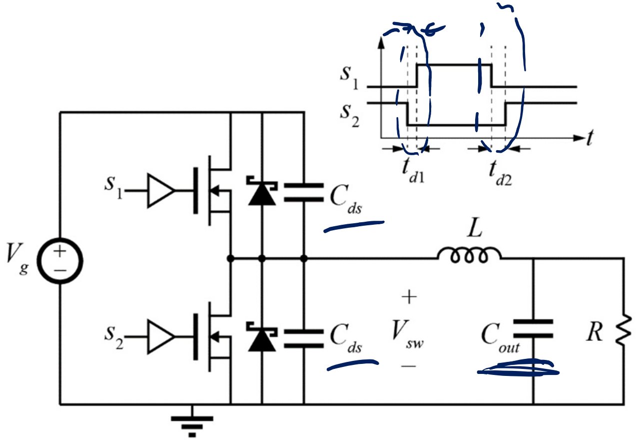
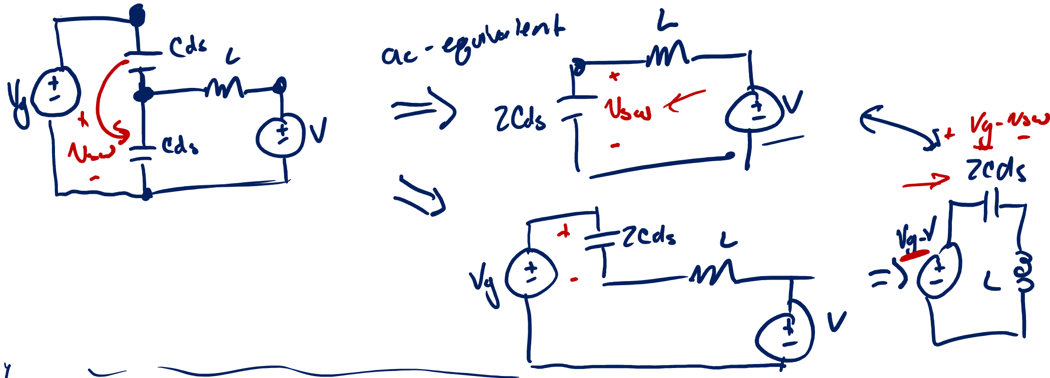
Time-Domain Analysis of Switching Transitions

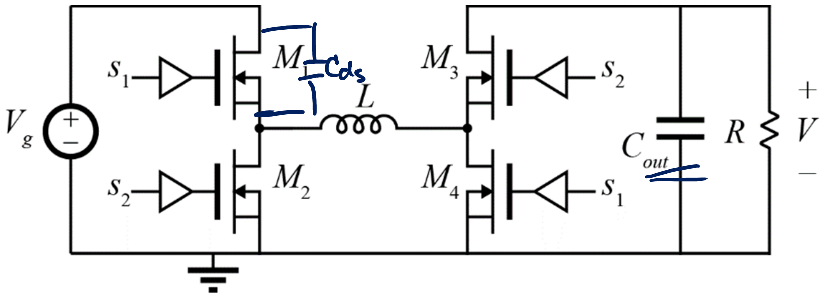
(1) Assume Cout is a filter element with small ripple

