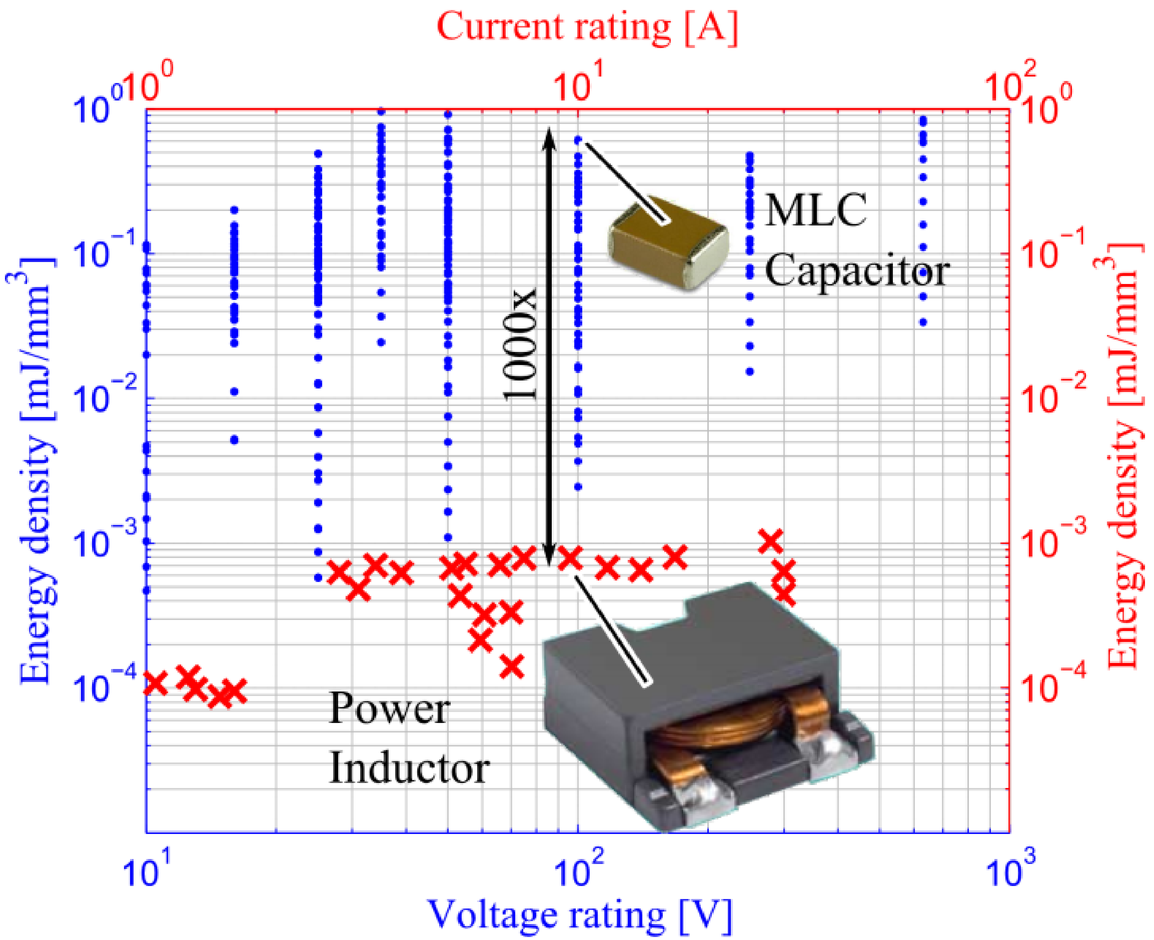
