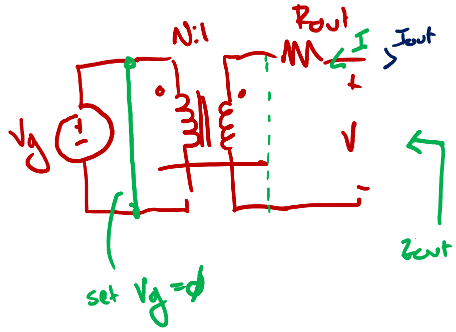
Charge Vector Analysis: Rules

- KVL & KCL apply
- for all caps $\int_0^{T_s} i_{cx}\,dt = \sum_{\text{all ints}} q_{cx} = \phi$ (cap-charge balance)

for a 2-subinterval converter $q_{cx}^{I} + q_{cx}^{II} = \phi \rightarrow q_{cx}^{I} = -q_{cx}^{II}$

same is true for q_{ax}

To find Rout:



$Z_{out} = R_{out} = \frac{V}{I}$

Tellegen's Theorem

For any valid circuit (KVL & KCL apply)

$$\sum_{i \in \text{elements}} V_i I_i = \phi$$

$V_i \rightarrow$ Voltage across

$I_i \rightarrow$ current through

Passive sign convention held on all elements

For our 2-interval SC converter

$$\underline{\bar{a}^{I}\bar{v}^{I}} + \underline{\bar{a}^{II}\bar{v}^{II}} = \phi \qquad \& \qquad \bar{a}^{I}\bar{v}^{I} = \bar{a}^{II}\bar{v}^{II} = \phi$$

$$\underbrace{V_g\left(a_{in}^{I} + a_{in}^{II}\right)}_{\text{set } V_g = \phi \text{ to find } R_{out}} + a_{c_1}^{I} v_{c_1}^{I} + a_{c_1}^{II} v_{c_1}^{II} + \dots \; V_{out}\underbrace{\left(a_{out}^{I} + a_{out}^{II}\right)}_{= 1 \text{ by our normalization}} = \phi$$

SSL Output Resistance

$$a_{c1}^{I} v_{c1}^{I} + a_{c1}^{II} v_{c1}^{II} + \underbrace{\cdots}_{\text{any other caps}} = -V_{out}$$

$$\sum_{i \in \text{all caps}} \underline{a_{ci}^{I}} v_{ci}^{I} + \underline{a_{ci}^{II}} v_{ci}^{II} = -V_{out} = \sum_{i \in \text{caps}} a_{ci}^{I} \underbrace{\left(v_{ci}^{I} - v_{ci}^{II} \right)}$$

$$\Delta v_{ci} = \frac{q_{ci}}{C_i} = \left(\frac{q_{ci}}{q_{out}}\right)\frac{q_{out}}{C_i} = a_{ci}^{I} \frac{q_{out}}{C_i}$$

For a 2-subinterval converter $\underline{a_{ci}^{I} = -a_{ci}^{II}}$

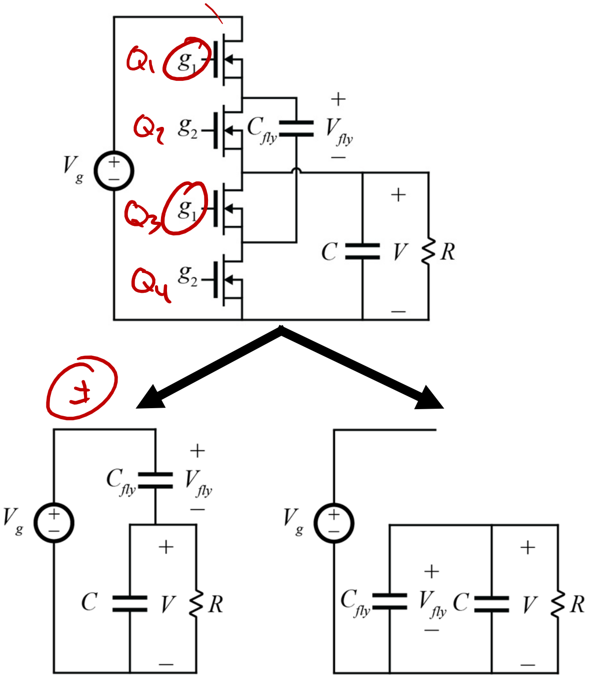
$$\left(\frac{1}{q_{out}} \frac{1}{f_s}\right) \quad -V_{out} = \sum_{i \in \text{caps}} \left(a_{ci}^{I}\right)^2 \frac{q_{out}}{C_i} \quad \left(\frac{1}{q_{out}} \frac{1}{f_s}\right)$$

