^2$$

$${J_1}^2 = -(1 - M_N)^2 + {J_2}^2 + (M_N - M_2)^2$$

$$\zeta = \frac{J_p + J_1}{1 - M_N}$$

$$\frac{\pi}{F} = \gamma + \phi + \zeta = \alpha + \zeta$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2M_2 + \frac{J_p - J_1}{2}\zeta\right)$$

$$\phi = \cos^{-1}\left(1 - \frac{{J_1}^2 + (M_1 - M_2)^2}{2(M_n - M_2)^2}\right)$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2M_2 + 1 - M_1 + \frac{J_p - J_1}{2}\zeta\right)$$

$${J_2}^2 = (1 - M_N)^2 + {J_p}^2 - (M_2 + M_N)^2$$

$${J_1}^2 = -(1 - M_N)^2 + {J_2}^2 + (M_N - M_2)^2$$

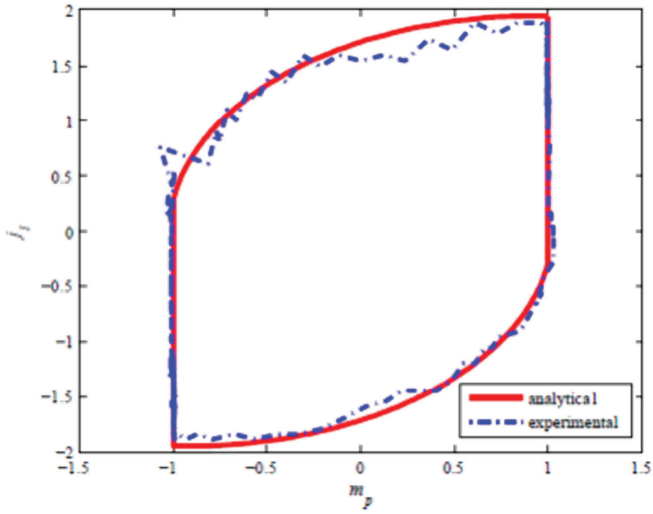
$$\zeta = \frac{J_p + J_1}{1 - M_N}$$

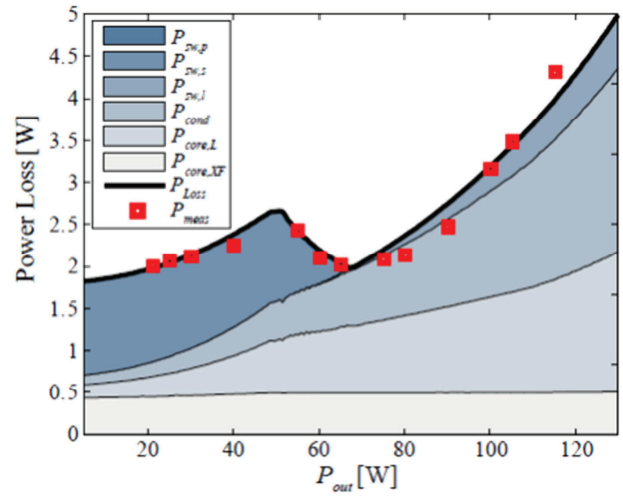
$$\frac{\pi}{F} = \gamma + \phi + \zeta = \alpha + \zeta$$

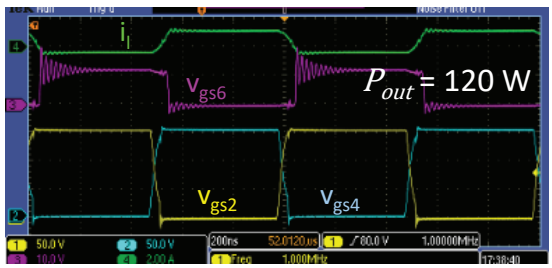
D Costinett, D. Maksimovic, and R Zane, "Design and Control for High Efficiency in High Step-Down Dual Active Bridge Converters Operating at High Switching Frequency," *Trans. On Pwr. Elec.* 2013

