Steady-State Large-Signal Analysis

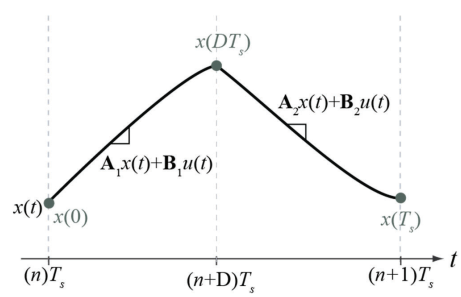
$$\boldsymbol{x}(T_s) = \left(\prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)\boldsymbol{x}(0) + \sum_{i=1}^{n_{sw}}\left\{\left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k}\right)\boldsymbol{A}_i^{-1}\left(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}\right)\boldsymbol{B}_i\right\}U$$

In steady-state, $\boldsymbol{x}(T_s) = \boldsymbol{x}(0)$

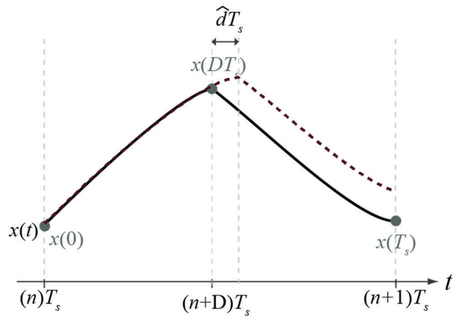
$$\boldsymbol{x}(T_s) = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}}\left\{\left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k}\right)\boldsymbol{A}_i^{-1}\left(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}\right)\boldsymbol{B}_i\right\}U$$

Gives explicit solution for steady-state operation of any switching circuit

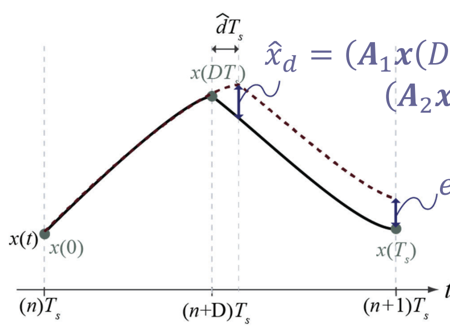
Small Signal Modeling



Small Signal Modeling



Small Signal Modeling



$$\hat{x}_d = (\boldsymbol{A}_1\boldsymbol{x}(DT_s) + \boldsymbol{B}_1 U)\hat{d}T_s - (\boldsymbol{A}_2\boldsymbol{x}(DT_s) + \boldsymbol{B}_2 U)\hat{d}T_s$$

$$e^{\boldsymbol{A}_2 D' T_s}\hat{x}_d$$

Complete Small Signal Model

This completes the small-signal model

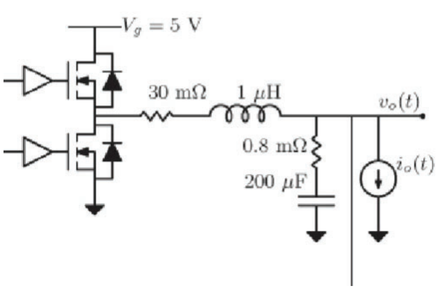
$$\hat{\boldsymbol{x}}[n+1] = \boldsymbol{\Phi}\hat{\boldsymbol{x}}[n] + \boldsymbol{\Psi}\hat{u}[n] + \boldsymbol{\Gamma}\hat{d}[n]$$

where

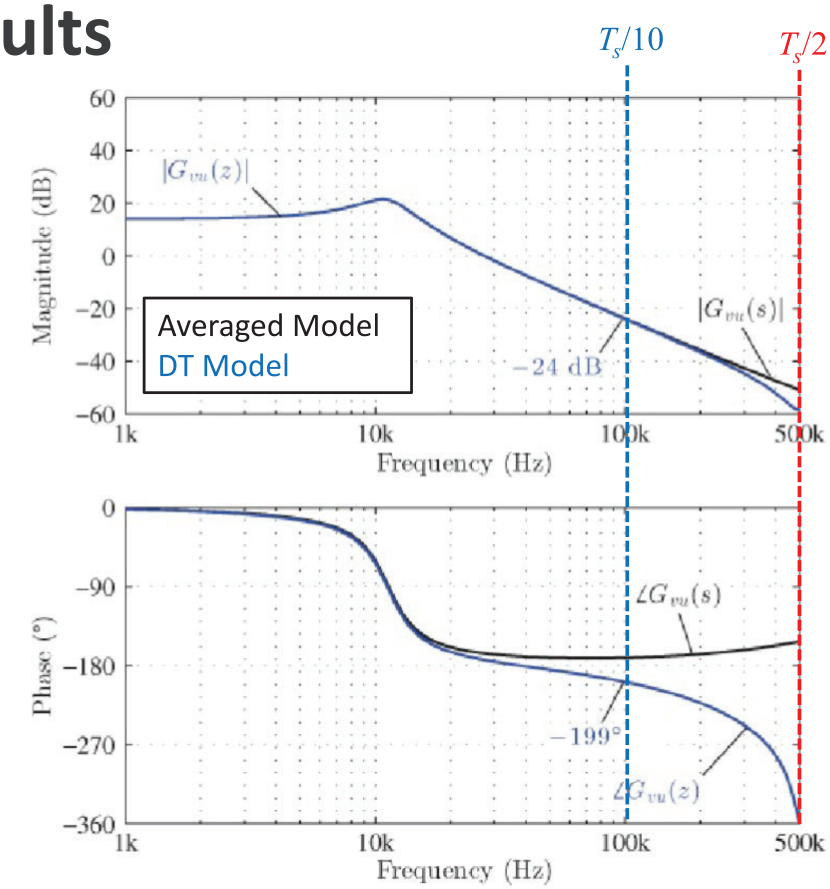
$$\boldsymbol{\Gamma} = e^{A_2 D' T_s}\big((\boldsymbol{A}_1 - \boldsymbol{A}_2)\boldsymbol{X}_D + (\boldsymbol{B}_1 - \boldsymbol{B}_2\,)U\big)T_s$$