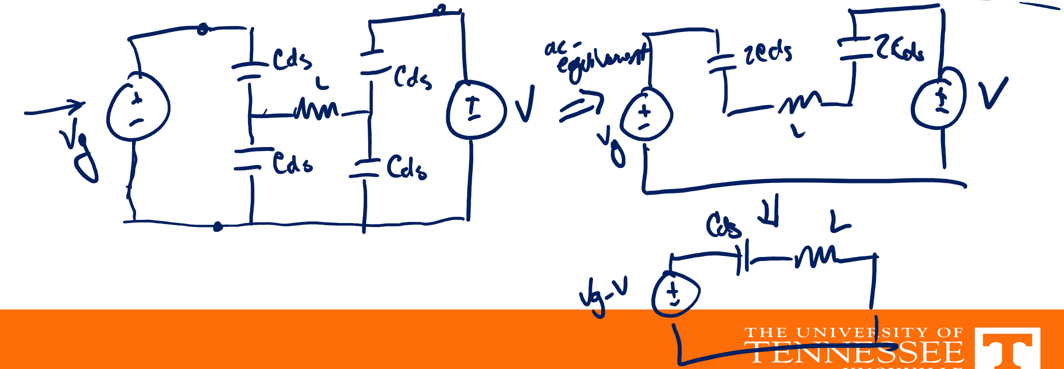
(2) Assume Cds is linear



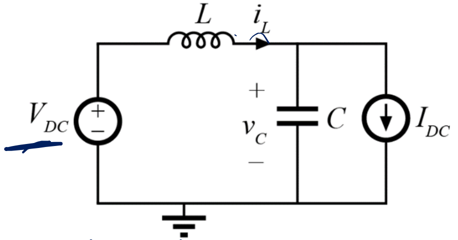
during dead time







Resonant Circuit Solution



Initial conditions :

$v_c(t=0) = V_0$

$i_L(t=0) = I_0$

$$① \quad C\frac{dv_c}{dt} = i_L - I_{DC}$$

$$② \quad L\frac{di_L}{dt} = V_{DC} - v_c$$

$$\rightarrow L\frac{d}{dt}\left(C\frac{dv_c}{dt} + I_{dc}\right) = V_{DC} - v_c$$

$$= LC\frac{d^2v_c}{dt^2} + v_c - V_{DC} = \emptyset$$

$$v_c = \begin{cases} A\sin(\theta) + B\cos(\theta), & \text{homogeneous} \\ v_c = V_{DC}, & \text{particular} \end{cases}$$

$$v_c(t) = V_{DC} + (V_0 - V_{DC})\cos\left(\frac{t}{\sqrt{LC}}\right) + (I_0 - I_{DC})\sqrt{\frac{L}{C}}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$i_L(t) = I_{DC} + (I_0 - I_{DC})\cos\left(\frac{t}{\sqrt{LC}}\right) + (V_{DC} - V_0)\sqrt{\frac{C}{L}}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

Normalization and Notation

Notation: $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$ $\quad R_0 = \sqrt{L/C}$

$$v_c(t) = V_{DC} + (V_0 - V_{DC})\cos(\omega_0 t) + R_0(I_0 - I_{DC})\sin(\omega_0 t)$$

$$i_L(t) = I_{DC} + (I_0 - I_{DC})\cos(\omega_0 t) + \frac{1}{R_0}(V_{DC} - V_0)\sin(\omega_0 t)$$

Normalization:

$m(t) = \frac{v(t)}{V_{base}}$

V_{base} → anything you want (constant)
(some choices better than others)

$j(t) = \frac{i(t)}{I_{base}}$

$I_{base} = \frac{V_{base}}{R_0}$

$\theta = \omega_0 t$

$V_{base} = V_{DC}$

$I_{base} = \dfrac{V_{DC}}{R_0}$

$$\begin{cases} v_c(t) = V_{DC} + (V_0 - V_{DC})\cos(\omega_0 t) + R_0(I_0 - I_{DC})\sin(\omega_0 t) \\ i_L(t) = I_{DC} + (I_0 - I_{DC})\cos(\omega_0 t) + \dfrac{1}{R_0}(V_{DC} - V_0)\sin(\omega_0 t) \end{cases}$$

$$R_0 \frac{I_0 - I_{DC}}{V_{DC}} = (J_0 - J_{DC})$$

$(M_{DC} = 1)$

$$\begin{cases} m_c(\theta) = \dfrac{v_c(t)}{V_{DC}} = M_{DC} + (M_0 - M_{DC})\cos\theta + R_0 \dfrac{I_0 - I_{DC}}{V_{DC}} \sin\theta \\ j_L(\theta) = \dfrac{i_L(t)}{I_{base}} = \dfrac{i_L(t)}{V_{DC}/R_0} = J_{DC} + (J_0 - J_{DC})\cos\theta + (M_{DC} - M_0)\sin\theta \end{cases}$$