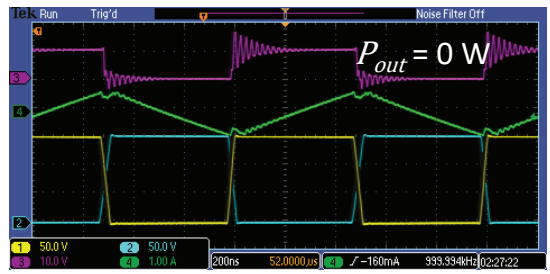
THE UNIVERSITY OF TENNESSEE KNOXVILLE

Model Validation







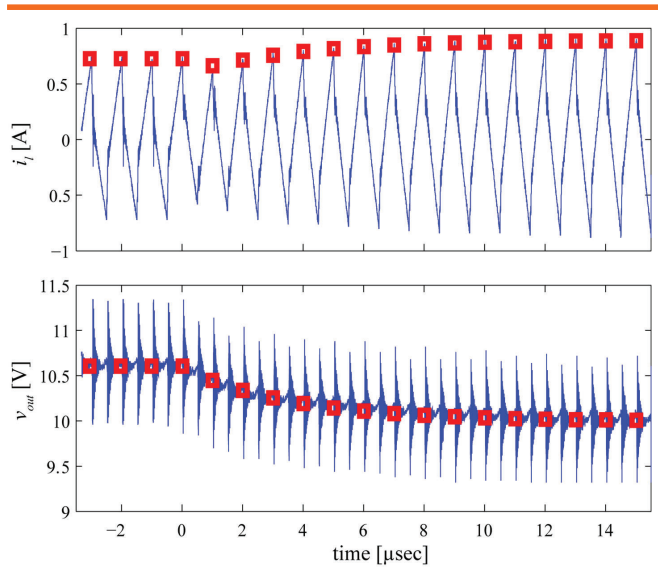


D Costinett, D. Maksimovic, and R Zane, "Design and Control for High fficiency in High Step-Down Dual Active Bridge Converters Operating at High Switching Frequency," *IEEE Trans. On Pwr. Elec.*, 2013

