$V_{base} = V_{DC}$

$I_{base} = \frac{V_{DC}}{R_0}$

$$\begin{cases} v_c(t) = V_{DC} + (V_0 - V_{DC})\cos(\omega_0 t) + R_0(I_0 - I_{DC})\sin(\omega_0 t) \\ i_L(t) = I_{DC} + (I_0 - I_{DC})\cos(\omega_0 t) + \frac{1}{R_0}(V_{DC} - V_0)\sin(\omega_0 t) \end{cases}$$

$\frac{R_0}{V_{base}}(I_0 - I_{DC}) = \frac{I_0 - I_{DC}}{I_{base}}$

$$\begin{cases} m_c(t) = \frac{v_c(t)}{V_{base}} = m_{DC} + \underline{(m_0 - m_{DC})}\cos(\theta) + \underline{(j_0 - j_{DC})}\sin(\theta) \\ j_L(t) = \frac{i_L(t)}{I_{base}} = j_{DC} + \underline{(j_0 - j_{DC})}\cos(\theta) + \underline{(m_{DC} - m_0)}\sin(\theta) \end{cases}$$

$A = m_0 - m_{DC}$

$B = j_0 - j_{DC}$

$$\begin{cases} m_c(t) = m_{DC} + A\cos\theta + B\sin\theta \\ j_L(t) = j_{DC} + B\cos\theta + (-A)\sin\theta \end{cases}$$

Try looking at $(m_c(t) - m_{DC})^2 + (j_L(t) - j_{DC})^2 =$

$$\left(A^2\cos^2\theta + B^2\sin^2\theta + \cancel{2AB\cos\theta\sin\theta}\right) + \left(B^2\cos^2\theta + A^2\sin^2\theta - \cancel{2AB\cos\theta\sin\theta}\right)$$

$$= A^2(\underbrace{\cos^2\theta + \sin^2\theta}_{1}) + B^2(\underbrace{\sin^2\theta + \cos^2\theta}_{1})$$

$$(m_c(t) - m_{DC})^2 + (j_L(t) - j_{DC})^2 = A^2 + B^2 = (m_0 - m_{DC})^2 + (j_0 - j_{DC})^2$$

Normalized Trajectory

$$(m_c(t) - M_{DC})^2 + (j_L(t) - J_{DC})^2 = (M_0 - M_{DC})^2 + (J_0 - J_{DC})^2$$

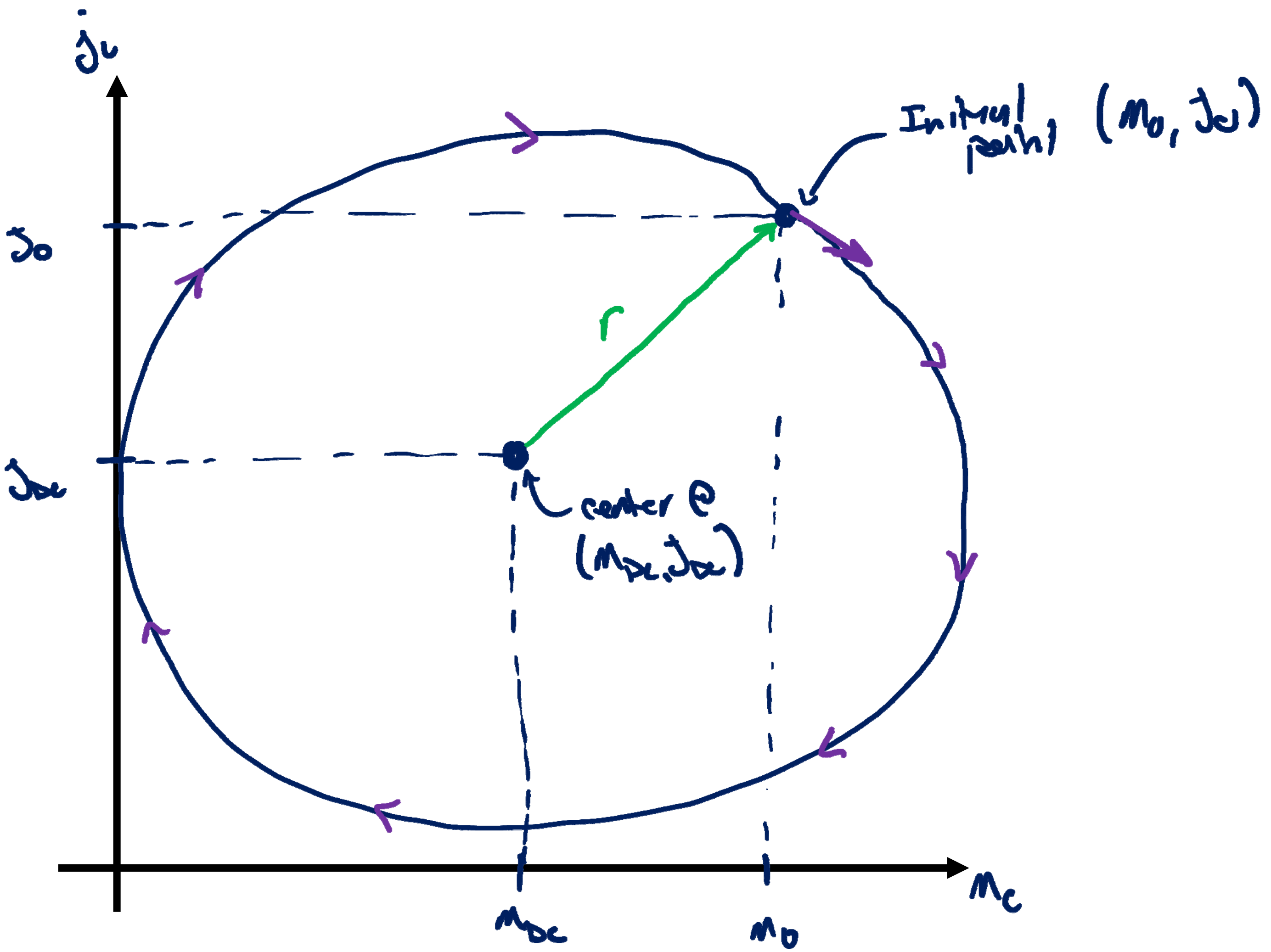
this is the equation for a circle in the $m_c - j_L$ plane

