Resonant Charging

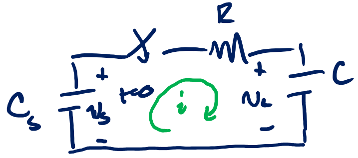
$$E_{loss} = \frac{1}{2}C(V_2 - V_1)^2 \frac{R}{R_0}\frac{\pi}{4} \longrightarrow \frac{R}{R_0} \longrightarrow$$ High-Q resonance for low loss

$$\frac{1}{R_0} = \sqrt{\frac{C}{L}} = \frac{1}{\sqrt{LC}} \cdot C = C\omega_0 = C\frac{\pi}{t_c} = \frac{1}{R_0}$$

$$\boxed{E_{loss} = \frac{1}{2}C(V_2 - V_1)^2 \frac{RC\pi^2}{4t_c}} \longrightarrow$$ $t_c \gg RC$ for low loss (still)

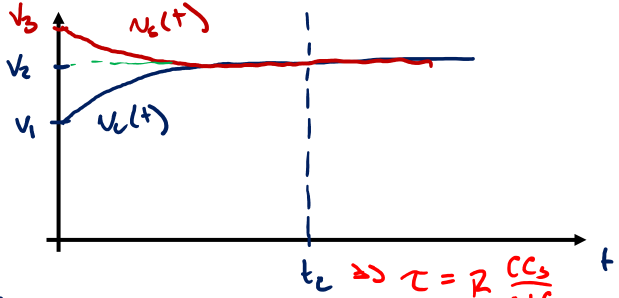
$$\frac{\pi^2}{4} \approx 2.5$$

Cap-Cap Charging



$v_c(t=0) = V_1$

$v_s(t=0) = V_3$



$t_c \gg \tau = R\frac{CC_s}{C+C_s}$

charge conservation

$$V_3 C_s + V_1 C = (C+C_s)V_2$$

$$V_3 = \left(\frac{C}{C_s}+1\right)V_2 - \frac{C}{C_s}V_1 = \frac{C}{C_s}(V_2-V_1) + V_2$$

$$\Delta E_c = \frac{1}{2}C\left(V_2^2 - V_1^2\right) \quad \text{(same)}$$

$$\Delta E_s = \frac{1}{2}C_s\left(V_3^2 - V_2^2\right)$$

$$= \frac{1}{2}C_s\left[(V_2-V_1)^2\left(\frac{C}{C_s}\right)^2 + \cancel{V_2^2} + 2\frac{C}{C_s}V_2(V_2-V_1) - \cancel{V_2^2}\right]$$

$$= \frac{1}{2}C_s\left[\left(\frac{C}{C_s}\right)^2\left(V_2^2 + V_1^2 - 2V_1V_2\right) + 2\frac{C}{C_s}\left(V_2^2 - V_1V_2\right)\right]$$

$$E_{loss} = \Delta E_s - \Delta E_c$$

$$= \frac{1}{2}\frac{C^2}{C_s}\left(V_2^2 + V_1^2 - 2V_1V_2\right) + C\left(V_2^2 - V_1V_2\right) - \frac{1}{2}CV_2^2 + \frac{1}{2}CV_1^2$$

$$= \left(\frac{1}{2}\frac{C^2}{C_s} + \frac{1}{2}C\right)V_2^2 + \left(\frac{1}{2}\frac{C^2}{C_s} + \frac{1}{2}C\right)V_1^2 + \left(-\frac{C^2}{C_s} - C\right)V_1V_2$$

$$= \frac{1}{2}C\left[\left(\frac{C}{C_s}+1\right)V_2^2 + \left(\frac{C}{C_s}+1\right)V_1^2 - 2\left(\frac{C}{C_s}+1\right)V_1V_2\right]$$

$$\boxed{E_{loss} = \frac{1}{2}C\left(\frac{C}{C_s}+1\right)\left(V_2-V_1\right)^2}$$