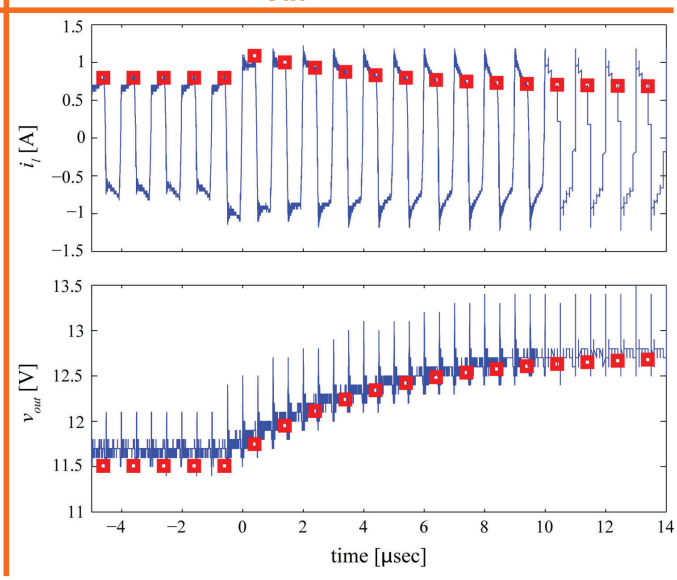
THE UNIVERSITY OF TENNESSEE KNOXVILLE

Discrete Time Model Validation

P_{out} = 30 W



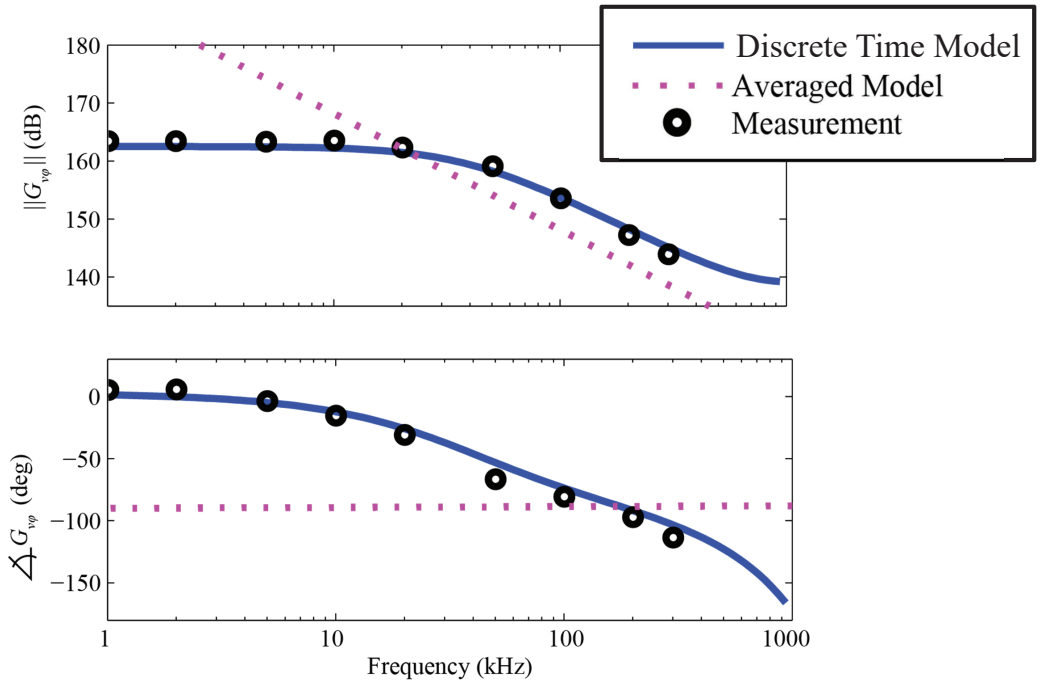
P_{out} = 90 W



D. Costinett, R. Zane, and D. Maksimovic, "Discrete-time small-signal modeling of a 1 MHz efficiency-optimized dual active bridge converter with varying load," in Proc. IEEE Workshop Contr. Modl. (COMPEL), june 2012, pp. 1 –7.

THE UNIVERSITY OF TENNESSEE KNOXVILLE

Discrete Time Dynamic Model Validation

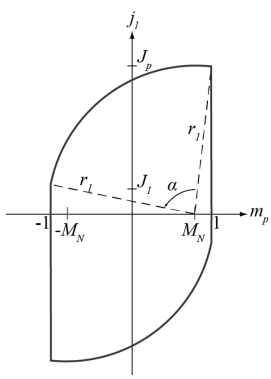


D. Costinett, R. Zane, and D. Maksimovic, "Discrete-time small-signal modeling of a 1 MHz efficiency-optimized dual active bridge converter with varying load," in Proc. IEEE Workshop Contr. Modl. (COMPEL), june 2012, pp. 1 –7.

THE UNIVERSITY OF TENNESSEE KNOXVILLE

Different Operating Modes

Mode I



$$J_1 = \sqrt{J_p^2 - 4M_N}$$

$$\alpha = \cos^{-1}\left(1 - \frac{(J_p - J_1)^2 + 4}{2J_p^2 + 2(1 - M_n)^2}\right)$$

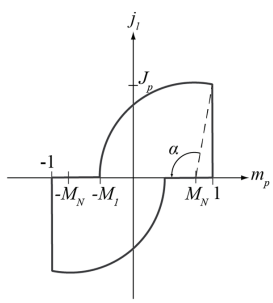
$$\beta = \frac{J_1 + J_2}{1 + M_N}$$

$$\zeta = \frac{J_p - J_2}{1 - M_N}$$

$$\frac{\pi}{F} = \alpha + \beta + \delta$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2 + \frac{J_p + J_2}{2}\zeta + \frac{J_1 - J_2}{2}\beta\ .\right)$$