with $\boldsymbol{X}_D = \boldsymbol{x}(DT_s)$ in steady-state

Example Results



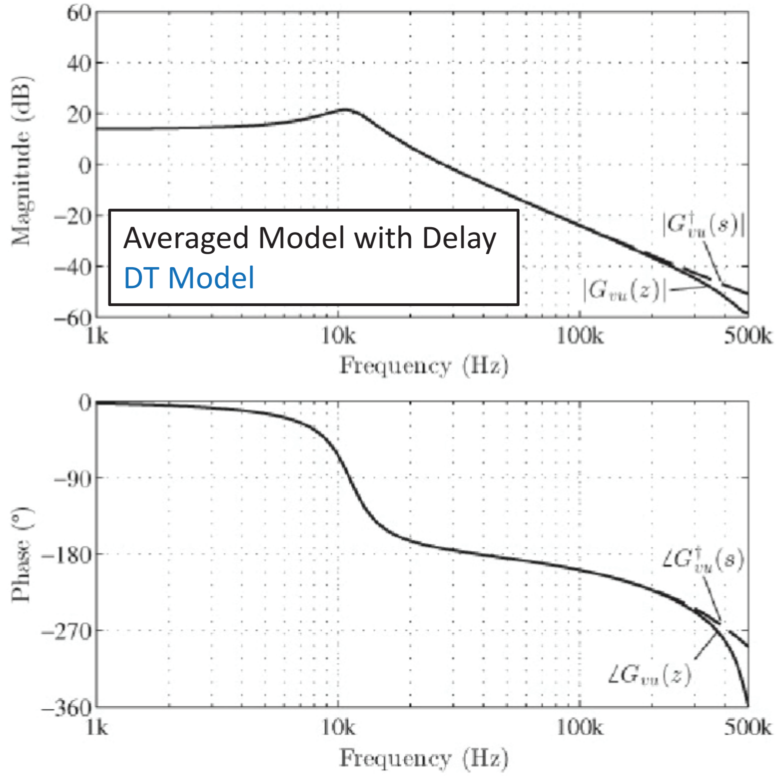
* Includes t_d=760ns of delay in feedback loop



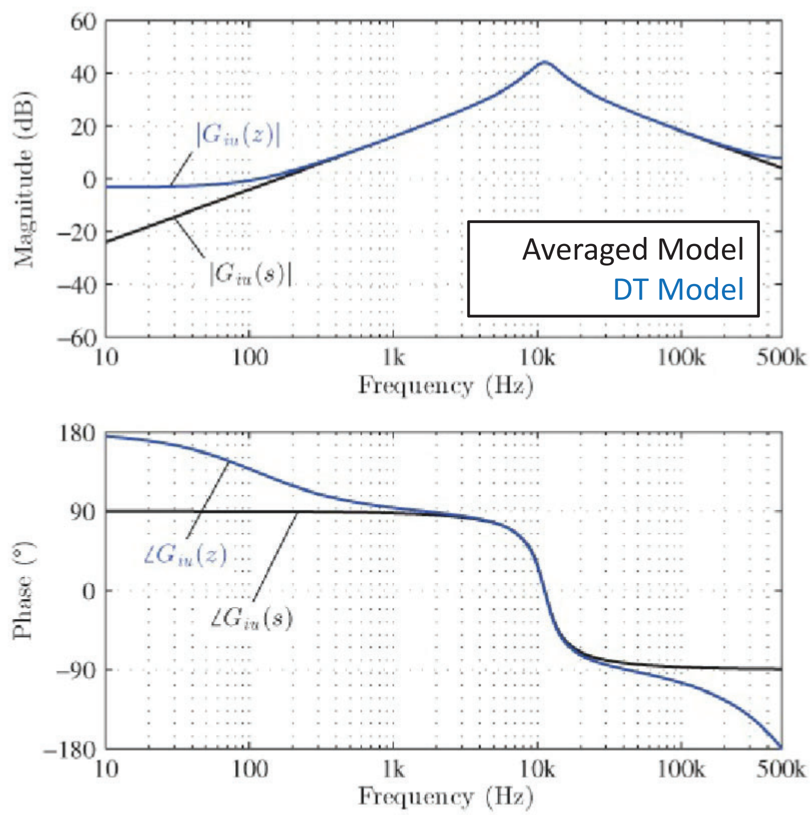
L. Corradini et. al. *Digital Control of High Frequency Switched-Mode Power Converters,* Section 3.2

Inclusion of Delay

$$G_{vu}^{\dagger}(s) = G_{vu}(s)e^{-st_d}$$



Current Control



Discrete Time Analysis: Results

$$\boldsymbol{X}_{ss} = \left(I - \prod_{i=n_{sw}}^{1} e^{\boldsymbol{A}_i t_i}\right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left(\prod_{k=n_{sw}}^{i+1} e^{\boldsymbol{A}_k t_k} \right) \boldsymbol{A}_i^{-1} \left(e^{\boldsymbol{A}_i t_i} - \boldsymbol{I}\right) \boldsymbol{B}_i \right\} U$$

- Valid for any switched circuit, as long as
  1. Inputs, U, are constant or slowly varying
  2. All times t_i are known
  3. Every subinterval can be described by a linear circuit
- Requires no dedicated analysis other than finding $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$
- Decisively **not** a design-oriented equation