State Estimation

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Operating States of a Power System

Power systems operate in one of three operating states:

Normal state:

Loads = Generation - Losses

Operational constraints are NOT violated.

- Secure normal: No Action
- Insecure normal: Preventive control action (SCOPF)

Emergency state:

Operating constraints are violated

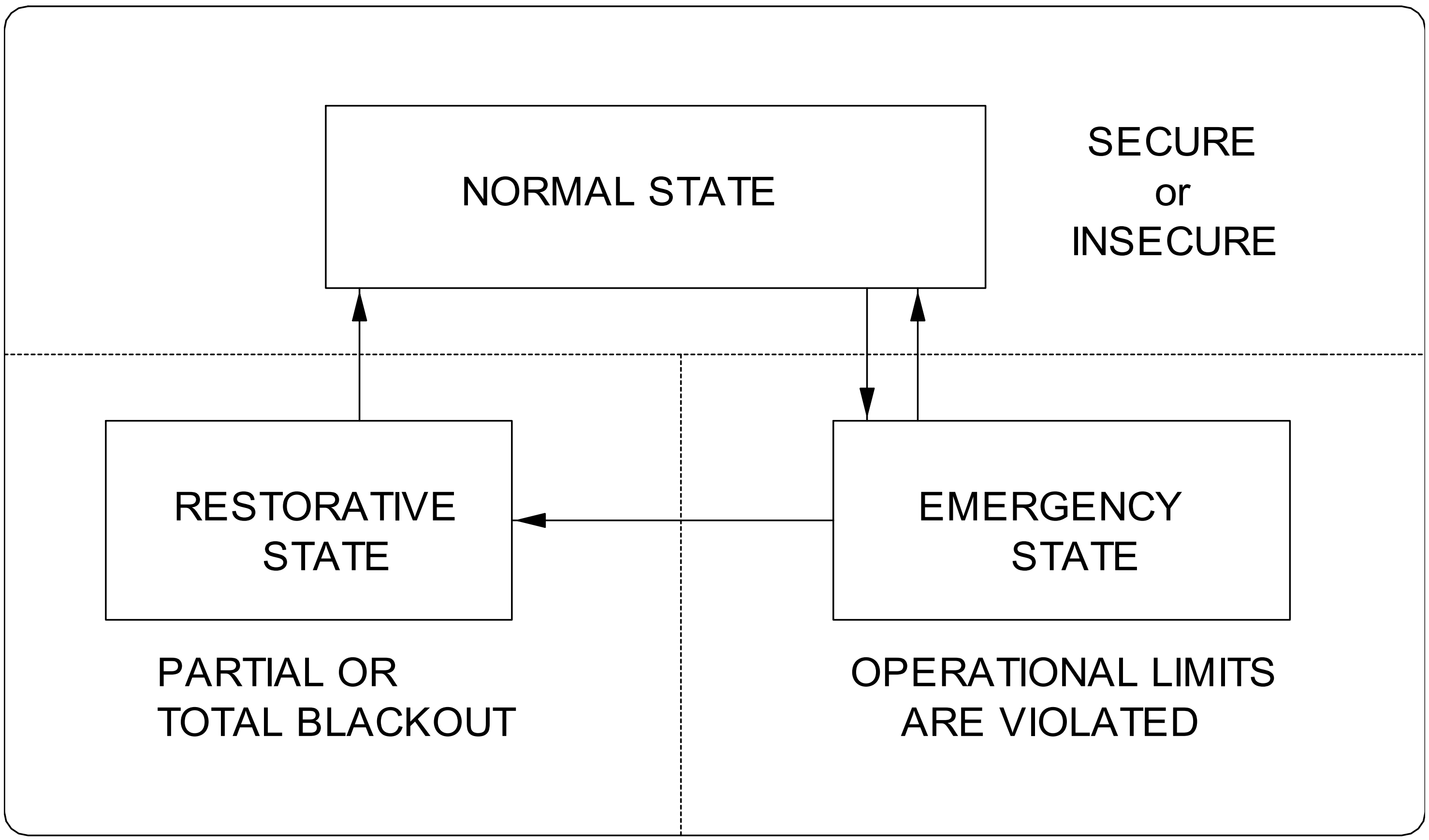
Requires immediate corrective action.

Restorative state:

Load versus generation balance is to be restored

Requires restorative control actions.

Operating States of a Power System



Classical Role of State Estimation

Facilitating Static Security Analysis

Security Analysis:

Monitoring the system, identifying its operating state, determining necessary preventive actions to make it secure.

Monitoring involves RTU's to measure and telemeter various quantities and a state estimator

Measured quantities:

Flows: line power flows
Phasor Magnitude: bus voltage and line current magnitudes
Phasor Angle: phase angle for bus voltage and line current
Injections: generator outputs and loads
Status: circuit breaker and switch status information, transformer tap positions

State Estimation Functions

Topology processor:
Creates one-line diagram of the system using the detailed circuit breaker status information.

Observability analysis:
Checks to make sure that state estimation can be performed with the available set of measurements.

