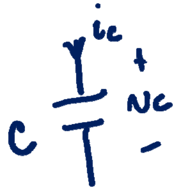
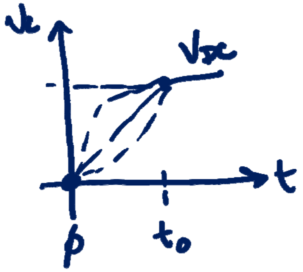
Modeling Nonlinear Capacitances





Charge

$$Q = \int_0^{t_0} i_c(t)\,dt$$

$$Q = \int_0^{t_0} C \frac{dv_c(t)}{dt}\,dt$$

$$Q = \int_\phi^{V_{DC}} C\,dv_c$$

$$Q = C \int_0^{V_{DC}} 1\,dv_c$$

$$Q = C V_{DC}$$

$$\boxed{Q = \int_0^{V_{DC}} C(v_c)\,dv_c}$$

Energy

$$E = \int_0^{t_0} i_c(t) \cdot v_c(t)\,dt$$

$$E = \int_0^{t_0} C \frac{dv_c(t)}{dt} \cdot v_c(t)\,dt$$

$$E = \int_0^{V_{DC}} C\, v_c\,dv_c$$

$$E = C\left[\frac{v_c^2}{2}\right]\Bigg|_0^{V_{DC}}$$

$$E = \frac{1}{2} C V_{DC}^2$$

Linear capacitance only

$$\boxed{E = \int_0^{V_{DC}} C(v_c) \cdot v_c\,dv_c}$$

Nonlinear $C(v_c)$

cannot simplify further without
(1) analytical expression for $C(v_c)$
(2) numerical result w/ datasheet Coss curve

D. Costinett, D. Maksimovic and R. Zane, "Circuit-Oriented Treatment of Nonlinear Capacitances in Switched-Mode Power Supplies," in *IEEE Transactions on Power Electronics*

Energy and Charge Equivalents

Linear "equivalent" capacitors only correct for a single characteristic

$$\begin{cases} Q = C_{eq,Q} V_{DC} \\ Q = \int_0^{V_{DC}} C(v_c)\, dv_c \end{cases}$$

$$\boxed{C_{eq,Q} = \frac{1}{V_{DC}} \int_0^{V_{DC}} C(v_c)\, dv_c}$$

Capacitor that has the same total charge at V_{DC} as our nonlinear cap

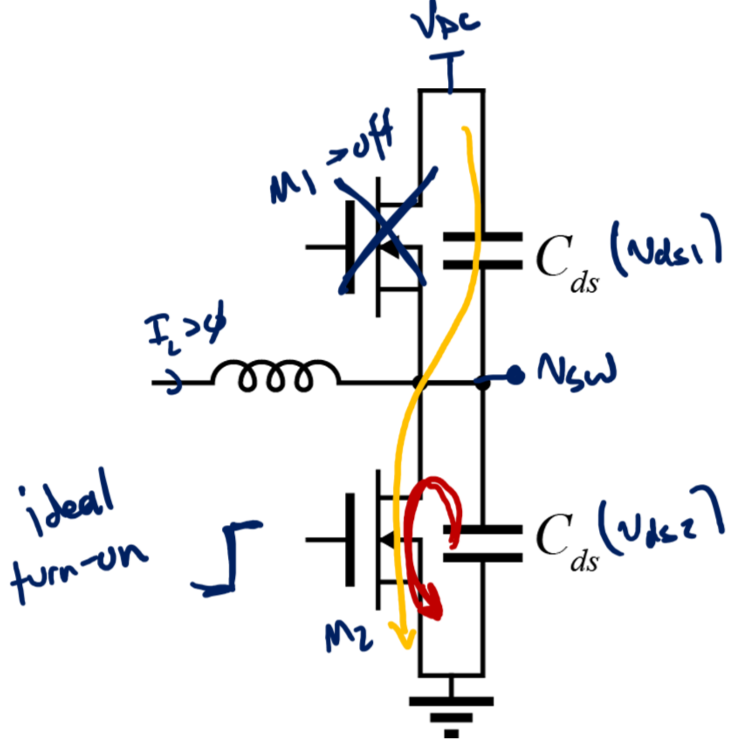
$$E = \frac{1}{2} C_{eq,E} V_{DC}^2 = \int_0^{V_{DC}} C(v_c) \cdot v_c \, dv_c$$

$$\boxed{C_{eq,E} = \frac{2}{V_{DC}^2} \int_0^{V_{DC}} C(v_c) \cdot v_c \, dv_c}$$

