$$E_{loss} = \underbrace{\frac{\pi}{4}\frac{R}{R_0}}\ \frac{1}{2}C(V_2-V_1)^2$$

$\frac{R}{R_0} \rightarrow$ High Q resonance will be more efficient than V-src charging

$$\frac{1}{R_0} = \omega_0 C, \qquad \omega_0 = \frac{\pi}{t_c}$$

$$E_{loss} = \frac{\pi^2}{4}\frac{RC}{t_c}\ \frac{1}{2}C(V_2-V_1)^2$$

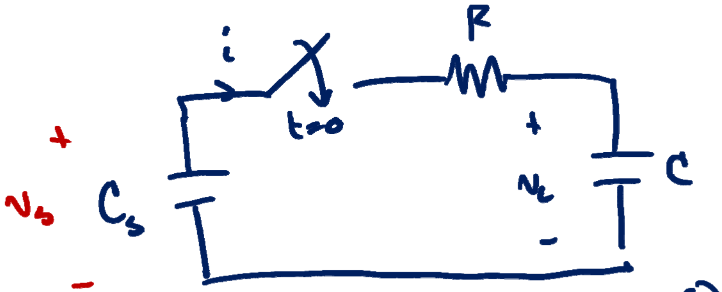
↳ if $t_c \gg RC$, less loss than V-src charging

if $t_c \not\gg RC$, V-src charging equations don't apply

Also may violate high-η approx in resonant charging

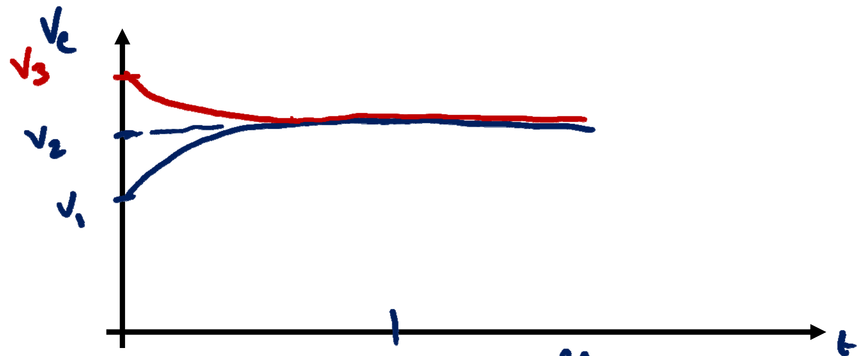
$$\frac{\pi^2}{4} \approx 2.5$$

Capacitor Charging: Cap-Cap



$v_s(t=0) = V_3$

$v_c(t=0) = V_1$



$t_c \text{ ss } R\frac{CC_s}{C+C_s} = \tau_{eq}$

since $\int i_s = \int i_c = \int i = Q_s = Q_c = Q$

$$V_3 C_s + V_1 C = V_2 (C_s + C)$$

$$V_3 = \frac{V_2 (C_s + C) - V_1 C}{C_s} = \frac{C}{C_s}(V_2 - V_1) + V_2$$

$$\Delta E_c = \frac{1}{2} C (V_2^2 - V_1^2)$$

$$\Delta E_s = \frac{1}{2} C_s (V_3^2 - V_2^2) = \frac{1}{2} C_s \left[\left(\frac{C}{C_s}\right)^2 (V_2 - V_1)^2 \cancel{+ V_2^2} + 2\frac{C}{C_s} V_2 (V_2 - V_1) \cancel{- V_2^2} \right]$$

$$= \frac{1}{2} C \left[\left(\frac{C}{C_s}\right)(V_2 - V_1)^2 + 2 V_2 (V_2 - V_1) \right]$$

$$E_{loss} = \Delta E_s - \Delta E_c = \frac{1}{2} C \left[\left(\frac{C}{C_s}\right)(V_2 - V_1)^2 + \overbrace{2V_2(V_2 - V_1) - V_2^2 + V_1^2}^{V_2^2 - 2V_2V_1 + V_1^2 = (V_2 - V_1)^2} \right] = \boxed{\frac{1}{2} C \left(\frac{C}{C_s} + 1\right)(V_2 - V_1)^2 = E_{loss}}$$

$$\underbrace{\frac{1}{2}C(V_2-V_1)^2}_{\text{same as V-src charging}}\underbrace{\left(1+\frac{C}{C_s}\right)}$$