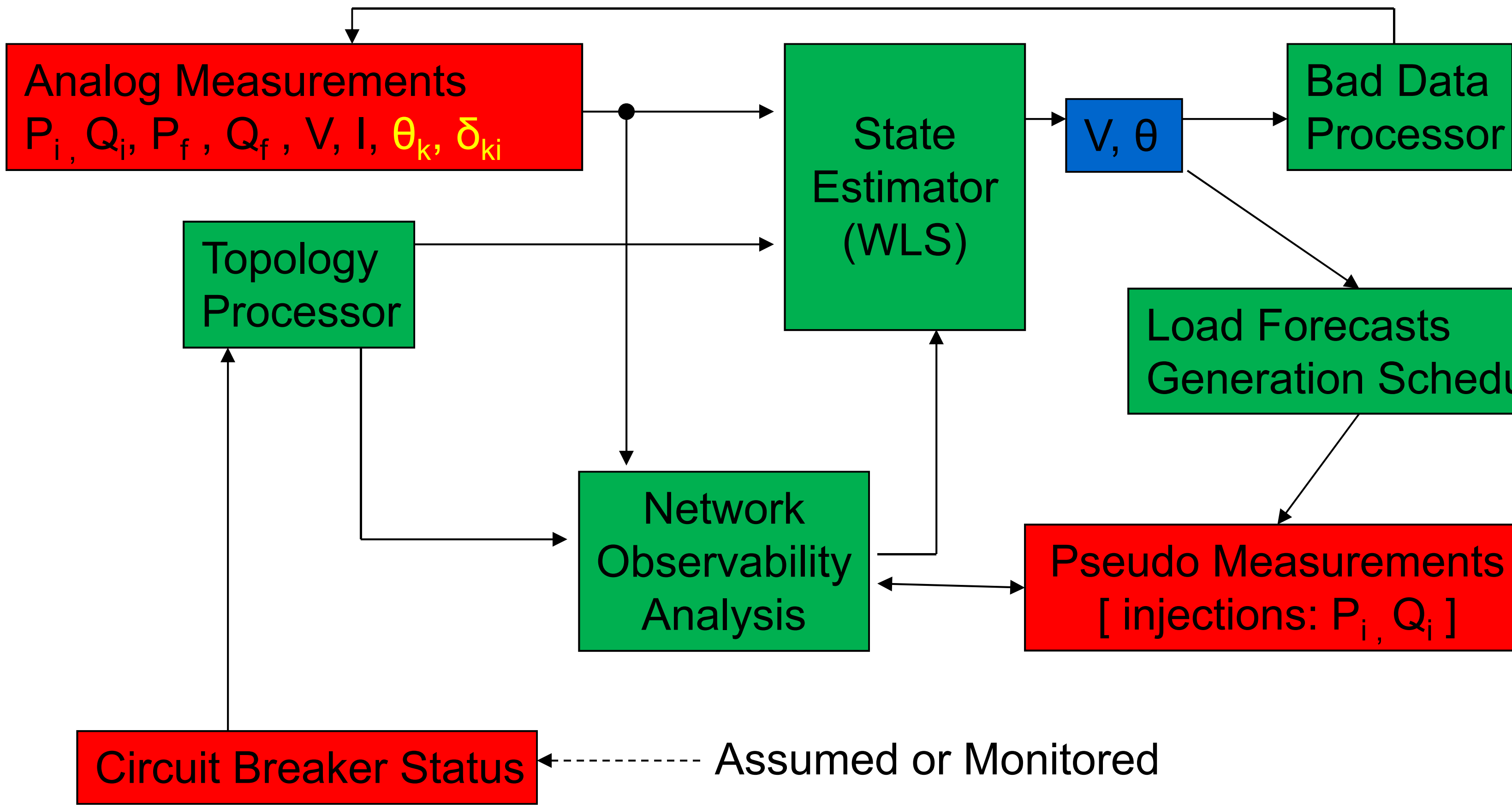
State estimation:
Estimates the system state based on the available measurements.

Bad data processing:
Checks for bad measurements. If detected, identifies and eliminates bad data.

Parameter and structural error processing:
Estimates unknown network parameters, checks for errors in circuit breaker status.