same energy @ V_{DC}

- $C_{eq,Q} = \langle C \rangle |_v$

C_{oss} Losses in a Half Bridge

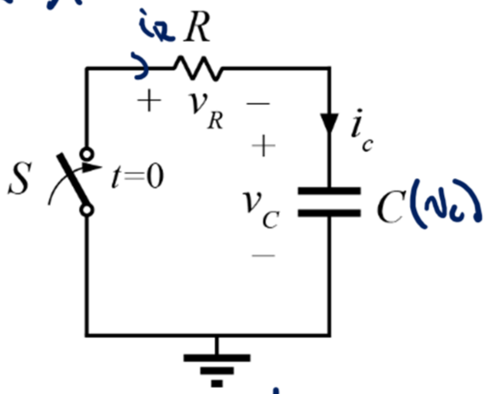


$v_{sw} = v_{ds2}$

$v_{ds1} = V_{DC} - v_{sw}$

before turn-on $v_{sw} = V_{DC}$

M_2 Energy Loss



i.c. $v_c(t=0) = V_{DC}$

$$E_R = \int_0^{\infty} v_R \cdot i_R \, dt$$

$$v_R = -v_c$$

$$i_R = i_c = C(v_c)\frac{dv_c}{dt}$$

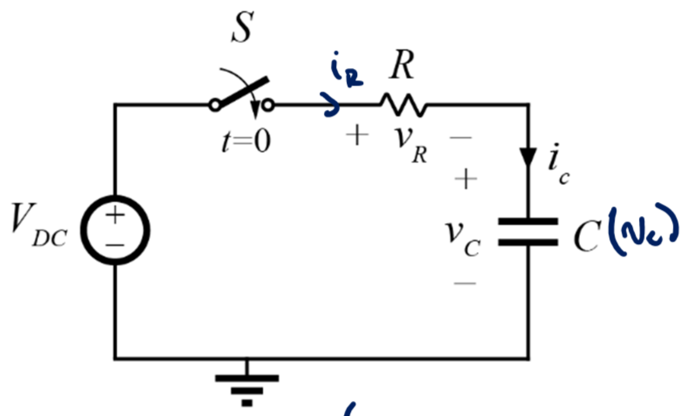
$$E_R = \int_0^{\infty} (-v_c)\, C(v_c)\, \frac{dv_c}{dt}\, dt$$

$$E_R = \int_{V_{DC}}^{0} (-v_c)\, C(v_c)\, dv_c$$

$$E_R = \int_0^{V_{DC}} C(v_c) \cdot v_c \, dv_c$$

$$\boxed{E_R = \frac{1}{2} C_{eq,E} V_{DC}^2}$$

M_1 Energy Loss



i.c. $v_c(t=0) = \phi$

$$v_R = V_{DC} - v_c$$
$$i_R = i_c = C(v_c)\frac{dv_c}{dt}$$

$$E_R = \int_0^{\infty} v_R \cdot i_R \, dt$$

$$E_R = \int_0^{\infty} (V_{DC} - v_c) \cdot C(v_c) \frac{dv_c}{dt} \, dt$$

$$E_R = \int_0^{V_{DC}} V_{DC} \cdot C(v_c) \, dv_c - \int_0^{V_{DC}} C(v_c) \cdot v_c \, dv_c$$

$$E_R = V_{DC} \cdot \underbrace{\int_0^{V_{DC}} C(v_c) \, dv_c}_{Q} - \underbrace{\int_0^{V_{DC}} C(v_c) \cdot v_c \, dv_c}_{E}$$

$$\boxed{E_R = V_{DC} \cdot \left(C_{eq,Q} \cdot V_{DC} \right) - \frac{1}{2} C_{eq,E} V_{DC}^{\,2}}$$

