State Plane Solution: Intervals 1 & 2

$V_{base} = V_g$

① $\left(\frac{1}{V_{base}}\right)\frac{V_g - V}{L} t_1 = (I_1 - (-I_4))\left(\frac{1}{V_{base}}\right)\frac{R_0}{R_0}$

$(1-M)\frac{t_1}{L} = (J_1 + J_4)\frac{1}{R_0}$

$$\boxed{(1-M)\,\theta_1 = (J_1 + J_4)}$$

Nonresonant: one equation

② $r_2^{\,2} = \boxed{J_2^{\,2} + M = J_1^{\,2} + (1-M)^2}$

$$\boxed{\beta = \pi - \tan^{-1}\left(\frac{J_2}{M}\right) - \tan^{-1}\left(\frac{J_1}{1-M}\right)}$$

Resonant interval: two equations $\left\{ \begin{array}{l} \text{relate start \& end points by radius} \\ \text{solve angle} \end{array} \right.$

$\frac{R_0}{L} = \frac{\sqrt{L/C_{eq}}}{L} = \sqrt{\frac{1}{L C_{eq}}} = \omega_0$

$\theta_1 = \omega_0 t_1$

θ_x is some nonresonant subinterval

$\alpha, \beta, \gamma, \delta$, etc $\rightarrow$ resonant subintervals

State Plane Solution: Intervals 3 & 4

(3) $-\frac{V}{L} t_3 = -J_3 - J_2$

$$\boxed{m\,\theta_3 = J_3 + J_2}$$

(4) $r_4^2 = \boxed{J_3^2 + m^2 = J_4^2 + (1-m)^2}$

$$\boxed{\delta = \pi - \tan^{-1}\left(\frac{J_3}{m}\right) - \tan^{-1}\left(\frac{J_4}{1-m}\right)}$$

State Plane Solution: Averaging Step

One small-ripple filter element : C_{out}

Cap-charge balance on C_{out} :

$$\langle i_{C_{out}} \rangle \big|_{T_s} = \phi \longrightarrow \langle i_L \rangle \big|_{T_s} = I_{out} = \frac{V}{R}$$

$$I_{out} = \langle i_L \rangle \big|_{T_s} = \frac{1}{T_s}\int_0^{T_s} i_L(t)\,dt = \frac{1}{T_s}\left[\int_{1} i_L(t)\,dt + \int_{2} i_L(t)\,dt + \int_{3} i_L(t)\,dt + \int_{4} i_L(t)\,dt\right]$$

$$\int_{1} i_L(t)\,dt = \frac{I_1 - I_4}{2} t_1$$

$$\int_{3} i_L(t)\,dt = \frac{I_2 - I_3}{2} t_3$$

$$\int_{2} i_L(t)\,dt = \int_{2} -i_C(t)\,dt = Q_{C2} = C_{eq}(\Delta V_c) = C_{eq} V_g$$

$$\int_{4} i_L(t)\,dt = \int_{4} -i_C(t)\,dt = Q_{C4} = -C_{eq} V_g$$

$$\left(\frac{1}{I_{base}}\right) I_{out} = \frac{1}{T_s}\left[\frac{I_1 - I_4}{2} t_1 + \cancel{C_{eq} V_g} + \frac{I_2 - I_3}{2} t_3 - \cancel{C_{eq} V_g}\right]\left(\frac{1}{I_{base}}\right)$$

$$J_{out} = \frac{1}{T_s}\left[\frac{J_1 - J_4}{2} t_1 + \frac{J_2 - J_3}{2} t_3\right]\left(\frac{\omega_0}{\omega_0}\right) \longrightarrow \boxed{J_{out} = \frac{F}{2\pi}\left[\frac{J_1 - J_4}{2}\theta_1 + \frac{J_2 - J_3}{2}\theta_3\right]}$$

$$F = \frac{f_s}{f_0}$$

Normalized Period

$(\omega_0)\left[t_1 + t_2 + t_3 + t_4\right] = T_s\,\omega_0 = \frac{1}{f_s}\,\omega_0$

$$\boxed{\theta_1 + \beta + \theta_3 + \delta = \frac{2\pi}{F}}$$

Equations: 8

Unknowns: θ_1, β, θ_3, δ, J_1, J_2, J_3, J_4

known/designed: J_{out}, M, F ← f_s, L, C

Different solutions possible $J_4 = \phi$ (min conduction loss w/ ZVS) are common trajectory → means F & therefore f_s are variable

Solving Numerically