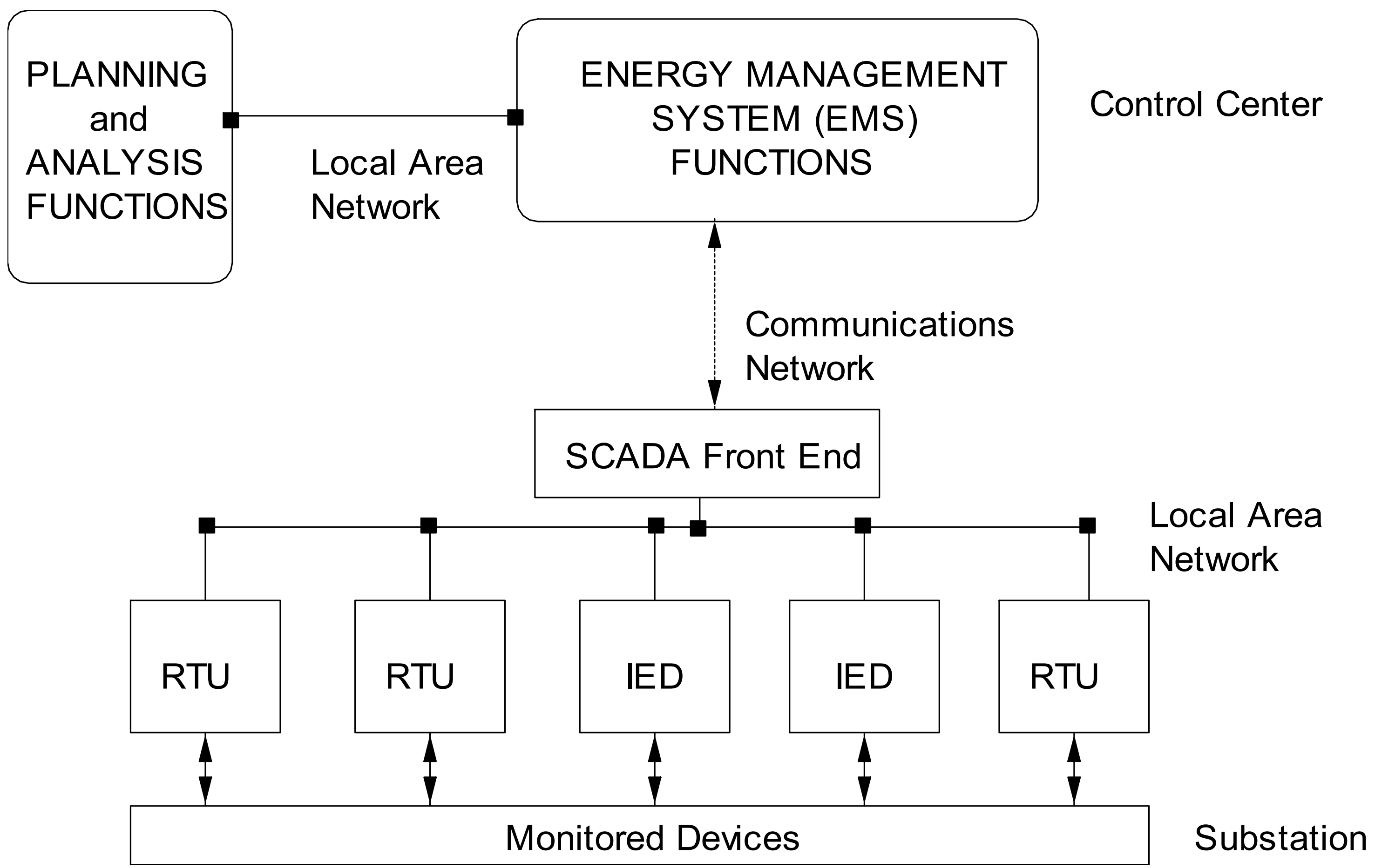
State Estimation and Related Functions

Weighted Least Squares (WLS) Estimator



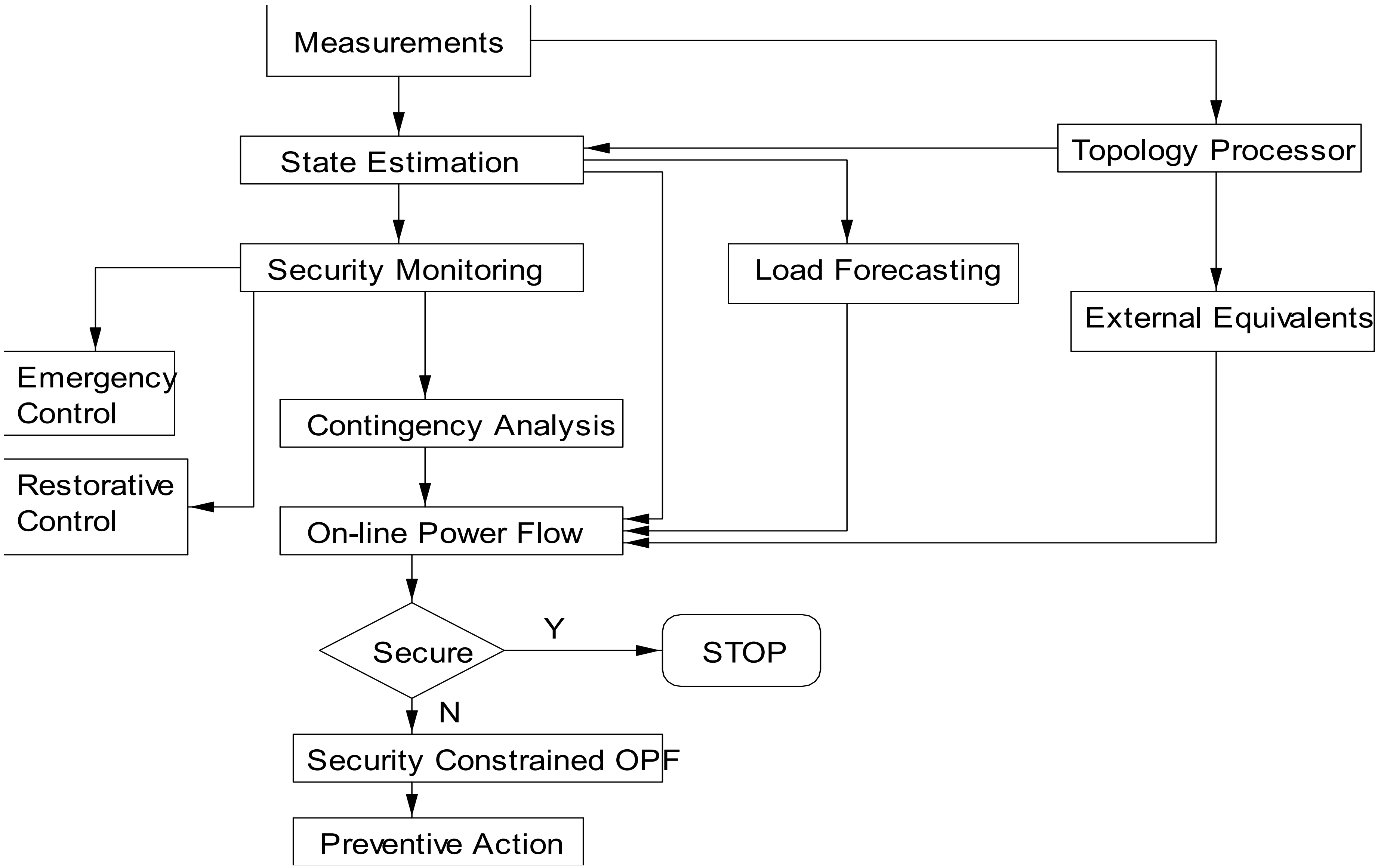
Communication Infrastructure

SCADA / EMS Configuration



Energy Management System Applications

SCADA / EMS Configuration

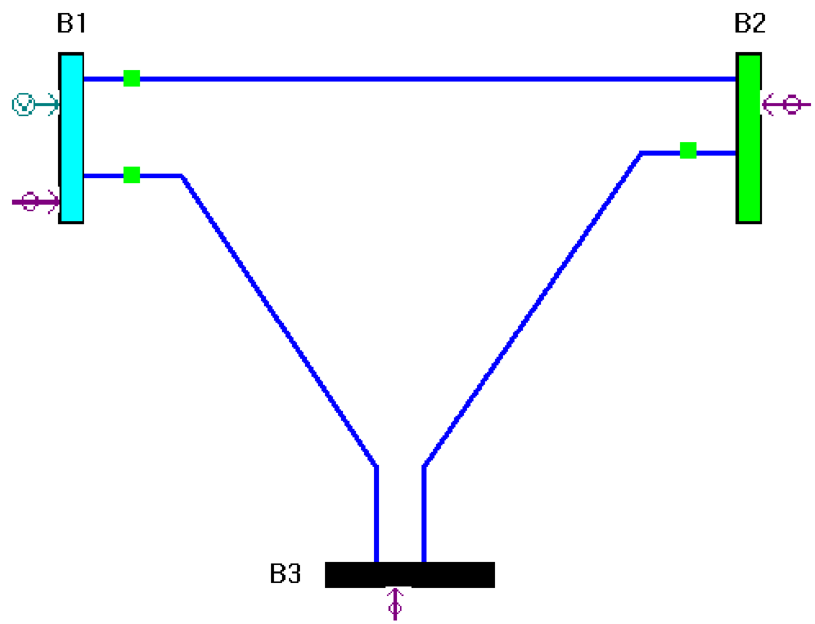