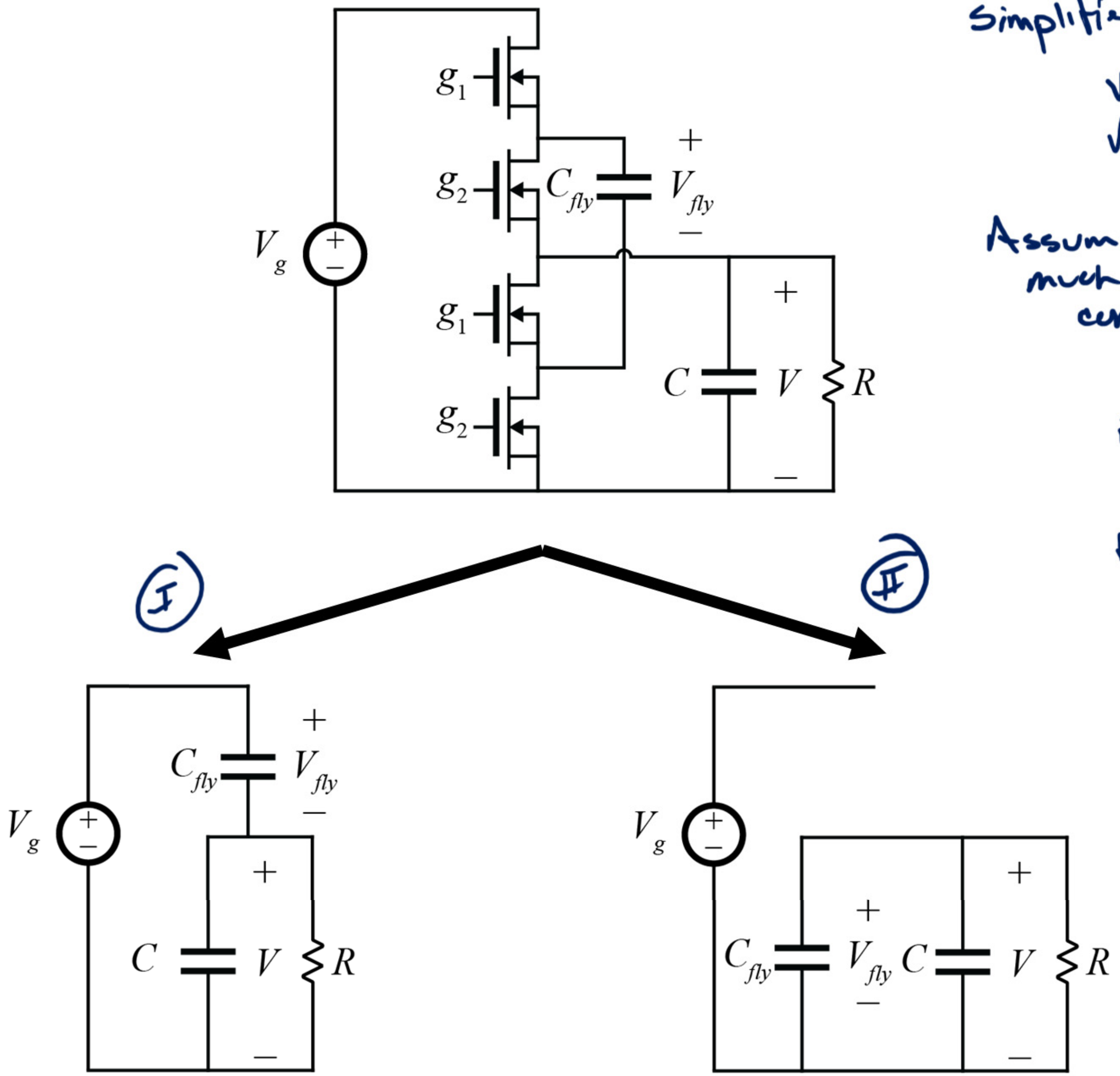
Switched Capacitor Converters