$$\frac{-V_{out}}{q_{out} f_s} = \sum_{i \in \text{caps}} \left(a_{ci}^{I}\right)^2 \frac{1}{C_i f_s}$$

$$\frac{-V_{out}}{I_{out}} = \boxed{R_{out} = \sum_{i \in \text{caps}} \left(a_{ci}\right)^2 \frac{1}{C_i f_s}}$$

for 2-subinterval converters

Output Resistance



$$\begin{array}{cccc} & a_{in} & a_{fly} & a_{out} \\ \bar{a}^{\mathrm{I}} = & \left[-\frac{1}{2}\right. & \frac{1}{2} & \left.\frac{1}{2}\right] \end{array}$$

$$\bar{a}^{\mathrm{II}} = \left[0 \quad -\frac{1}{2} \quad \frac{1}{2}\right]$$

$$R_{out} = \sum_{caps} (a_c)^2 \frac{1}{C_i f_s}$$

$$= \left(\frac{1}{2}\right)^2 \frac{1}{C_{fly} f_s} = \boxed{\frac{1}{4 C_{fly} f_s} = R_{o,SSL}}$$

In FSL:

$$\begin{array}{ccccc} & a_{Q_1} & a_{Q_2} & a_{Q_3} & a_{Q_4} \\ \bar{a}_r = & \left[1/2\right. & 1/2 & 1/2 & \left.1/2\right] \end{array}$$

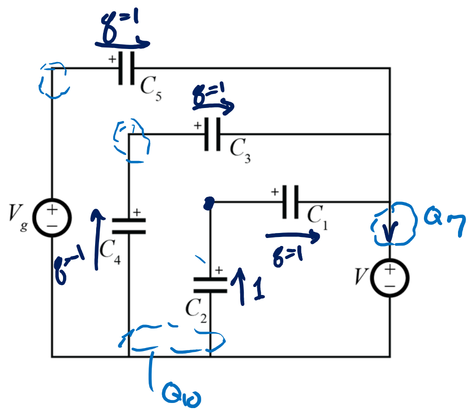
$$R_{o,FSL} = \sum_{i \in FETs} 2 (a_{r,i})^2 R_{on,i}$$

$$\boxed{R_{o,FSL} = 2 R_{on}}$$

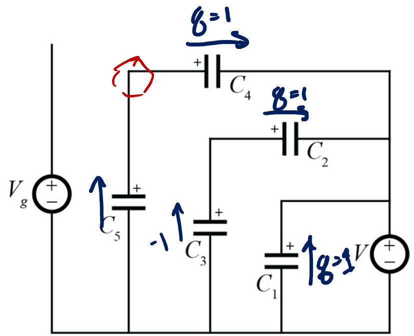
$$(R_{on,i} = R_{on})$$

Dickson Charge Vector Analysis

I



II



$$a^{I} = \begin{array}{c} \begin{array}{ccccccc} q_{in} & q_{c1} & q_{c2} & q_{c3} & q_{c4} & q_{c5} & q_{out} \end{array} \\ \left[\begin{array}{ccccccc} -1 & 1 & -1 & 1 & -1 & 1 & 3 \end{array} \right] \end{array} / q_{out}$$

$$a^{I} = \left[-\tfrac{1}{6} \quad \underbrace{\tfrac{1}{6} \quad -\tfrac{1}{6} \quad \tfrac{1}{6} \quad -\tfrac{1}{6} \quad \tfrac{1}{6}}_{\bar{a}_c} \quad \tfrac{1}{2} \right]$$

$q_{out} = 6$

$$a^{II} = [\, \emptyset \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 3 \,] / q_{out}$$

$$a^{II} = \left[\emptyset \quad \tfrac{-1}{6} \quad \tfrac{1}{6} \quad \tfrac{-1}{6} \quad \tfrac{1}{6} \quad \tfrac{-1}{6} \quad \tfrac{1}{2} \right]$$

$$\underbrace{a_{in} = -\tfrac{1}{6} \qquad a_{out} = 1}_{M = \frac{1}{6}, \quad 6:1 \text{ Dickson converter}}$$