Power System State Estimation

Problem Statement

- [z] : Measurements
  P-Q injections
  P-Q flows
  V magnitude, I magnitude

- [x] : States
  V, θ, Taps (parameters)



- **EXAMPLE:**

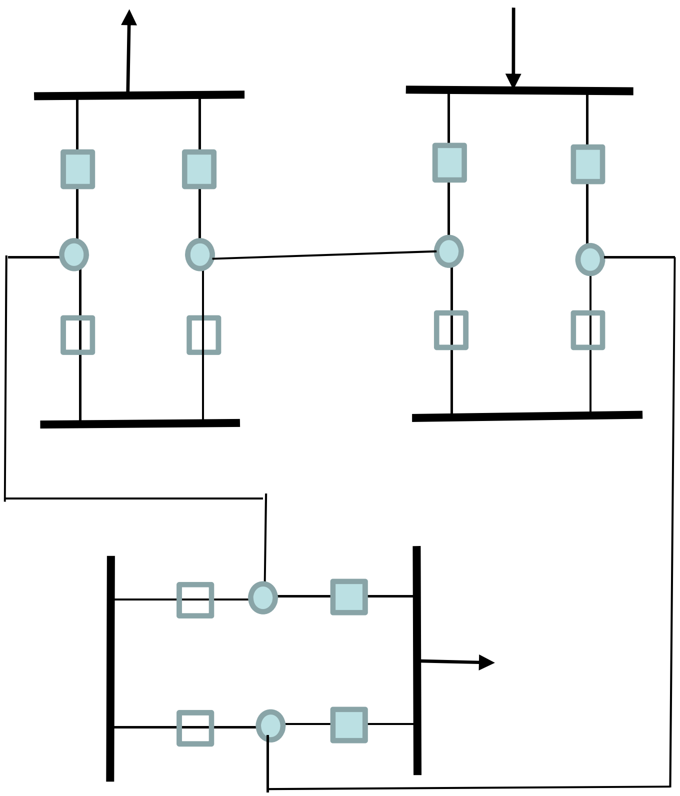
- [z] = [P12; P13; P23; P1; P2; P3; V1; Q12; Q13; Q23; Q1; Q2; Q3]
  m = 13 (no. of measurements)