Mode II



$$M_1 = \sqrt{J_p^2 + (1 - M_N)^2} - M_N$$

$$\alpha = \cos^{-1}\left(1 - \frac{J_p^2 + (1 + M_1)^2}{2(M_1 - M_n)^2}\right)$$

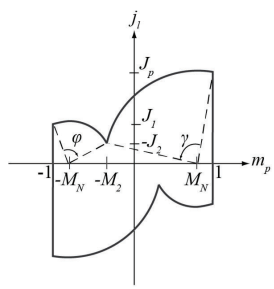
$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(1 + M_1 + \frac{J_p + J_2}{2}\zeta - \frac{J_2}{2}\beta\right)$$

$$\beta = \frac{J_1 + J_2}{1 + M_N}$$

$$\zeta = \frac{J_p - J_2}{1 - M_N}$$

$$\frac{\pi}{F} = \alpha + \beta + \delta$$

Mode III



$$\gamma = \cos^{-1}\left(1 - \frac{(J_p + J_2)^2 + (1 + M_2)^2}{2(1 - M_n)^2 + 2J_p^2}\right)$$

$$\phi = \cos^{-1}\left(1 - \frac{(J_1 + J_2)^2 + (1 - M_2)^2}{2(M_n - M_2)^2 + 2J_2^2}\right)$$

$$J_2^2 = (1 - M_N)^2 + J_p^2 - (M_2 + M_N)^2$$

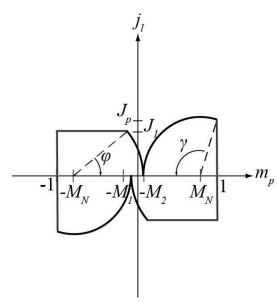
$$J_1^2 = -(1 - M_N)^2 + J_2^2 + (M_N - M_2)^2$$

$$\zeta = \frac{J_p + J_1}{1 - M_N}$$

$$\frac{\pi}{F} = \gamma + \phi + \zeta = \alpha + \zeta$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2M_2 + \frac{J_p - J_1}{2}\zeta\right)$$

Mode IV



$$\phi = \cos^{-1}\left(1 - \frac{J_1^2 + (M_1 - M_2)^2}{2(M_n - M_2)^2}\right)$$

$$J = \frac{\langle i_o \rangle}{I_{base}} = \frac{F}{n_t \pi}\left(2M_2 + 1 - M_1 + \frac{J_p - J_1}{2}\zeta\right)$$

$$J_2^2 = (1 - M_N)^2 + J_p^2 - (M_2 + M_N)^2$$

$$J_1^2 = -(1 - M_N)^2 + J_2^2 + (M_N - M_2)^2$$

$$\zeta = \frac{J_p + J_1}{1 - M_N}$$

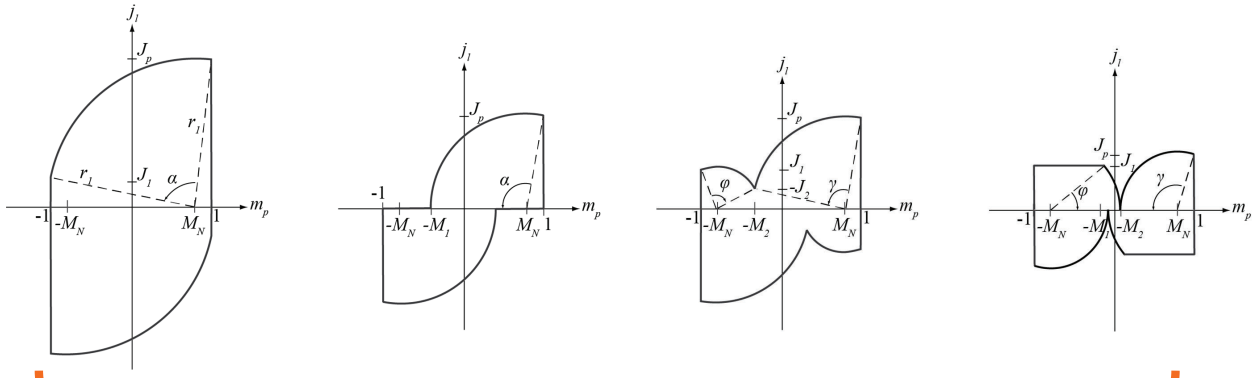
$$\frac{\pi}{F} = \gamma + \phi + \zeta = \alpha + \zeta$$