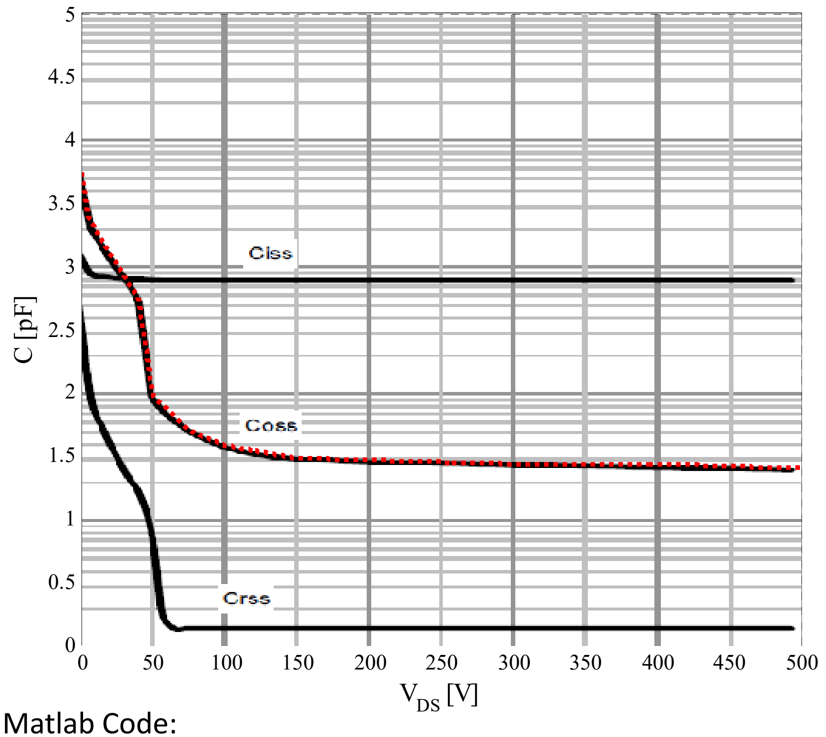
Total Half Bridge C_{oss} Loss

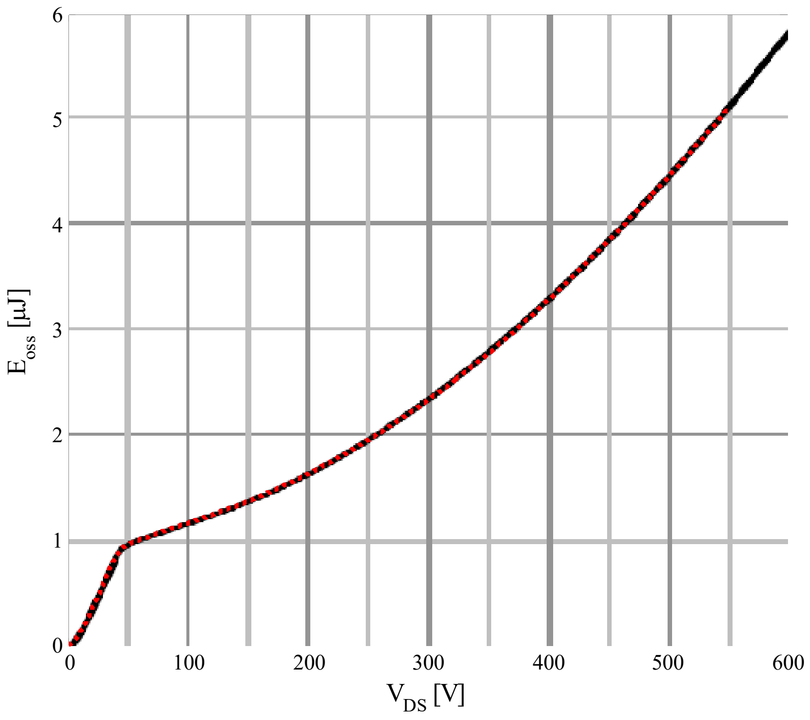
$$\boxed{E_{M_2} = \frac{1}{2} C_{eq,E,M_2} V_{DC}^2 + V_{DC}^2 C_{eq,Q,M_1} - \frac{1}{2} C_{eq,E,M_1} V_{DC}^2}$$

If M_1 & M_2 are the same device $C_{eq,E,M_2} = C_{eq,E,M_1}$

$\underline{E_{M_2} = V_{DC}^2 C_{eq,Q}}$ → Energy loss depends only on charge-equivalent capacitor

Energy Equivalent





<u>Matlab Code:</u>

```
Vdc = 550;

Vds = [0    5    10   40  50 75 100 150 200 300 400 500 600];
Coss = [5500 2500 1900 550 95 50 38  30  29  27  27  25  24]*1e-12;

vx = 0.01:.01:Vdc;
Cx = 10.^interp1(Vds,log10(Coss),vx,'linear');

E = cumtrapz(vx, Cx.*vx);
Ceq_e = 2*(E)./vx.^2;
```

Nonlinear Capacitance Extraction

- http://web.eecs.utk.edu/~dcostine/personal/PowerDeviceLib/DigiTest/index.html

Datasheet Reported Capacitance

Dynamic characteristics

Input capacitance	C_{iss}	V_{GS}=0 V, V_{DS}=100 V, f=1 MHz	-	790	-	pF
Output capacitance	C_{oss}		-	38	-	
Effective output capacitance, energy related[6]	$C_{o(er)}$	V_{GS}=0 V, V_{DS}=0 V to 480 V	-	36	-	
Effective output capacitance, time related[7]	$C_{o(tr)}$		-	96	-	
Turn-on delay time	$t_{d(on)}$	V_{DD}=400 V, V_{GS}=10 V, I_D=5.2 A, R_G=3.3 Ω	-	10	-	ns
Rise time	t_r		-	5	-	
Turn-off delay time	$t_{d(off)}$		-	40	-	
Fall time	t_f		-	5	-	

[6] $C_{o(er)}$ is a fixed capacitance that gives the same stored energy as C_{oss} while V_{DS} is rising from 0 to 80% V_{DSS}.

[7] $C_{o(tr)}$ is a fixed capacitance that gives the same charging time as C_{oss} while V_{DS} is rising from 0 to 80% V_{DSS}.