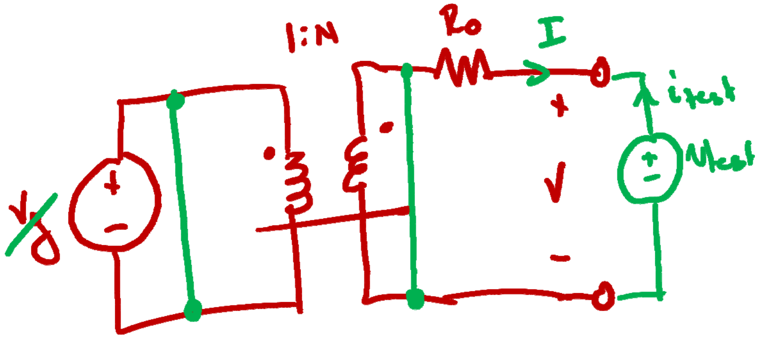
Charge Vector Analysis: Rules

- KVL & KCL equations apply
- Cap-Q balance, for all caps $\quad \int_0^{T_s} i_{cx}\,dt \rightarrow Q_{cx} = \emptyset$ in steady state

In a 2-subinterval converter

$$q_{cx}^{I} + q_{cx}^{II} = \emptyset \quad \rightarrow \quad q_{cx}^{I} = -q_{cx}^{II}$$