D Costinett, D. Maksimovic, and R Zane, "Design and Control for High Efficiency in High Step-Down Dual Active Bridge Converters Operating at High Switching Frequency," *Trans. On Pwr. Elec.* 2013

THE UNIVERSITY OF TENNESSEE KNOXVILLE

Different Operating Modes

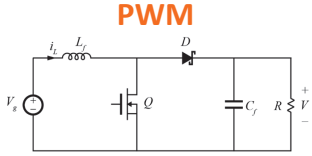
Mode I | Mode II | Mode III | Mode IV



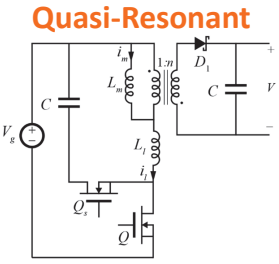
$$\boldsymbol{X}_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k} \right) \boldsymbol{A}_i^{-1} (e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}) \boldsymbol{B}_i \right\} U$$

Different Topologies

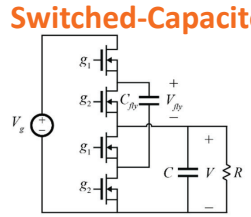
PWM



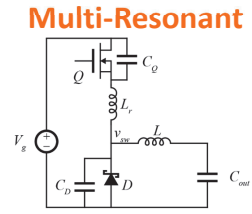
Quasi-Resonant



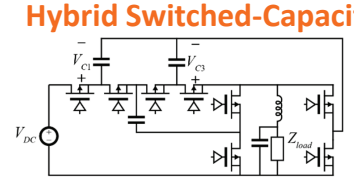
Switched-Capacitor



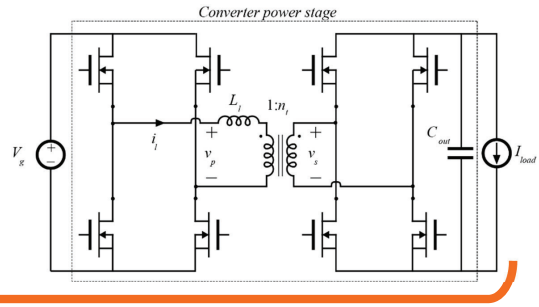
Multi-Resonant



Hybrid Switched-Capacitor



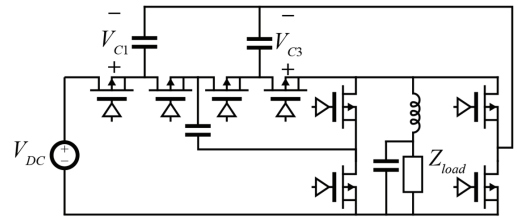
AC-Link



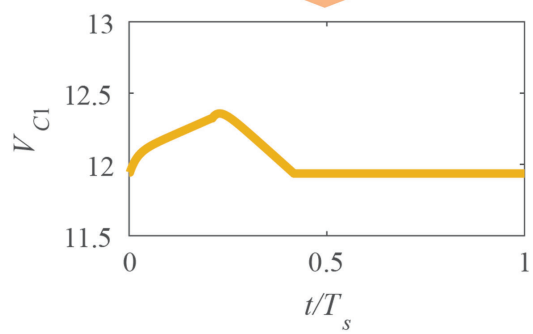
$$\boldsymbol{X}_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k} \right) \boldsymbol{A}_i^{-1} (e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}) \boldsymbol{B}_i \right\} U$$

Numerical Approach: HDSC Example

- 4:1 Hybrid Dickson Switched-Capacitor Converter
- 48-to-5 V, 0-100 A output
- Including C_{oss}, 13 states, 3 subintervals