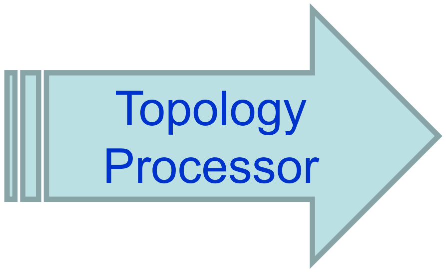
- [x] = [V1; V2; V3; θ2; θ3]
  n = 5 (no. of states)

Network Model

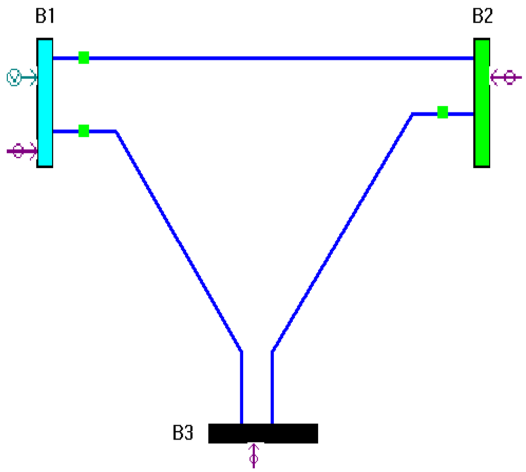
Bus/branch and bus/breaker Models

Bus/Breaker





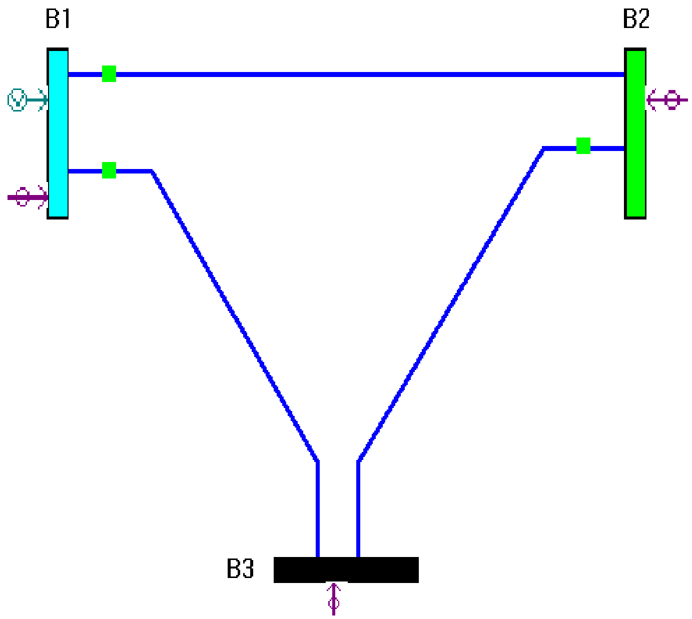
Bus/Branch



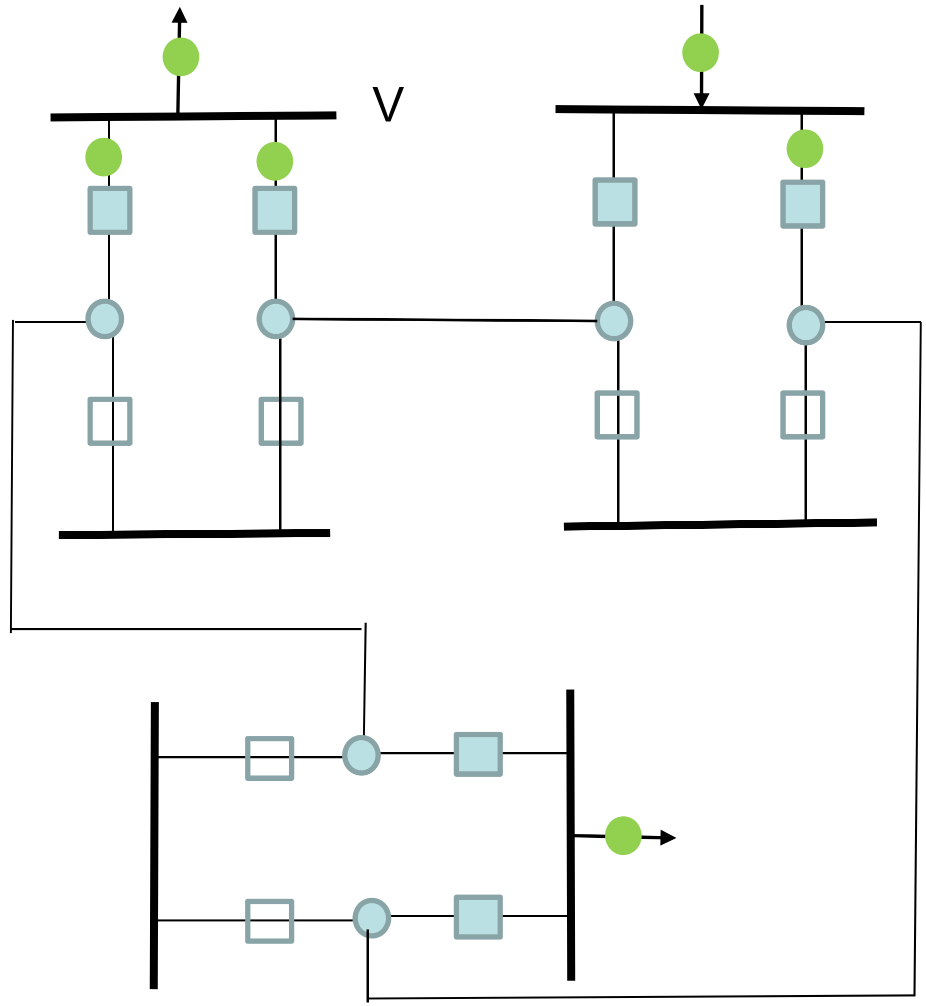
Measurements

Bus/branch and bus/breaker Models

Bus/branch

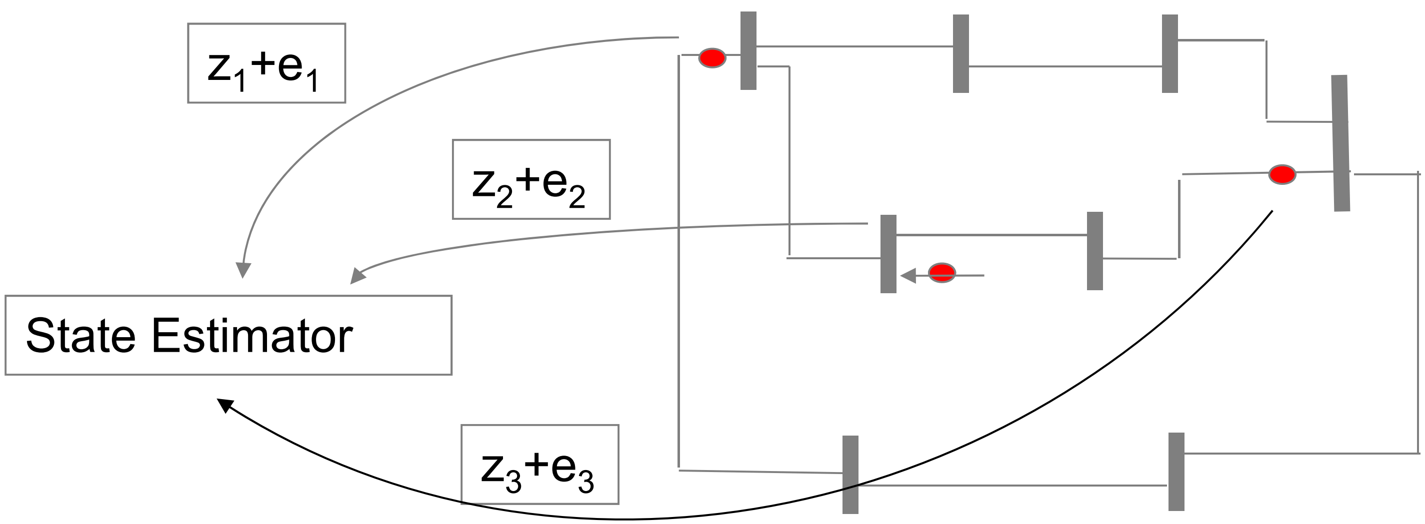


Bus/Breaker



Measurement Model

$[z_m] = [h([x])] + [e]$



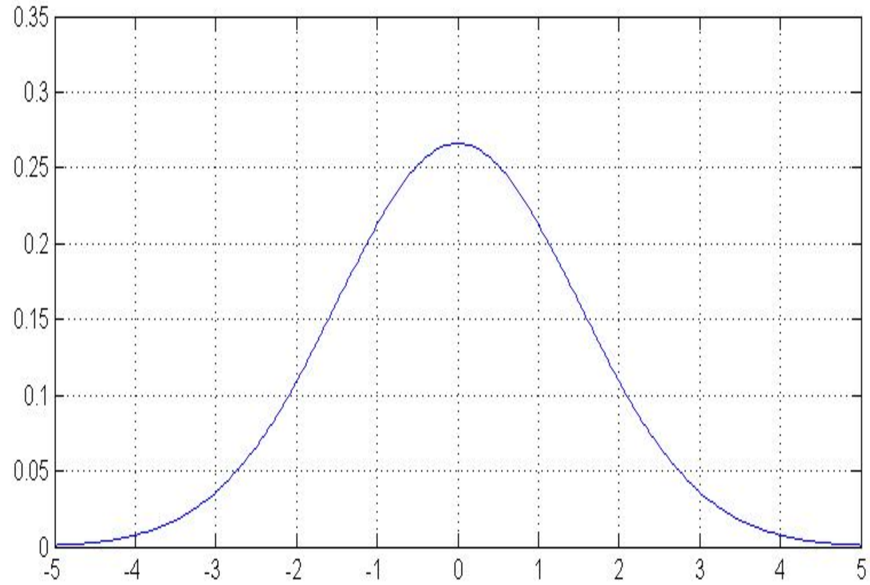
z_i : true measurement

e_i : measurement error

$e_i = e_s + e_r$

systematic   random

Measurement Model



Assumptions

- $e_i \sim N(0, \sigma_i^2)$
- Holds true if:

$$e_s = 0, \; e_r \sim N(0, \sigma_i^2)$$

- If $e_s \neq 0$, then $E(e_i) \neq 0$,

i.e. SE will be biased !

Maximum Likelihood Estimator (MLE)

Likelihood Function

Consider the random variables $X_1, X_2, \ldots, X_n$ with a p.d.f of $f(X \mid \theta)$, where θ is unknown.

The joint p.d.f of a set of random observations

$$x = \{ x_1, x_2, \ldots , x_n \}$$

will be expressed as:

$$f_n(x \mid \theta) = f (x_1 \mid \theta)\, f(x_2 \mid \theta) \ldots f(x_n \mid \theta)$$