if $C_s \gg C$, same as V-src charging

if $C_s \not\gg C$, higher loss than V-src charging

if $C = C_s \rightarrow$ 2x loss of V-src charging

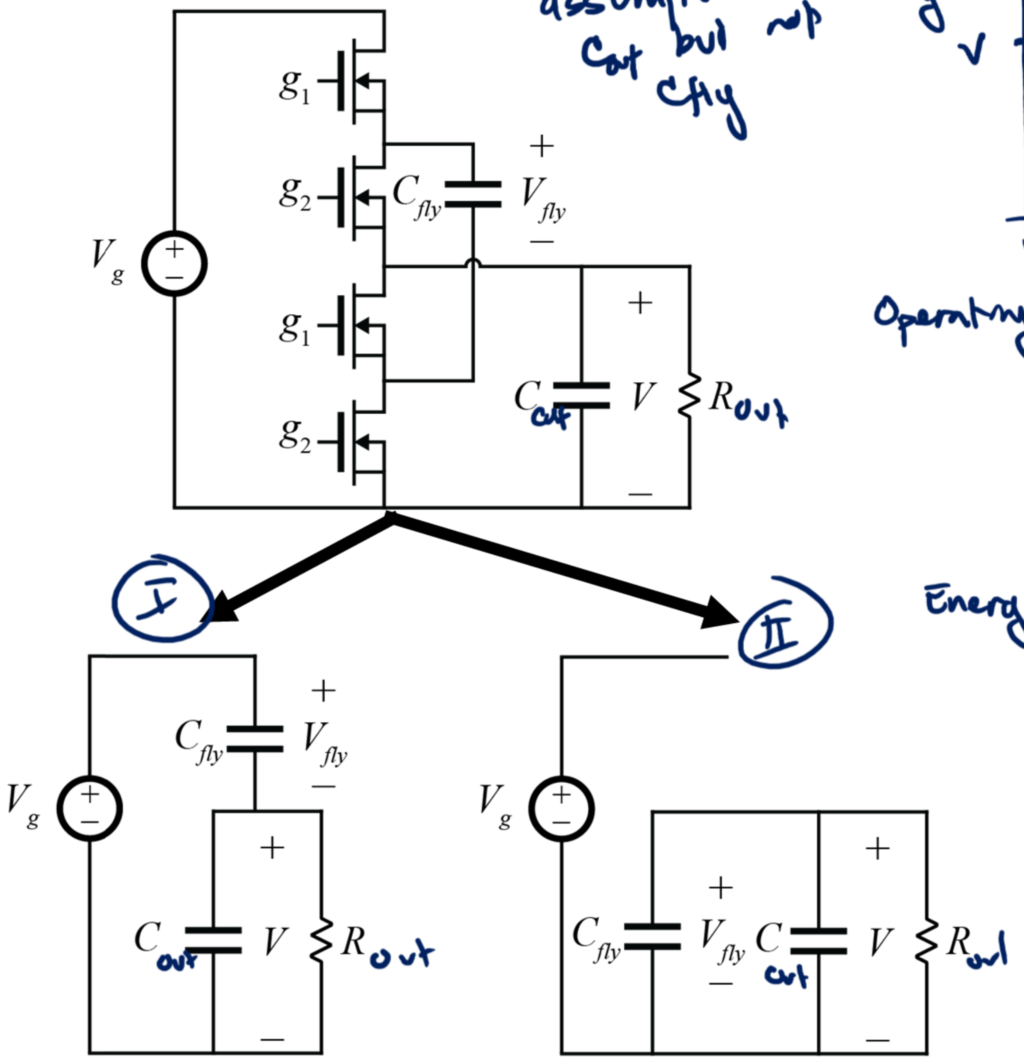
Comparison of Capacitor Charging

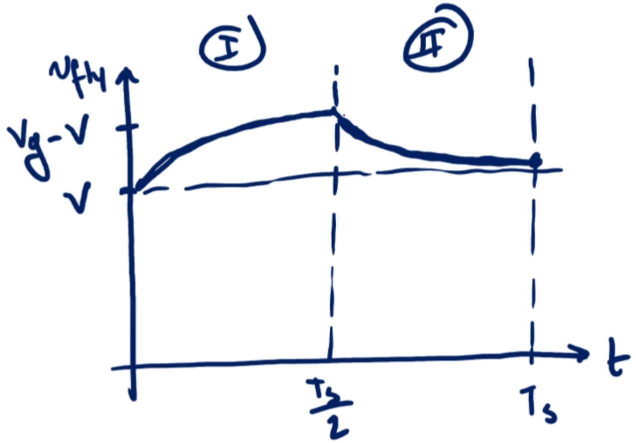
for C charging from V_1 to V_2 in time t_c