A 2:1 SC Converter



Simplified analysis

$V \approx$ constant ←

$V_{fly} \approx$ constant

Assume all subinterval times are much longer than any R-C time constants w/ parasitic resistances

in I $\quad V + V_{fly} = V_g$

in II $\quad V_{fly} = V$

$$2V = V_g$$

$$\boxed{\frac{V}{V_g} = \frac{1}{2}}$$

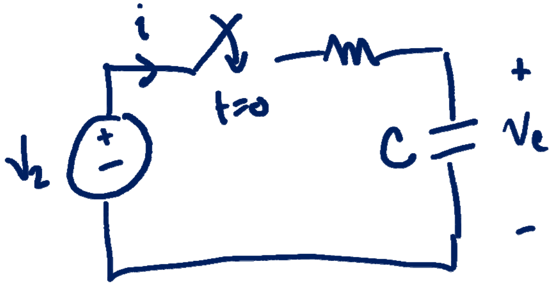
SC Converters

- Fixed conversion ratio
  - Continuous regulation impossible[1] (except linear)
- Not lossless, even with ideal elements
- Can be very small, fully integrated
  - Discrete caps can be smaller than inductors for comparable energy storage
  - Integrated caps are much better than on-chip inductors
- **Resonant** versions can reduce loss
- **Hybrid** versions can allow regulation

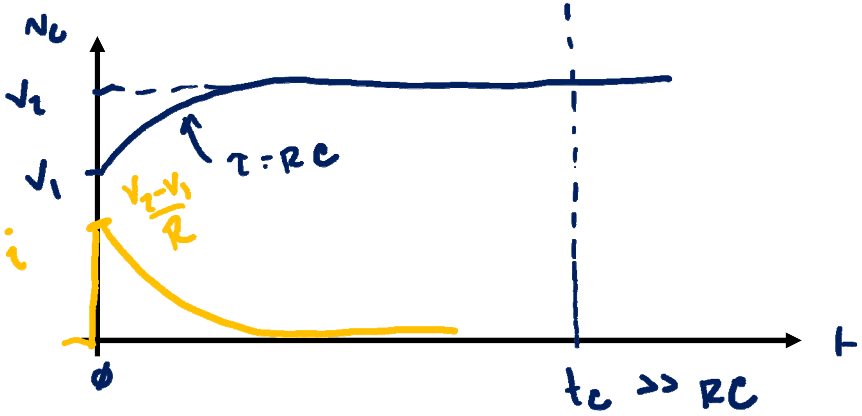
add some inductance

[1]D. Wolaver, "Fundamental study of dc to dc conversion systems," dissertation, 1969

Capacitor Charging: Voltage Source



$v_c(t=0) = V_1$



After $t_e >> RC$

$\Delta E_c = \frac{1}{2}CV_2^2 - \frac{1}{2}CV_1^2$

$\boxed{\Delta E_c = \frac{1}{2}C(V_2^2 - V_1^2)}$

$E_V = \int_0^{t_e} i \cdot V_2 \, dt = V_2 Q_c = \boxed{V_2 C (V_2 - V_1) = E_V}$

$E_{loss} = E_V - \Delta E_c = V_2 C (V_2 - V_1) - \frac{1}{2}C(V_2^2 - V_1^2)$

$= CV_2^2 - CV_2V_1 - \frac{1}{2}CV_2^2 + \frac{1}{2}CV_1^2$

$= \frac{1}{2}CV_2^2 - CV_2V_1 + \frac{1}{2}CV_1^2$

$= \boxed{\frac{1}{2}C(V_2 - V_1)^2 = E_{loss} = \frac{1}{2}C(\Delta V_c)^2}$

$$\eta_c = \frac{\Delta E_c}{E_V} = \frac{\frac{1}{2}C(V_2^2 - V_1^2)}{V_2 C (V_2 - V_1)}$$

$$\Delta V_c = (V_2 - V_1)$$

$$V_1 = V_2 - \Delta V_c$$

$$= \frac{\frac{1}{2}CV_2^2 - \frac{1}{2}C(V_2 - \Delta V_c)^2}{V_2 C \Delta V_c} = \frac{\cancel{\frac{1}{2}CV_2^2} - \cancel{\frac{1}{2}CV_2^2} + CV_2\Delta V_c - \frac{1}{2}C\Delta V_c^2}{V_2 C \Delta V_c}$$