same for a_{cx}

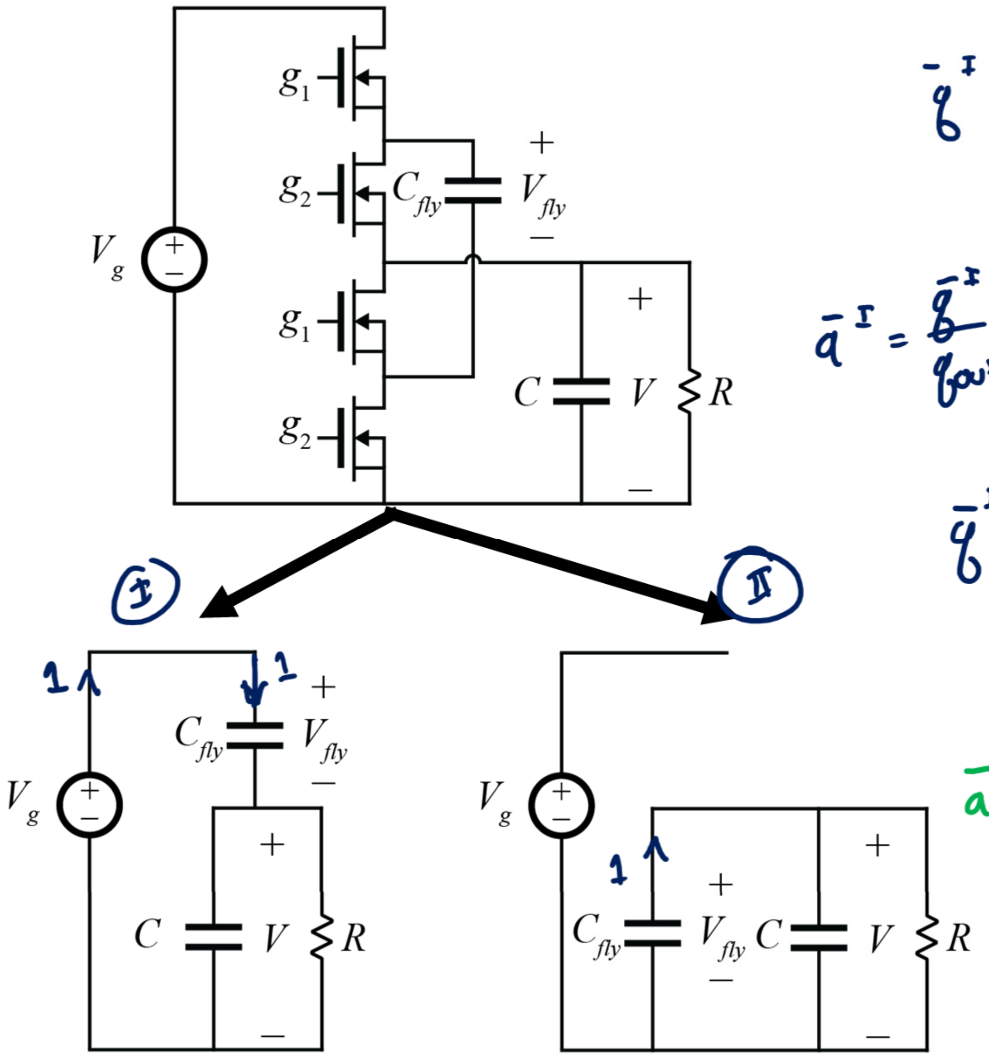


to find Ro:

shut off all indep. sources

$$R_o = \frac{v_{test}}{i_{test}} = \frac{V}{-I}\Bigg|_{v_g=\emptyset}$$

2:1 Converter Charge Vector Analysis



pick value arbitrarily

$q_{out} = q_{out}^{I} + q_{out}^{II} = 2$

$$\bar{q}^{I} = \left[\, q_{in}^{I} \quad q_{fly}^{I} \quad q_{out}^{I} \,\right]$$

$$= \left[\, -1 \leftarrow 1 \rightarrow 1 \,\right]$$

$$\bar{a}^{I} = \frac{\bar{q}^{I}}{q_{out}} = \left[\, -\tfrac{1}{2} \quad \tfrac{1}{2} \quad \tfrac{1}{2} \,\right]$$

$$\bar{q}^{II} = \left[\, q_{in}^{II} \quad q_{fly}^{II} \quad q_{out}^{II} \,\right]$$

$$= \left[\, \varnothing \quad -1 \rightarrow 1 \,\right]$$

$$\bar{a}^{II} = \left[\, \varnothing \quad -\tfrac{1}{2} \quad \tfrac{1}{2} \,\right]$$

$q_{in} = a_{in}^{I} + a_{in}^{II} = -\frac{1}{2}$

$q_{out} : a_{out}^{I} + a_{out}^{II} = 1$

$M = \frac{1}{2} = \frac{V}{V_g} = \frac{I_g}{I_o}$

$$\boxed{M = \frac{V}{V_g} = \frac{-a_{in}}{a_{out}}}$$

Tellegen's Theorem

For any valid circuit

$$\sum_{\text{all elements}} V_i I_i = \phi$$

V_i = Voltage across

I_i = current through

Passive sign convention is required

for our 2-subinterval switched cap converter

$$\bar{a}^{I}\vec{v}^{I} + \bar{a}^{II}\vec{v}^{II} = \phi \qquad \$ \qquad \bar{a}^{I}\vec{v}^{I} = \bar{a}^{II}\vec{v}^{II} = \phi$$

SSL Output Resistance

Apply Tellegen's Theorem:

$$\bar{a}^{I}\bar{v}^{II} + a^{II}v^{II} = \phi$$

$$\underbrace{\cancel{(a_{in}^{I} + a_{in}^{II})V_g}}_{\text{by application of test circuit for } R_o} \xrightarrow{} \phi + \bar{a}_c^{I}\bar{v}_c^{I} + \bar{a}_c^{II}\bar{v}_c^{II} + \underbrace{(a_{out}^{I} + a_{out}^{II})}_{= 1 \text{ by normalization}} V = \phi$$

$$\bar{a}_c^{I}\bar{v}_c^{I} + \bar{a}_c^{II}\bar{v}_c^{II} = -V = \sum_{\text{all caps}} a_{ci}^{I}v_{ci}^{I} + a_{ci}^{II}v_{ci}^{II}$$