	E_{loss}	Assumptions
Voltage Source	$\frac{1}{2}C(V_2-V_1)^2$	$t_c \gg RC = \tau$
Current Source	$\frac{1}{2}C(V_2-V_1)^2 \frac{2RC}{t_c}$	X
Resonant	$\frac{1}{2}C(V_2-V_1)^2 \frac{\pi^2 RC}{4t}$ or $\frac{1}{2}C(V_2-V_1)^2 \frac{R}{R_0}\frac{\pi}{4}$	High-Q resonance so state plane solution applies $R_0 \gg R$
Cap - Cap	$\frac{1}{2}C(V_2-V_1)^2\left(\frac{C}{C_s}+1\right)$	$t_c \gg R\left(\frac{CC_s}{C+C_s}\right) = \tau_{eq}$

2:1 SC Revisited

still apply small-ripple assumption on C_{out} but not C_{fly}





Operating with $t_c \gg RC_{fly}$, $t_c = \frac{T_s}{2} \gg RC_{fly}$

$R \rightarrow$ resistance in charging path

$R = 2 \times r_{on} + ESR_c + R_{par}$

$$\text{Energy loss} = \begin{cases} \text{I} & \frac{1}{2} C_{fly} (V_g - V - V)^2 \\ \text{II} & \frac{1}{2} C_{fly} (V_g - V - V)^2 \end{cases}$$

$$E_{loss} = C_{fly} (V_g - 2V)^2$$

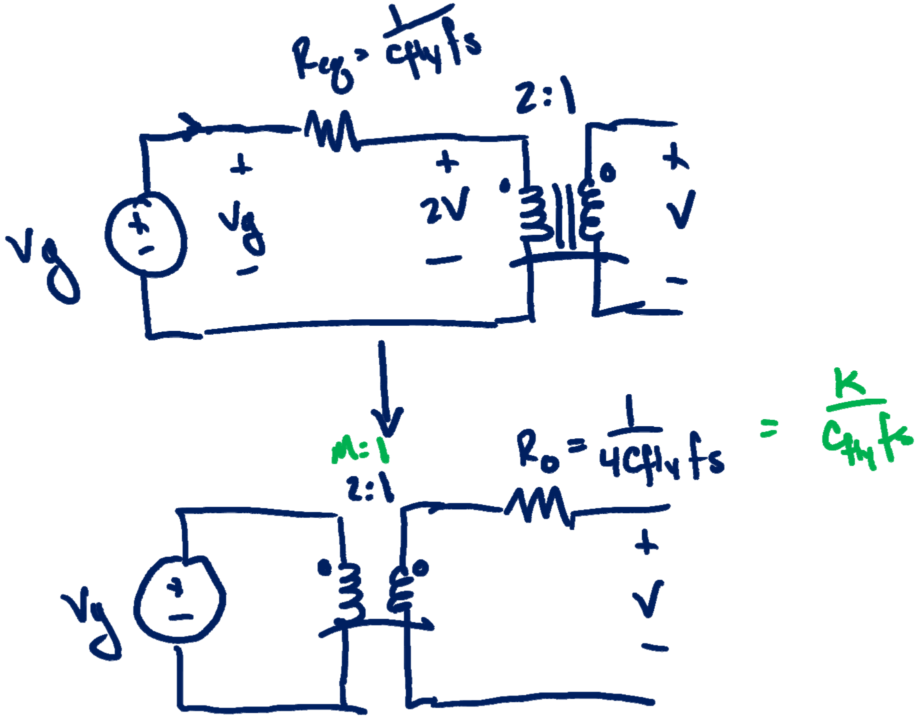
$$P_{loss} = C_{fly} (V_g - 2V)^2 f_s$$

$$P_{in} = V_g C_{fly} (V_g - 2V) f_s$$

Equivalent Circuit Model

$$P_{loss} = C_{fly}(V_g - 2V)^2 f_s$$

$$P_{in} = V_g C_{fly}(V_g - 2V) f_s$$



- No regulation possible except losses
- For high η, want small R_o → ↑C_{fly} & ↑f_s
- For $t_c \gg R C_{fly}$
- as long as above is true, $R = 2r_{on} + ESR_c$, R_{par} doesn't affect η