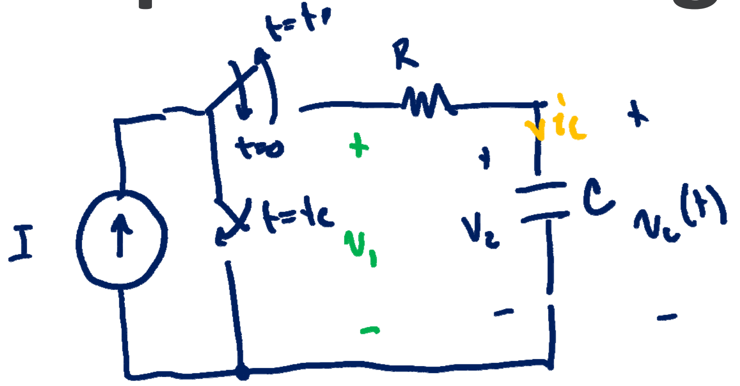
$$= 1 - \frac{\frac{1}{2}\cancel{C}\Delta V_c^{\cancel{2}}}{V_2\cancel{C}\cancel{\Delta V_c}}$$

$$\boxed{\eta_c = 1 - \frac{\Delta V_c}{2V_2}}$$

if $\Delta V_c \ll V_2$, $\eta_c \to 1$

- No dependence on R other than $t_c \gg RC$

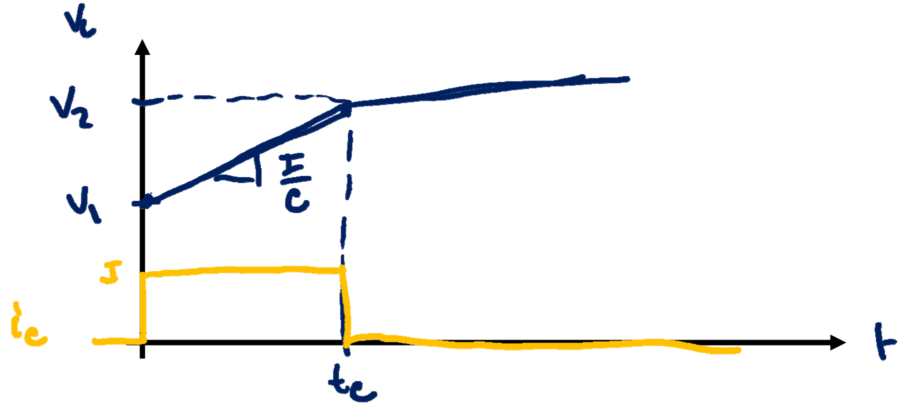
Capacitor Charging: Current Source



$$V_c(t=0) = V_1$$

$$\frac{I}{C} t_c = V_2 - V_1$$

$$I = \frac{C(V_2 - V_1)}{t_c}$$



$$\Delta E_C = \frac{1}{2} C (V_2^2 - V_1^2)$$

$$E_I = \int_0^{t_c} I v_1 \, dt = I \int_0^{t_c} (V_c + IR) \, dt = \underbrace{\frac{1}{2} C (V_2^2 - V_1^2)}_{\Delta E_C} + \underbrace{I^2 R t_c}_{E_R}$$

$$= \frac{1}{2} C (V_2^2 - V_1^2) + \frac{C^2 (V_2 - V_1)^2}{t_c^2} R t_c$$

$$= \frac{1}{2} C (V_2^2 - V_1^2) + \frac{1}{2} C (V_2 - V_1)^2 \underbrace{\frac{2RC}{t_c}}_{}$$

scale factor relative to where charging

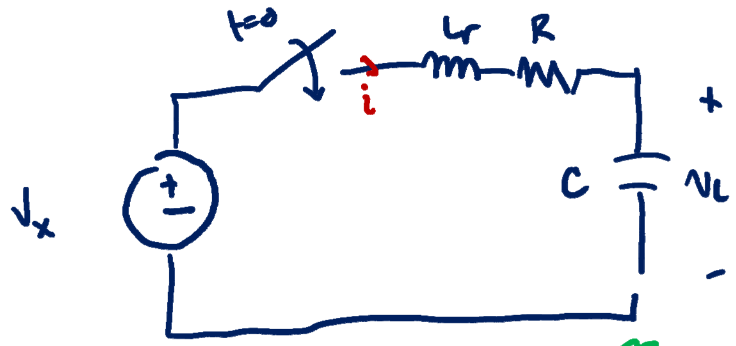
Voltage source $E_{loss} = \frac{1}{2}C(V_2-V_1)^2$