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

here

$$r = \sqrt{(M_0 - M_{DC})^2 + (J_0 - J_{DC})^2}$$

- DC op point defines center
- IC defines initial point & radius
- IC on 1st derivatives defines direction

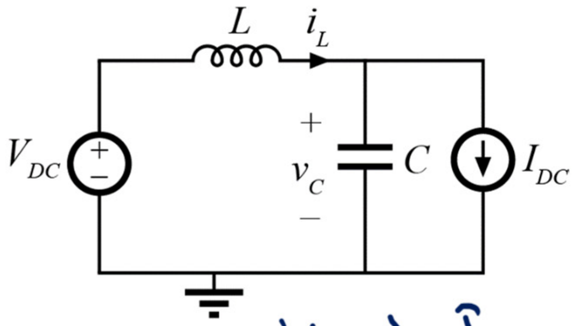


state plane:

for 1-L, 1-C circuits w/ DC bias

transforms diff EQ analysis to geometry

State Plane Analysis



I.C.'s $i_L(t=0) = I_0$

$v_C(t=0) = V_0$

DC solution is $v_C = V_{DC}$ & $i_L = I_{DC}$

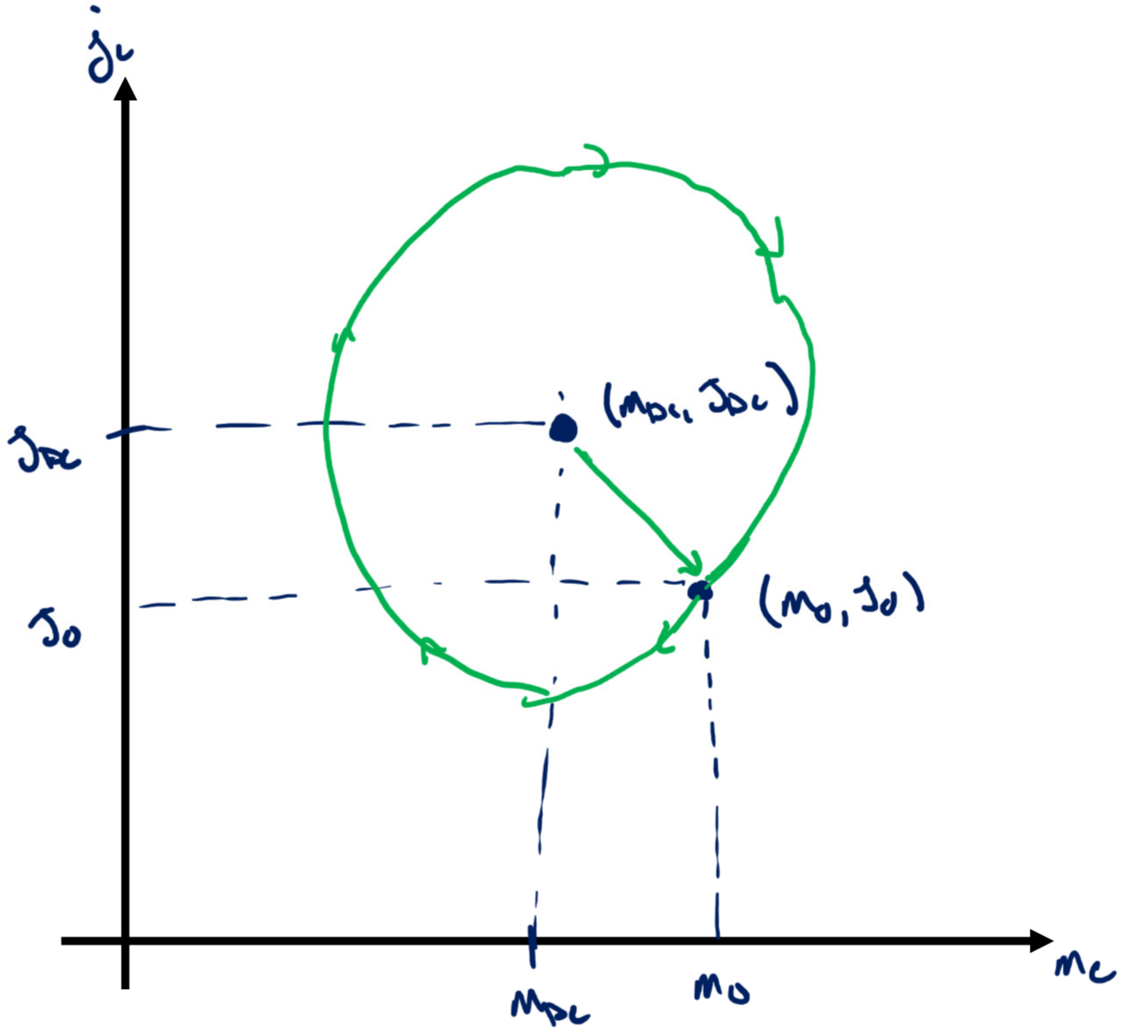
$V_{base} = \underline{\qquad}$ anything

$I_{base} = \dfrac{V_{base}}{R_0}$

Direction from IC on 1st derivative

$m_0 > m_{DC}$, j_L initially decreasing

$j_0 < j_{DC}$, m_C initially decreasing



[1] R. Oruganti and F. C. Lee, “Resonant Power Processors, Part I – State Plane Analysis”, Industry Applications, IEEE Tran. on, vol. 21, no. 6, nov 1985.

[2] D. P. Atherton, Nonlinear Control Engineering. London: Van Nostrand Reinhold, 1982, Ch. 2.

Example Analysis

What is $i_{L,ph}$?

$$j_{L,ph} = j_{DC} + r$$

$$= j_{DC} + \sqrt{(j_0 - j_{DC})^2 + (m_0 - m_{DC})^2}$$

$$i_{L,ph} = I_{base} \cdot j_{L,ph} = I_{DC} + \frac{V_{base}}{Z_0}\sqrt{(j_0 - j_{DC})^2 + (m_0 - m_{DC})^2}$$