This joint p.d.f is referred to as the ***Likelihood Function***.

The value of θ, which will maximize the function $fn(x \mid \theta)$ will be called the ***Maximum Likelihood Estimator (MLE)*** of θ.

Maximum Likelihood Estimator (MLE)

Maximum Likelihood Estimator

Normal (Gaussian) Density Function, f(z)

$$f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\tfrac{1}{2}(\tfrac{z-\mu}{\sigma})^2\}$$

Likelihood Function, $f_m(z)$

$$f_m(z) = f_m(z_1) f_m(z_2) \cdots f_m(z_m)$$

Log-Likelihood Function, *L*

$$L = \log f_m(z) = \sum_{i=1}^{m} \log f(z_i)$$

$$= -\tfrac{1}{2} \sum_{i=1}^{m} (\tfrac{z_i - \mu_i}{\sigma_i})^2 - \tfrac{m}{2} \log 2\pi - \sum_{i=1}^{m} \log \sigma_i$$

Maximum Likelihood Estimator (MLE)

Weighted Least Squares (WLS) Estimator

Given the set of observations $z_1, z_2, \ldots, z_n$ MLE will be the solution to the following:

$$\text{Maximize } f_m(z)$$

OR

$$\text{Minimize } \sum_{i=1}^{m} \left(\frac{z_i - \mu_i}{\sigma_i} \right)^2$$

Defining a new variable "r", measurement residual:

$$\text{Minimize} \sum_{i=1}^{m} W_{ii} r_i^{\ 2} \qquad W_{ii} = \frac{1}{\sigma_i^2}$$