Dickson Output Resistance

$$R_{out} = \sum_{i \in caps} (a_{c,i})^2 \frac{1}{C_i f_s}$$

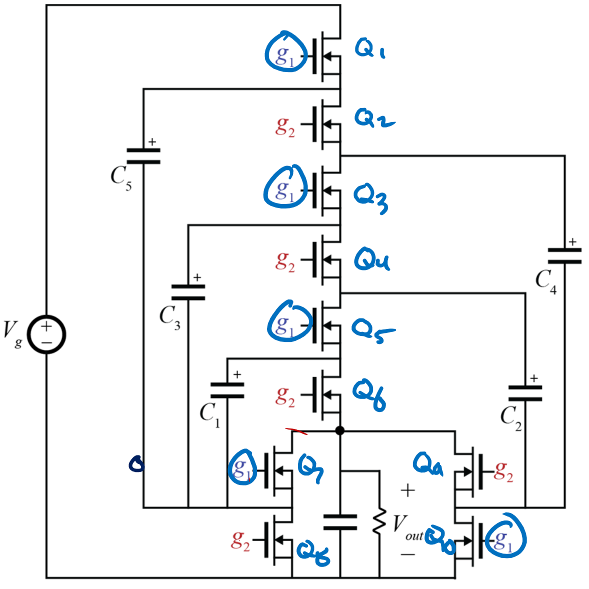
Assume all $C_i = C_{fly}$

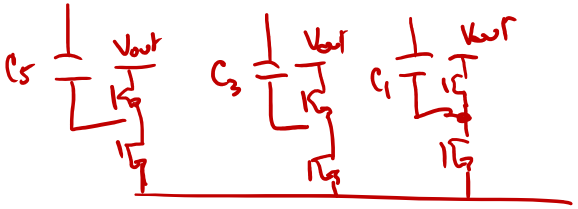
$$\boxed{R_{o,SSL} = \frac{5}{36} \frac{1}{C_{fly} f_s}}$$

In FSL

$a_r = [$

Dickson Converter





$$a^{I} = \left[-\tfrac{1}{6} \quad \tfrac{1}{6} \quad -\tfrac{1}{6} \quad \tfrac{1}{6} \quad -\tfrac{1}{6} \quad \tfrac{1}{6} \quad \tfrac{1}{2} \right]$$

$$a^{II} = \left[\emptyset \quad -\tfrac{1}{6} \quad \tfrac{1}{6} \quad -\tfrac{1}{6} \quad \tfrac{1}{6} \quad -\tfrac{1}{6} \quad \tfrac{1}{2} \right]$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \\ a_r = [& 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/2 & 1/2 & 1/3 & 1/3 &] \end{array}$$

$$R_{o,FSL} = \sum_{i \in FETs} (a_{ri})^2 R_{on,i} \cdot 2$$

$$R_{on,i} = R_{on}$$

$$= 2R_{on}\left(\frac{1}{6} + \frac{1}{2} + \frac{2}{9} \right)$$

$$R_{o,FSL} = 2R_{on}\left(\frac{16}{18} \right) = R_{on}\frac{32}{18} = \boxed{R_{on}\frac{16}{9}}$$

Charge Vector Analysis in FSL

$\bar{a}_r = \frac{\bar{q}_r}{q_{out}}$ where q_{ri} are charges through transistors (or any resistances)

For a 2-subinterval converter transistor only has nonzero q_{ri} in one of the subintervals

a_{ri} found as some linear combination of a_{ci}

$R_{o,FSL}$ found by:

i_{ri} = current in transistor i when conducting → Assumed constant in FSL

$i_{ri} = q_{ri}\left(\frac{2}{T_s}\right)$ → Assume 2-subinterval converter w/ $D = 50\%$

$P_{ri} = (i_{ri})^2 R_{on,i} \cdot \frac{1}{2} = \left(q_{ri}\frac{2}{T_s}\right)^2 R_{on,i} \cdot \frac{1}{2}$

$P_{ri} = (a_{ri})^2\, 4 \cdot \frac{1}{2} f_s^2 R_{on,i}\, q_{out}^2$

$P_{ri} = I_{out}^2 (a_{ri})^2 R_{on,i} \cdot 2$

$P_{tot} = I_{out}^2 \sum_{i \in FETs} 2(a_{ri})^2 R_{on,i}$ →

$$\boxed{R_{o,FSL} = \sum_{i \in FETs} 2(a_{ri})^2 R_{on,i}}$$