for 2 subintervals $a_{ci}^{I} = -a_{ci}^{II}$ by cap-Q balance

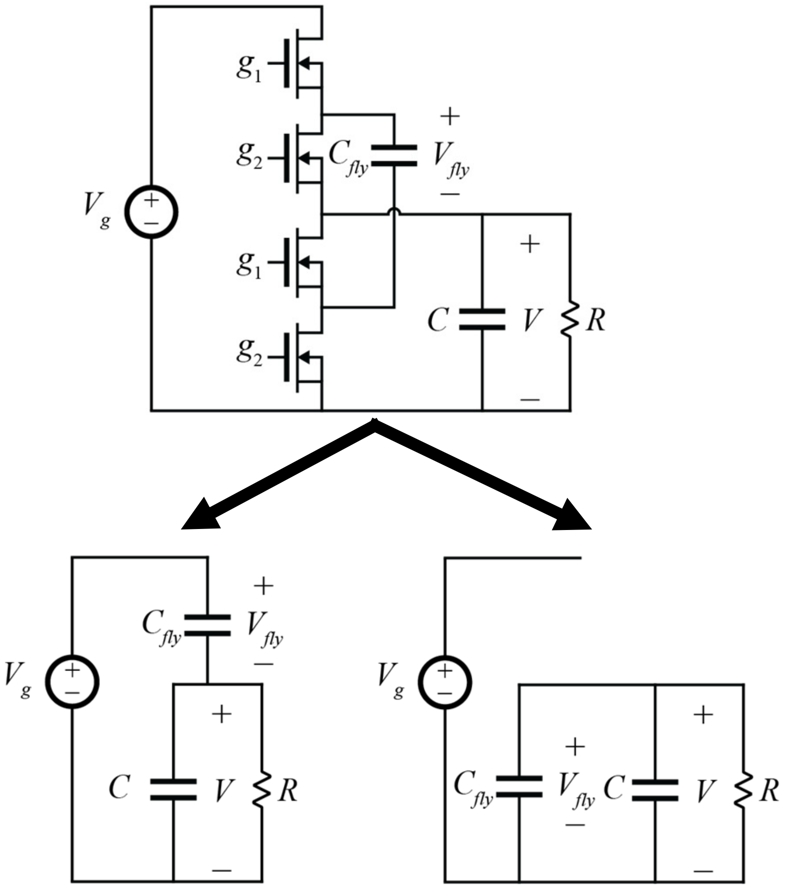
$$\frac{1}{I_{out}}(-V) = \sum_{\text{all caps}} a_{ci}^{I}\underbrace{(v_{ci}^{I} - v_{ci}^{II})}_{\Delta v_{ci} = \frac{q_{ci}}{C_i}}\left(\frac{1}{I_{out}}\right)$$

$I_{out} = q_{out} f_s$

$$\frac{-V}{I} = R_o = \sum_{\text{all caps}} a_{ci}^{I}\left(\frac{q_{ci}}{C_i}\right)\frac{1}{q_{out} f_s} = \boxed{\sum_{\text{all flying caps}} (a_{ci})^2 \frac{1}{C_i f_s} = R_o}$$