$$\text{Subject to } z_i = h_i(x) + r_i \quad i = 1,..,m \qquad \mu_i = E(z_i) = h_i(x)$$

The solution of the above optimization problem is called the **weighted least squares (WLS)** estimator for **x**.

Maximum Likelihood Estimator (MLE)

Weighted Least Squares (WLS) Estimator

Linear case:

$$\text{Minimize} \sum_{i=1}^{m} W_{ii} r_i^{\,2}$$

$$\text{Subject to} [z] = [H]\cdot[x] + [r]$$

Solution is given by:

$$[\hat{x}] = [G^{-1}]\cdot[H^T]\cdot[W]\cdot[z]$$

$$[G] = [H^T]\cdot[W]\cdot[H]$$

$$W_{ii} = \frac{1}{\sigma_i^2} \qquad W = diag\{W_{ii}\}$$

Measurement Model

Given a set of measurements, [z]

and the correct network topology/parameters:

$$[z] = [h([x])] + [e]$$

Measurements:

Known !
They are measured
Contain errors

True System States:

Unknown !
Can be measured
or estimated

Measurement Errors:

Unknown !
Can not be directly measured
or computed

Measurement Model

Following the state estimation, the estimated state will be denoted by $[\hat{x}]$:

$$[z] = [h([\hat{x}])] + [r]$$

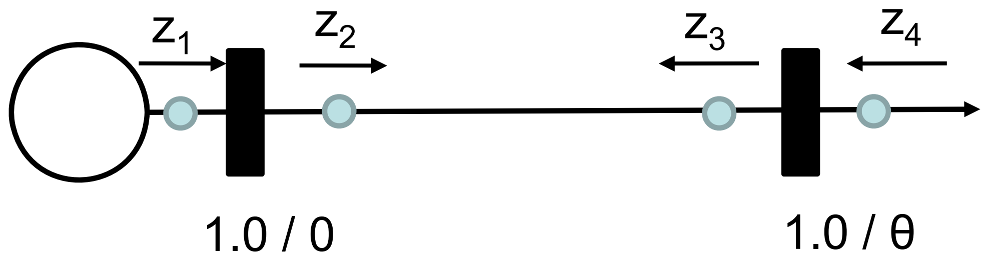
Measurements: They are measured Contain errors