$\hookrightarrow$ Add'l term relative to V-src charging

if $C_s \gg C$ $\rightarrow$ back to V-src charging solution

if $C_s = C$ $\rightarrow$ 2x losses from V-src charging

if $C_s \ll C$ $\rightarrow$ high losses

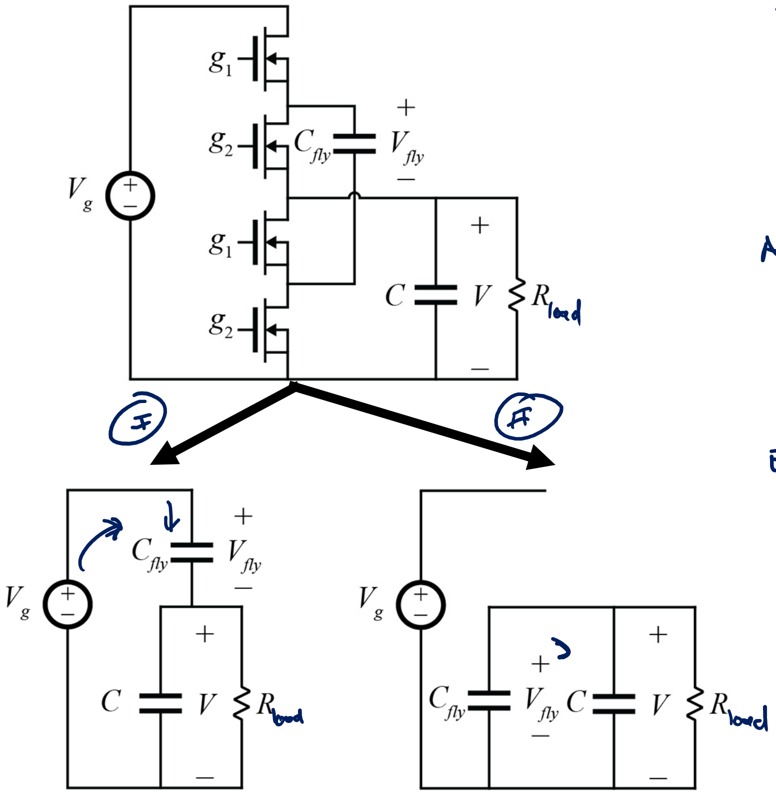
Comparison of Capacitor Charging

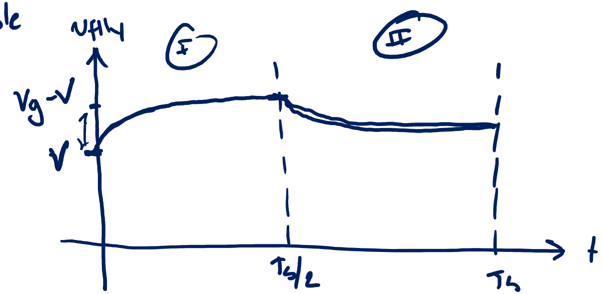
for charging C from V_1 to V_2 in time t_c

	E_{loss}	Assumptions
Voltage Source	$\frac{1}{2}C(V_2-V_1)^2$	$\underline{\underline{t_c \gg RC}} = \tau$
Current Source	$\frac{1}{2}C(V_2-V_1)^2 \frac{2RC}{t_c}$	X
Resonant	$\frac{1}{2}C(V_2-V_1)^2 \frac{\pi^2 RC}{4 t_c}$	High-Q resonance $R_0 \gg R \longleftrightarrow t_c \gg \pi RC$
Cap-Cap	$\frac{1}{2}C(V_2-V_1)^2\left(\frac{C}{C_s}+1\right)$	$t_c \gg R\frac{CC_s}{C+C_s} = \tau$

