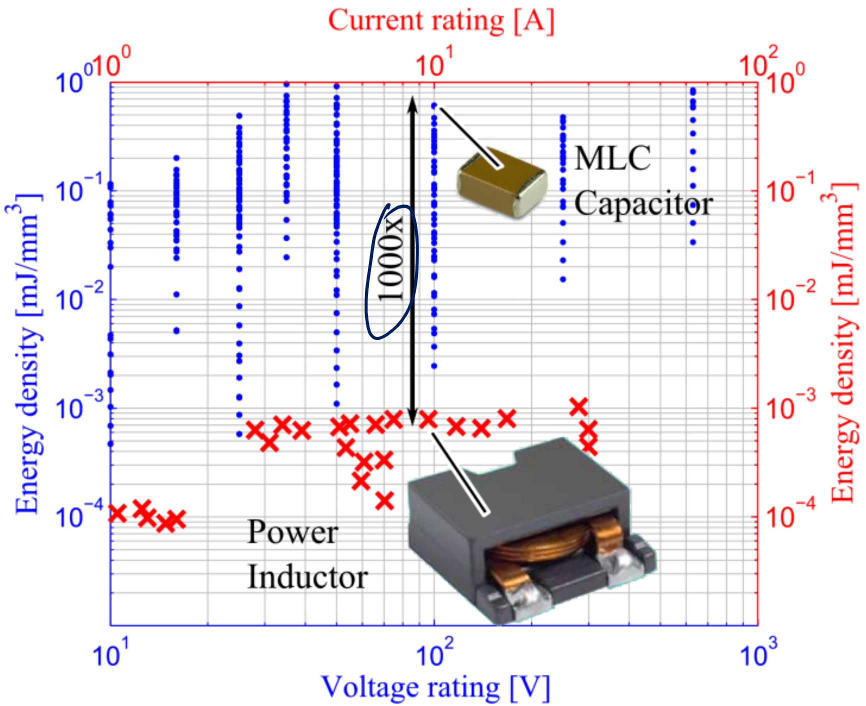
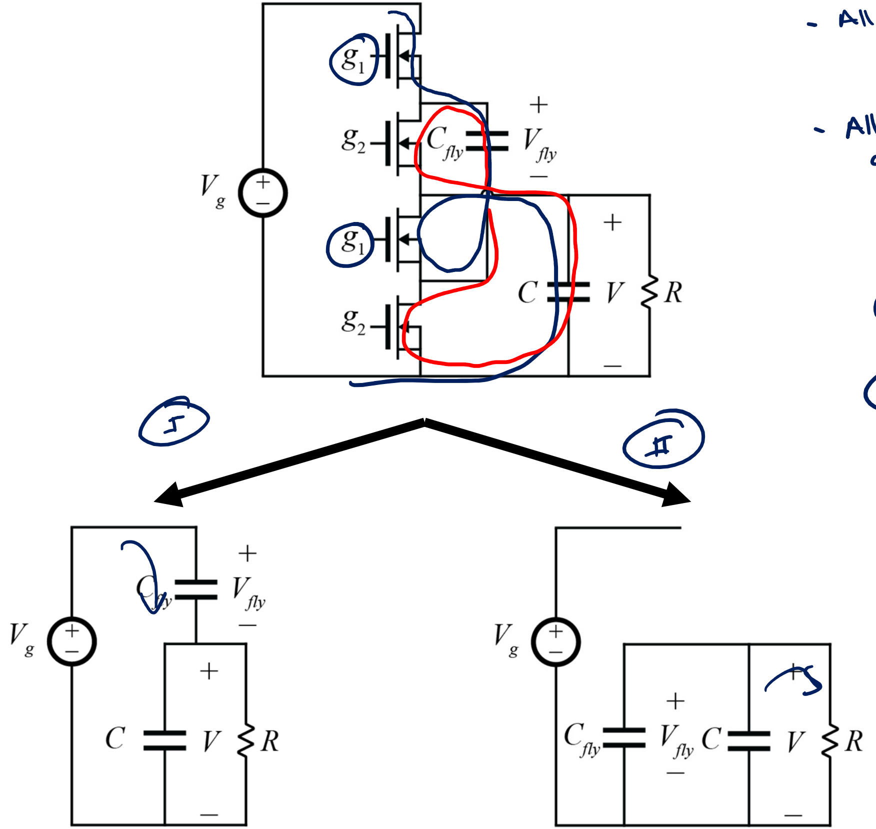
Switched Capacitor Converters



A 2:1 SC Converter



Assume:

- All C's are large, $V \approx$ const → $V_{fly} \approx$ const
- All time intervals long enough for any R-C time constants to converge to final value