I - source $E_{loss} = \frac{1}{2}C(V_2-V_1)^2 \frac{2RC}{t_c}$

if $t_c \gg 2RC$ current source charging has lower loss

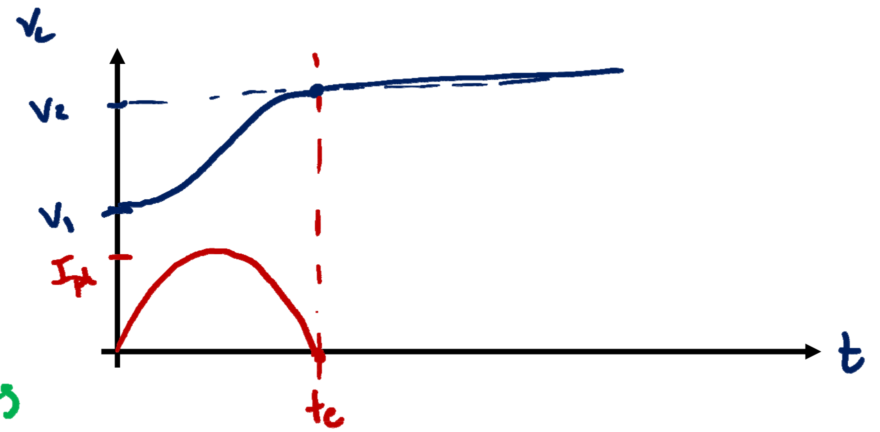
if $t_c \not\gg 2RC$, unknown → voltage source charging expressions were developed for $t_c \gg RC$

Capacitor Charging: Resonant



$v_c(t=0) = V_1$

$V_x = \frac{V_1+V_2}{2}$



state plane: apply high-η approx
solve waveforms w/ $R = \emptyset$ then calculate losses

$$\Delta E_c = \frac{1}{2}C(V_2^2 - V_1^2)$$

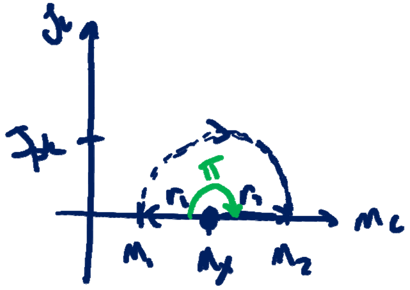
$$E_{loss} = E_R = I_{rms}^2 R t_c = \frac{I_{pk}^2}{2} R t_c$$

$$= \frac{1}{R_0^2}\frac{R}{2} t_c \frac{(V_2-V_1)^2}{4} = \frac{1}{R_0^2}\frac{\pi}{\omega_0}\frac{R}{2}\frac{(V_2-V_1)^2}{4}$$

$$\boxed{E_{loss} = \frac{\pi R}{4 R_0}\, \frac{1}{2}C(V_2-V_1)^2}$$

if $\frac{R}{R_0} = \frac{1}{Q}$ is small, higher η than v-src charging

state plane:



$J_{pk} = r_1 = m_x - m_1 = m_2 - m_x = \frac{m_2-m_1}{2}$

$I_{pk} = J_{pk}\frac{V_{base}}{R_0}$

$I_{pk} = \frac{1}{R_0}\frac{V_2-V_1}{2}$

$t_c = \frac{\pi}{\omega_0}$

$R_0 \omega_0 = \sqrt{\frac{L}{C}}\frac{1}{\sqrt{LC}} = \frac{1}{C}$