General for any 2-subinterval switched cap converter in SSL

Output Resistance

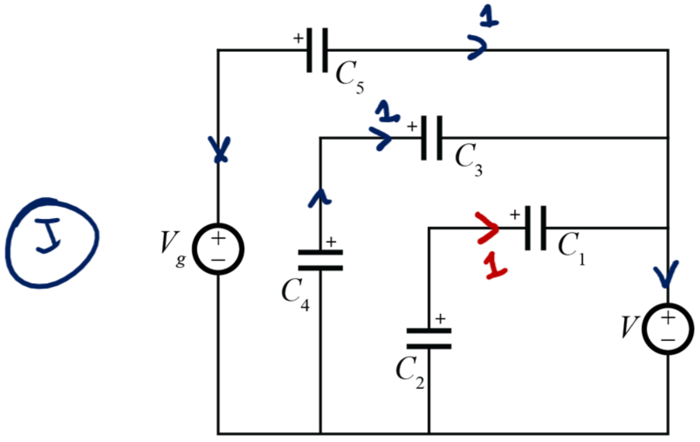


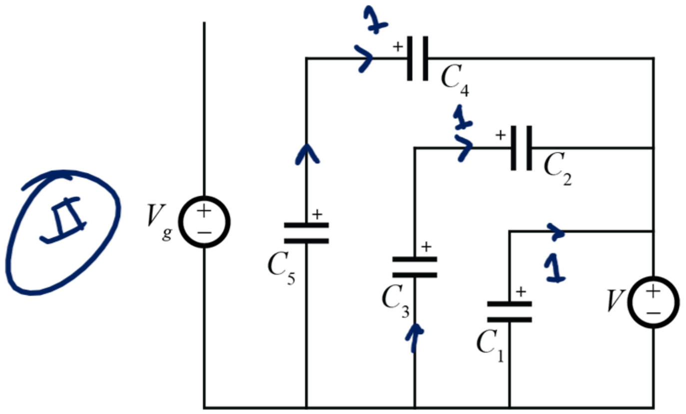
$$\bar{a}^{\mathrm{I}} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad \bar{a}^{\mathrm{II}} = \begin{bmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$R_o = \sum_{\text{all flying caps}} \left(\bar{a}_{c,i}\right)^2 \frac{1}{c_i f_s} = \frac{1}{4} \frac{1}{C_{fly} f_s} = \frac{1}{4 C_{fly} f_s} \quad \checkmark$$

$$\boxed{R_{o,SSL} = \frac{1}{4 C_{fly} f_s}}$$

Dickson Charge Vector Analysis





$$q_{out} = q_{out}^{I} + q_{out}^{II} = 6$$

$$\bar{a}^{I} = \frac{\bar{q}^{I}}{q_{out}} = \begin{array}{ccccccc} a_{in} & a_{c1} & a_{c2} & a_{c3} & a_{c4} & a_{c5} & a_{out} \\ \end{array}$$
$$\left[\begin{array}{ccccccc} -1 & 1 & -1 & 1 & -1 & 1 & 3 \end{array}\right] / q_{out}$$

$$\bar{a}^{I} = \left[\begin{array}{ccccccc} -\frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} \end{array}\right.$$

$$\bar{a}^{II} = \frac{\bar{q}^{II}}{q_{out}} = \left[\begin{array}{ccccccc} \phi & -1 & 1 & -1 & 1 & -1 & 3 \end{array}\right] / q_{out}$$

$$\bar{a}^{II} = \left[\begin{array}{ccccccc} \phi & \frac{-1}{6} & \frac{1}{6} & -\frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{2} \end{array}\right]$$

$$M = \frac{-a_{in}}{a_{out}} = \frac{1}{6} \checkmark \quad 6:1 \text{ Dickson}$$

Dickson SSL Output Resistance

$$R_{o,SSL} = \sum_{\substack{\text{flying}\\ \text{caps}}} (a_{ci})^2 \frac{1}{c_i f_s} = \sum_{i=1}^{5} \left(\frac{1}{6}\right)^2 \frac{1}{C_{fly} f_s} = \boxed{\frac{5}{36\, C_{fly} f_s} = R_{o,SSL}}$$

Assume all caps have $C_i = C_{fly}$, same capacitance

Charge Vector Analysis in FSL

$\bar{a}_r = \frac{\bar{q}_r}{q_{out}} \Rightarrow$ q_{ri} is the charge flowing through transistor i

Can be extended to any resistor in the circuit

by limiting to transistors, we're assuming r_{on} dominates

a_{ri} are generally a linear combination of a_{ci} from before

$R_{o,FSL}$ found by:

In FSL, currents are nearly constant over the subinterval

$i_{ri} = q_{ri} \cdot \frac{2}{T_s}$ → for a 2-subinterval converter @ $D = 50\%$