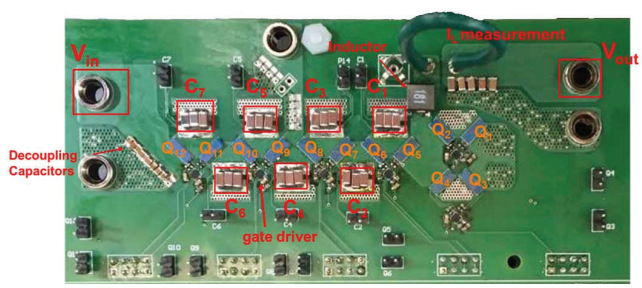


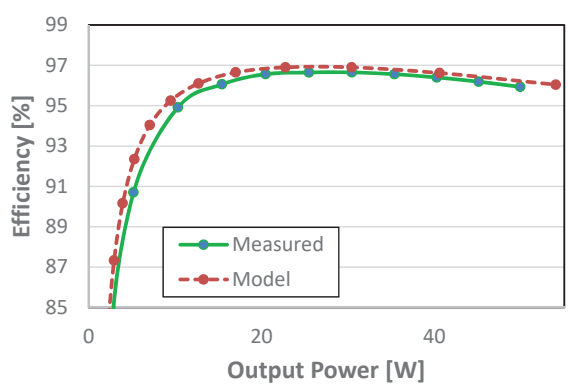
$$\boldsymbol{X}_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k} \right) \boldsymbol{A}_i^{-1} (e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}) \boldsymbol{B}_i \right\} U$$



Model Validation

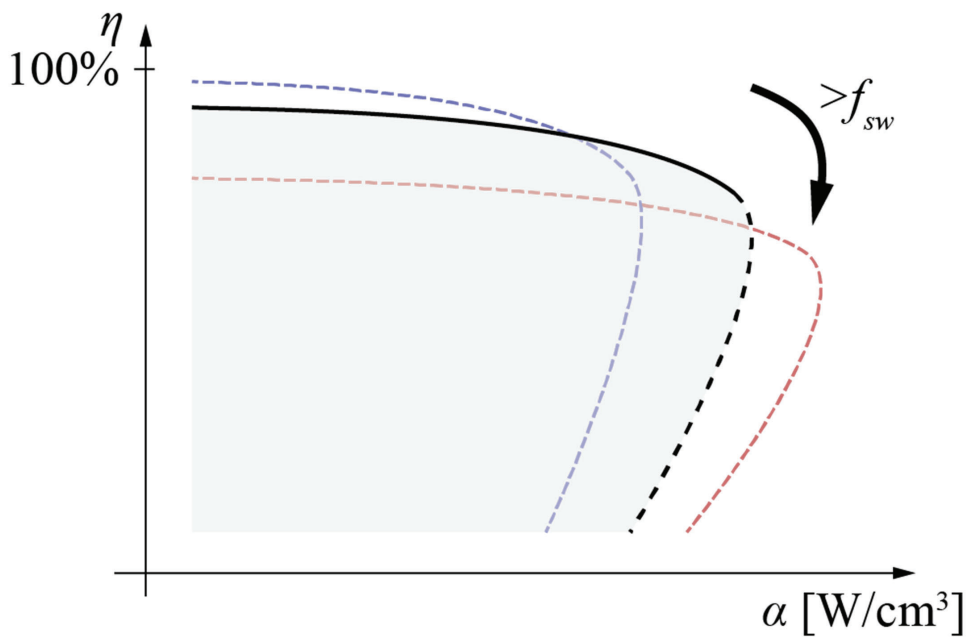
- Constructed 8:1 HDSC converter
  - Measured 96.7% peak efficiency at 30W
  - Model predicts 96.9% at 30.4W
- Model includes capacitor ESR





COURSE CONCLUSIONS

HF Power Electronics – When and Why



Thank you for all your hard work, and good luck with finals!

THE UNIVERSITY OF TENNESSEE KNOXVILLE