2:1 SC Revisited

Remove small-ripple on C_{fly}





Assume $T_s/2 \gg RC_{fly}$

R = equivalent resistance

$= ESR_{cfly} + 2R_{on}$

Energy loss:

$$\begin{cases} \text{I} & E_{loss} = \frac{1}{2} C_{fly} (V_g - V - V)^2 \\ \text{II} & E_{loss} = \frac{1}{2} C_{fly} (V_g - V - V)^2 \end{cases}$$

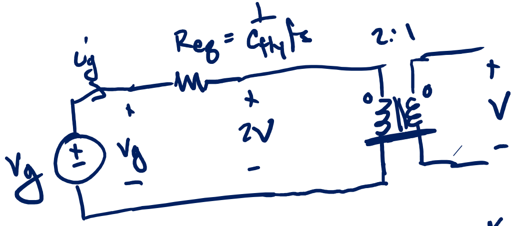
$$P_{loss} = C_{fly} (V_g - 2V)^2 f_s$$

$$P_{in} = V_g C_{fly} (V_g - 2V) f_s$$

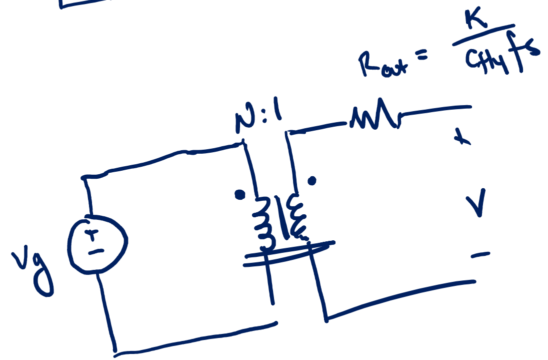
Equivalent Circuit Model

$$P_{loss} = \underbrace{C_{fly} f_s}_{} (V_g - 2V)^2$$

$$P_{in} = V_g \underbrace{C_{fly} f_s}_{} (V_g - 2V)$$



$\left(\frac{1}{4 C_{fly} f_s} \text{ in the 2:1 case} \right)$



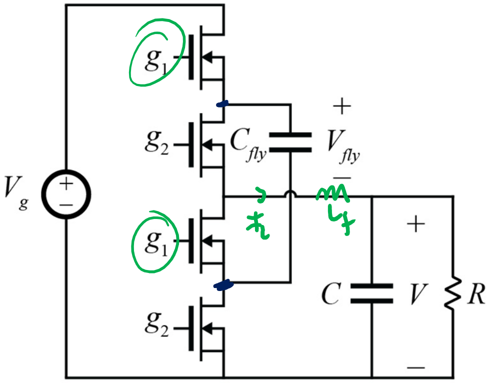
For high η, want small R_{out}

→ Large C_{fly}

→ High f_s

can continue on until $\frac{T_s}{2} = t_c \cancel{\gg} RC_{fly}$

2:1 – Current Loaded



2:1 SC, current loaded

3-level Buck converter @ D = 50%

2:1 : FCML converter

Add L_f → large, filter element with small ripple

(I) $E_{loss} = \frac{1}{2} C_{fly} (V_2 - V_1)^2 \frac{2RC_{fly}}{t_c}$

(II) $E_{loss} = \frac{1}{2} C_{fly} (V_2 - V_1)^2 \frac{2RC_{fly}}{t_c}$

$t_c = \frac{T_s}{2}$

total $P_{loss} = \underline{C_{fly}^2 (V_2 - V_1)^2} \frac{4R}{T_s} f_s$

$[C_{fly}(V_2 - V_1)]^2 = (\Delta Q_{fly})^2 = \left(I_L \frac{T_s}{2}\right)^2$

$P_{loss} = I_L^2 \frac{\cancel{T_s^2}}{\cancel{4}} \left(\cancel{4} R \frac{1}{\cancel{T_s^2}}\right) = \underline{\underline{I_L^2 R}}$

$R = ESR_c + 2R_{on} + R_L + \ldots$