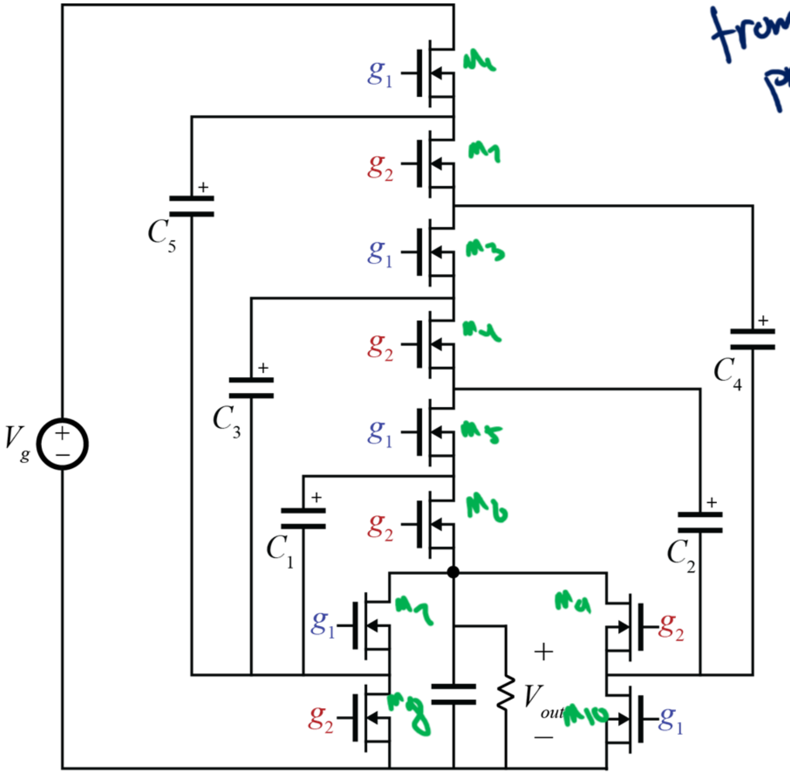
$$P_{ri} = (i_{ri})^2 R_i \frac{1}{2} = \left(q_{ri} \frac{2}{T_s}\right)^2 R_i \frac{1}{2}$$

$$P_{ri} = \left(a_{ri} \cdot q_{out} \frac{2}{T_s}\right)^2 R_i \frac{1}{2}$$

$$P_{ri} = a_{ri}^2 \, 4 \tfrac{1}{2} R_i \cdot q_{out}^2 f_s^2 = a_{ri}^2 \cdot 2 \cdot R_i I_{out}^2$$

$$P_{total} = \sum_{\text{all } R_i} (a_{ri})^2 \cdot 2 R_i I_{out}^2 \longrightarrow \boxed{R_{o,FSL} = \sum_{\text{all } R_i} 2 (a_{ri})^2 R_i}$$

Dickson Converter FSL Output Resistance



from previous

$$\left\{ \begin{array}{l} \bar{a}^{I} = \left[-\tfrac{1}{6} \;\; \tfrac{1}{6} \;\; -\tfrac{1}{6} \;\; \tfrac{1}{6} \;\; -\tfrac{1}{6} \;\; \tfrac{1}{6} \;\; \tfrac{1}{2} \right] \\ \bar{a}^{II} = \left[0 \;\; -\tfrac{1}{6} \;\; \tfrac{1}{6} \;\; -\tfrac{1}{6} \;\; \tfrac{1}{6} \;\; -\tfrac{1}{6} \;\; \tfrac{1}{2} \right] \end{array} \right.$$

$$\begin{array}{cccccccccc} M_1 & M_2 & M_3 & M_4 & M_5 & M_6 & M_7 & M_8 & M_9 & M_{10} \end{array}$$

$$\bar{a}_r = \left[\tfrac{1}{6} \;\; \tfrac{1}{6} \;\; \tfrac{1}{6} \;\; \tfrac{1}{6} \;\; \tfrac{1}{6} \;\; \tfrac{1}{6} \;\; \tfrac{1}{2} \;\; \tfrac{1}{2} \;\; \tfrac{1}{3} \;\; \tfrac{1}{3} \right]$$