$A = M_0 - M_{DC}$

$B = J_0 - J_{DC}$

$$\begin{cases} m_c(\theta) = M_{DC} + A\cos\theta + B\sin\theta \\ j_L(\theta) = J_{DC} + B\cos\theta - A\sin\theta \end{cases}$$

Try looking at $(m_c(\theta) - M_{DC})^2 + (j_L(\theta) - J_{DC})^2 =$

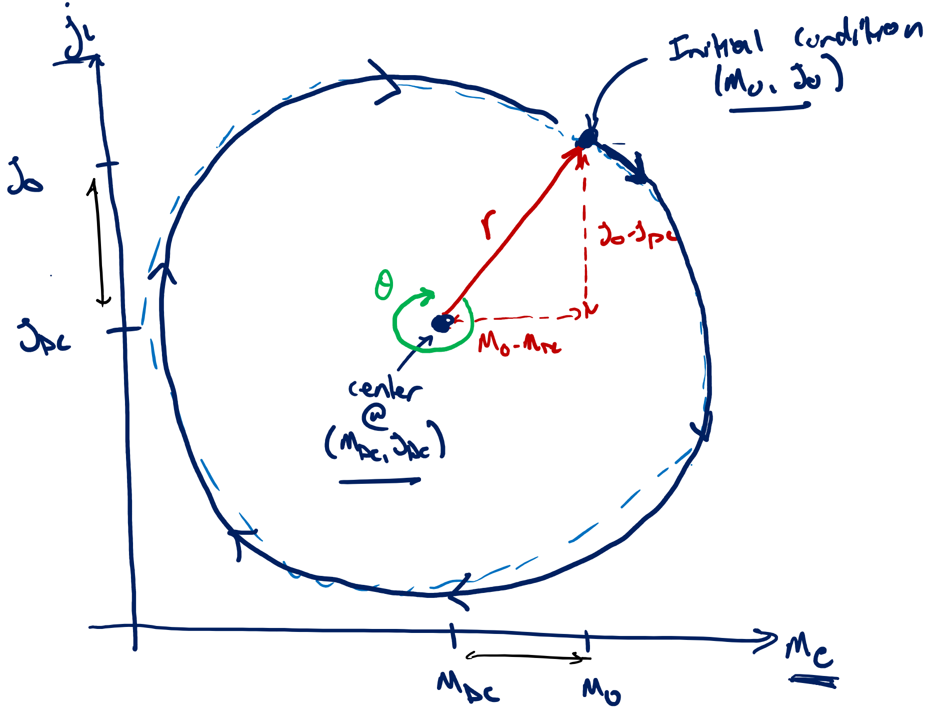
$= A^2\cos^2\theta + B^2\sin^2\theta + \cancel{2AB\cos\theta\sin\theta} + B^2\cos^2\theta + A^2\sin^2\theta - \cancel{2AB\cos\theta\sin\theta}$

$= A^2(\cos^2\theta + \sin^2\theta) + B^2(\sin^2\theta + \cos^2\theta)$

$= A^2 + B^2 = (M_0 - M_{DC})^2 + (J_0 - J_{DC})^2$

$$(m_c(\theta)-m_{pc})^2 + (j_L(\theta)-J_{pc})^2 = (M_0 - M_{pc})^2 + (J_0 - J_{pc})^2$$

This is the equation for a circle $x^2+y^2=r^2$ $r=\sqrt{(M_0-M_{pc})^2+(J_0-J_{pc})^2}$



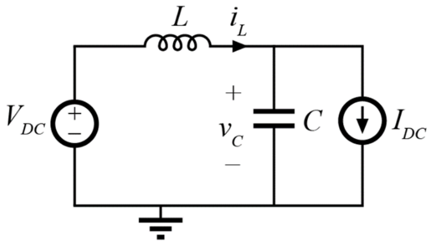
Is a defined direction associated with the state plane

- Defined by I.C. on first derivatives of j_L & m_c

State Plane

- Transforms Diff EQ solution into trig solution
- Limited to 1-L, 1-C subcircuits

State Plane Analysis



$V_{base} = $ ______

$I_{base} = \frac{V_{base}}{R_0}$

DC solution:

$v_c = V_{DC} \rightarrow m_c = M_{DC}$

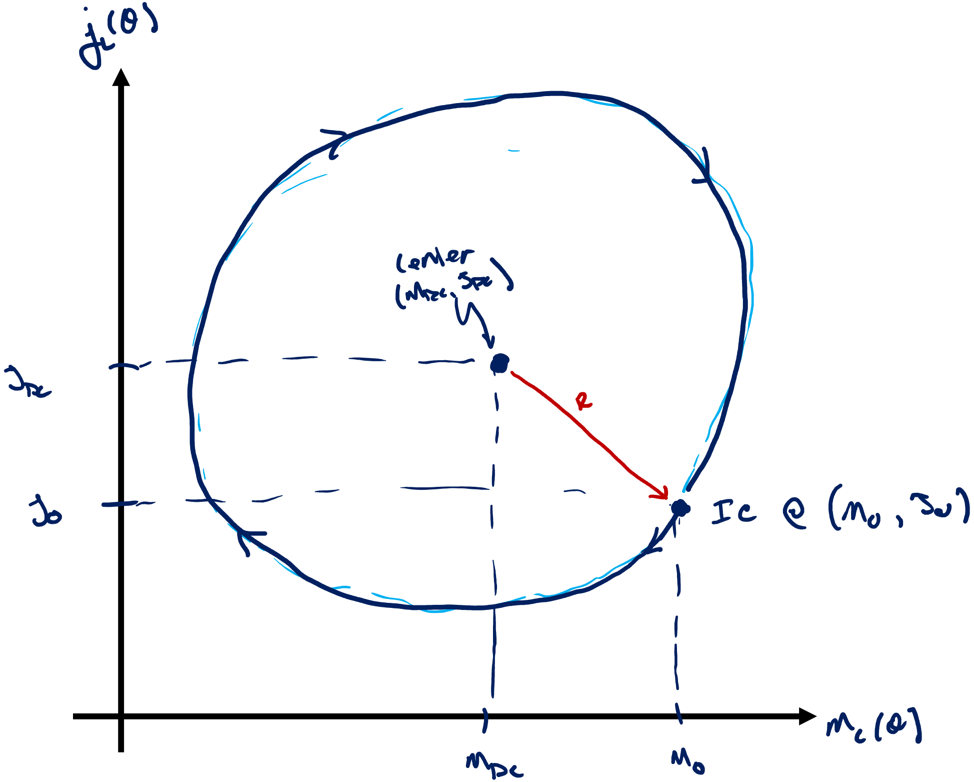
$i_L = I_{DC} \rightarrow j_L = J_{DC}$

Initial Conditions:

$v_c(0) = V_0 \rightarrow m_c = M_0$

$i_L(0) = I_0 \rightarrow j_L = J_0$

Direction from IC on first derivatives



[1] R. Oruganti and F. C. Lee, "Resonant Power Processors, Part I – State Plane Analysis", Industry Applications, IEEE Tran. on, vol. 21, no. 6, nov 1985.

[2] D. P. Atherton, Nonlinear Control Engineering. London: Van Nostrand Reinhold, 1982, Ch. 2.

Example Analysis

What is $i_{L,pk}$?

$$j_{L,pk} = \frac{i_{L,pk}}{I_{base}} = \frac{i_{L,pk}}{V_{base}/R_0}$$

$$j_{L,pk} = J_{pk} + r$$

$$j_{L,pk} = J_{pk} + \sqrt{(M_0 - M_{pk})^2 + (J_0 - J_{pk})^2}$$

$$i_{L,pk} = I_{pk} + \frac{V_{base}}{R_0}\sqrt{(M_0 - M_{pk})^2 + (J_0 - J_{pk})^2}$$

When does $i_{L,pk}$ occur?

$$\theta = \omega_0 t \qquad t = \frac{\theta}{\omega_0}$$

$$\theta_{pk} = \frac{\pi}{2} + \tan^{-1}\left(\frac{J_{pk} - J_0}{M_{pk} - M_0}\right)$$

$$t_{pk} = \frac{\pi}{2\omega_0} + \frac{1}{\omega_0}\tan^{-1}\left(\frac{J_{pk} - J_0}{M_{pk} - M_0}\right)$$