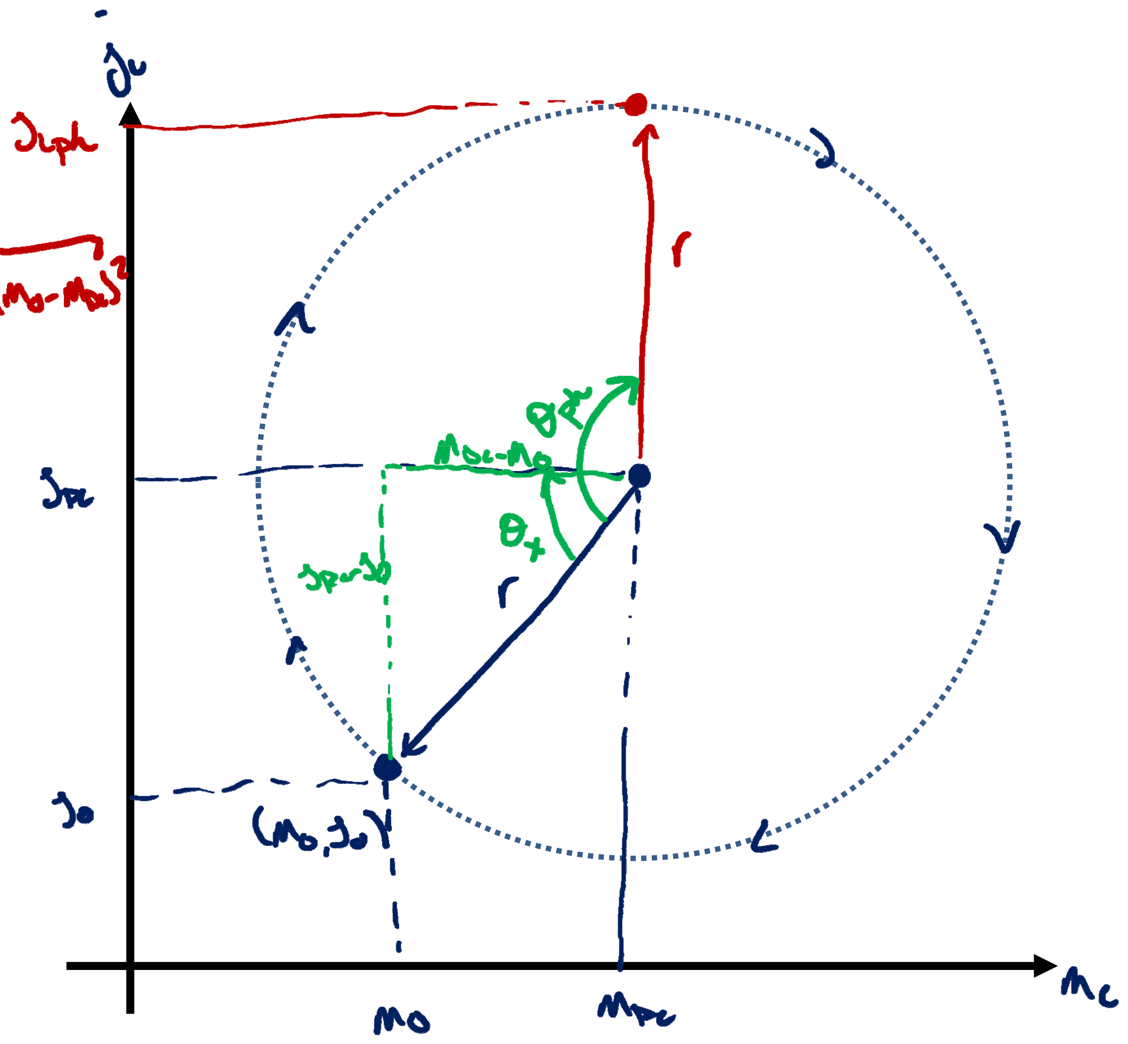
At what time does $i_{L,ph}$ occur?

$$\theta_{ph} = \omega_0 t_{ph}$$

$$\theta_{ph} = \frac{\pi}{2} + \theta_x$$

$$\theta_{ph} = \frac{\pi}{2} + \tan^{-1}\left(\frac{j_{DC} - j_0}{m_{DC} - m_0}\right)$$

$$t_{ph} = \frac{\theta_{ph}}{\omega_0} = \frac{\pi}{2\omega_0} + \frac{1}{\omega_0}\tan^{-1}\left(\frac{j_{DC} - j_0}{m_{DC} - m_0}\right)$$



State Plane Algorithm

Manipulate circuit into 1-L, 1-C & DC source equivalent
- May require some linearize & approximation

(1) Normalize

$$\begin{cases} m_c(t) = \dfrac{v_c(t)}{V_{base}} \\ j_L(t) = \dfrac{i_L(t)}{I_{base}} \\ \theta_x = \omega_0 t_x \end{cases}$$

V_{base} picked for convenience

$$I_{base} = \frac{V_{base}}{R_0}$$

$$R_0 = \sqrt{\frac{L}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

(2) plot on $m_c - j_L$ axes the normalized resonant trajectory
- center at the dc solution (m_{DC}, j_{DC})
- initial point at IC (m_0, j_0)
- initial direction from IC or 1st derivatives of $v_c(t)$ and/or $i_L(t)$

(3) Solve any quantities of interest using geometry & trig

(4) Denormalize back to i, v, & t

DCM Buck Converter Example (M=1/2)