I $\quad V_g = V + V_{fly}$

II $\quad V_{fly} = V$

$$V_g = 2V \rightarrow \boxed{V = \tfrac{1}{2} V_g}$$

SC Converters

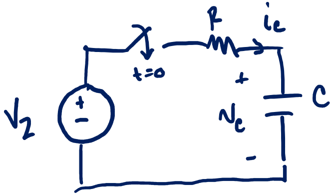
- Fixed conversion ratio
  - Continuous regulation impossible[1] (except linear)
- Not lossless, even with ideal elements
- Can be very small, fully integrated

  - Discrete caps may be smaller & more energy dense than inductors
  - On-chip capacitors are significantly better than on-chip inductors

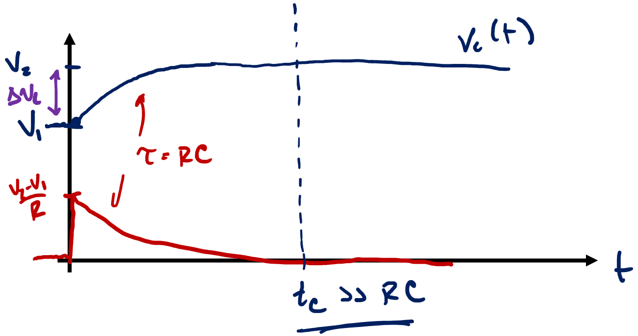
- **Resonant** versions can reduce loss
- **Hybrid** versions can allow regulation

[1]D. Wolaver, "Fundamental study of dc to dc conversion systems," dissertation, 1969

Capacitor Charging: Voltage Source



$v_c(t=0) = V_1$



At / after $t_c \gg RC$

$$\Delta E_c = \tfrac{1}{2} C V_2^2 - \tfrac{1}{2} C V_1^2$$

$$\boxed{\Delta E_c = \tfrac{1}{2} C \left(V_2^2 - V_1^2\right)}$$

$$\Delta E_V = \int_0^{t_c} V_2 \, i_c \, dt = V_2 \int_{V_1}^{V_2} C \, dv_c = \boxed{V_2 \overbrace{C\left(V_2 - V_1\right)}^{\Delta Q_c} = \Delta E_V}$$

$$E_{loss} = \Delta E_V - \Delta E_c = C V_2 (V_2 - V_1) - \tfrac{1}{2} C \left(V_2^2 - V_1^2\right)$$

$$= C V_2^2 - C V_2 V_1 - \tfrac{1}{2} C V_2^2 + \tfrac{1}{2} C V_1^2$$

$$= \tfrac{1}{2} C V_2^2 + \tfrac{1}{2} C V_1^2 - (2)\tfrac{1}{2} C V_2 V_1$$

$$\boxed{E_{loss} = \tfrac{1}{2} C \left(V_2 - V_1\right)^2} = \tfrac{1}{2} (\Delta v_c)^2$$

$$V_2 - V_1 = \Delta V_c$$

$$\eta = \frac{\Delta E_c}{\Delta E_V} = \frac{\frac{1}{2}\cancel{C}(V_2^2 - V_1^2)}{V_2 \cancel{C}(V_2 - V_1)} = \frac{\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2}{V_2^2 - V_2 V_1}$$

$$= \frac{1}{2}\,\frac{V_2^2 - (V_2 - \Delta V_c)^2}{V_2^2 - V_2(V_2 - \Delta V_c)} = \frac{1}{2}\,\frac{\cancel{V_2^2} - \cancel{V_2^2} - (\Delta V_c)^2 + 2V_2\Delta V_c}{\cancel{V_2^2} - \cancel{V_2^2} + V_2 \Delta V_c}$$

$$= \frac{1}{2}\left(2 - \frac{\Delta V_c}{V_2}\right)$$

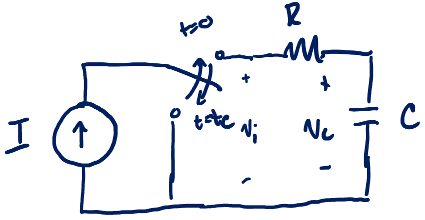
$$\boxed{\eta = 1 - \frac{\Delta V_c}{2V_2}} \longrightarrow$$

if $\Delta V_c \ll V_2 \rightarrow$ high efficiency possible

No dependence on R

except $t_c \gg RC$

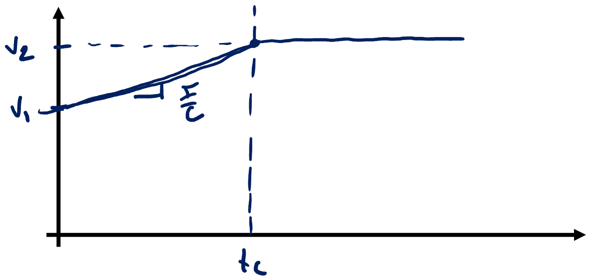
Capacitor Charging: Current Source



$v_c(t=0) = V_1$

$\frac{I}{C} t_c = (V_2 - V_1)$

$t_c = \frac{C(V_2 - V_1)}{I}$



$\boxed{\Delta E_c = \frac{1}{2} C (V_2^2 - V_1^2)}$ (same)

$$\Delta E_I = \int_0^{t_c} v_i I\, dt = \int_0^{t_c} (v_c + IR) I\, dt = I\left[\frac{V_2 + V_1}{2} t_c + IR\, t_c\right]$$

$$= \cancel{I} \frac{V_2 + V_1}{2} \frac{C(V_2 - V_1)}{\cancel{I}} + I^{\cancel{2}} R \frac{C(V_2 - V_1)}{\cancel{I}}$$

$$= \frac{1}{2} C (V_2^2 - V_1^2) + IRC(V_2 - V_1)$$

$$= \frac{1}{2} C (V_2^2 - V_1^2) + RC\left[C \frac{V_2 - V_1}{t_c}\right](V_2 - V_1) = \boxed{\underbrace{\frac{1}{2} C (V_2^2 - V_1^2)}_{\Delta E_c} + \overbrace{\underbrace{\frac{1}{2} C (V_2 - V_1)^2}_{\text{Eloss in V-src case}} \underbrace{\frac{2RC}{t_c}}_{\text{Add'l scale factor}}}^{\text{Eloss}}}$$

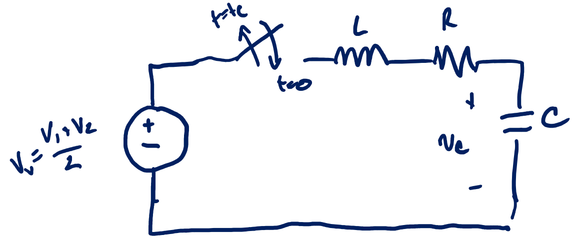
V-src charging : $E_{loss} = \frac{1}{2}C(V_2 - V_1)^2$

I-src charging : $E_{loss} = \frac{1}{2}C(V_2 - V_1)^2 \frac{2RC}{t_c}$

if $\underline{\underline{t_c \gg RC}}$ → Current source charging has lower loss

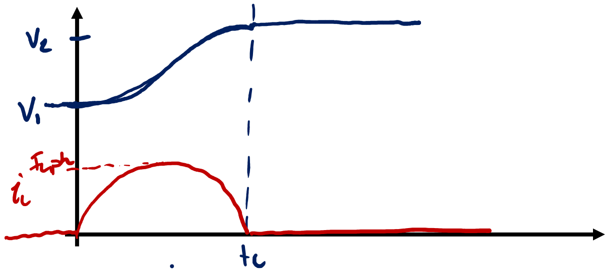
if $t_c \not\gg RC$ → Don't know, V-src charging analysis is only valid for $t_c \gg RC$

Capacitor Charging: Resonant



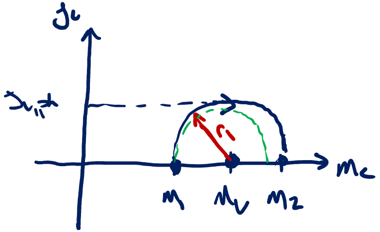
$v_c(t=0) = V_1$

$i_L(t=0) = \emptyset$



High-efficiency approx: Assume we can use waveforms with $R=\emptyset$ then calculate losses afterwards

$$\boxed{\Delta E_c = \frac{1}{2} C \left(V_2^2 - V_1^2\right)}$$ (same)



$\alpha = \pi$

$R_0 = \sqrt{\frac{L}{C}}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$t_c = \frac{\pi}{\omega_0}$

$J_{L,pk} = r_1 = m_v - m_1$

$I_{L,pk} = \frac{1}{R_0}\left(\frac{V_1+V_2}{2} - V_1\right)$

$= \frac{1}{R_0}\left(\frac{V_2-V_1}{2}\right)$

$E_{loss} = I_{L,rms}^2 R t_c = \frac{I_{L,pk}^2}{2} R t_c$

$= \frac{1}{2} R \frac{1}{R_0^2} \frac{(V_2-V_1)^2}{4} t_c$

$= \frac{1}{2} R \frac{1}{R_0^2} \frac{\pi}{\omega_0} \frac{1}{4} (V_2-V_1)^2$

E_{loss} in V_{src} case

$= \frac{1}{8} \frac{R\pi}{R_0} C (V_2-V_1)^2 = \boxed{\frac{1}{2} C (V_2-V_1)^2 \cdot \frac{R\pi}{4R_0}}$

$\frac{1}{R_0}\frac{1}{\omega_0} = \sqrt{\frac{C}{L}}\sqrt{LC} = C$