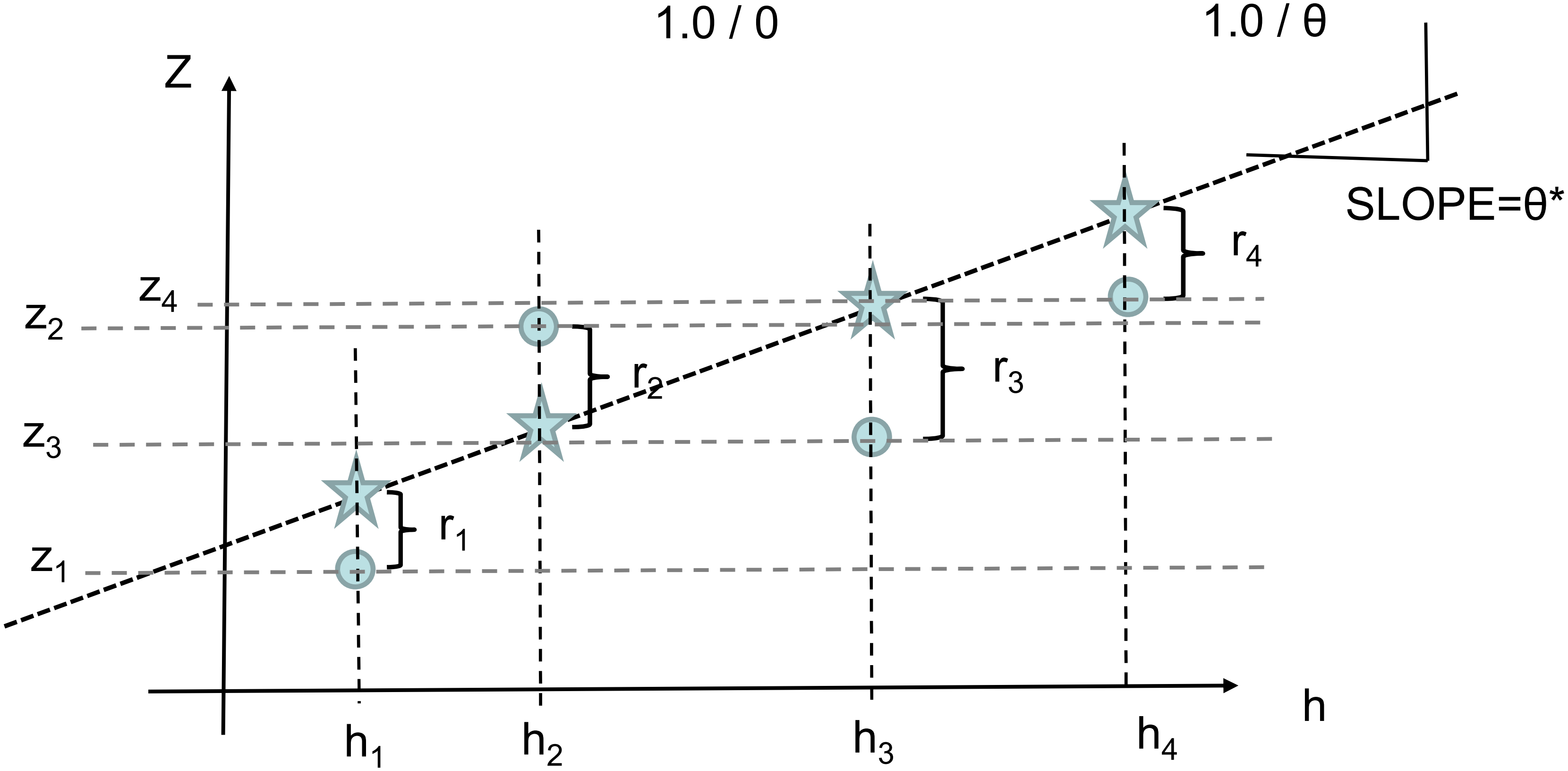
Estimated System States

Measurement Residuals: Computed

Simple Example

$Z = h\,\theta + e$





☆ : ESTIMATED MEASUREMENT ○ : MEASURED VALUE

r_i : MEASUREMENT RESIDUAL = $Z - h\,\theta^*$

Weighted Least Squares (WLS) Estimation

$$\textbf{\textit{Minimize}} \quad \omega_1 r_1^2 + \omega_2 r_2^2 + \omega_3 r_3^2 + \omega_4 r_4^2$$

What are weights, w_i ?

$$\omega_i = \frac{1.0}{\sigma_i^2}$$

How are they chosen ?

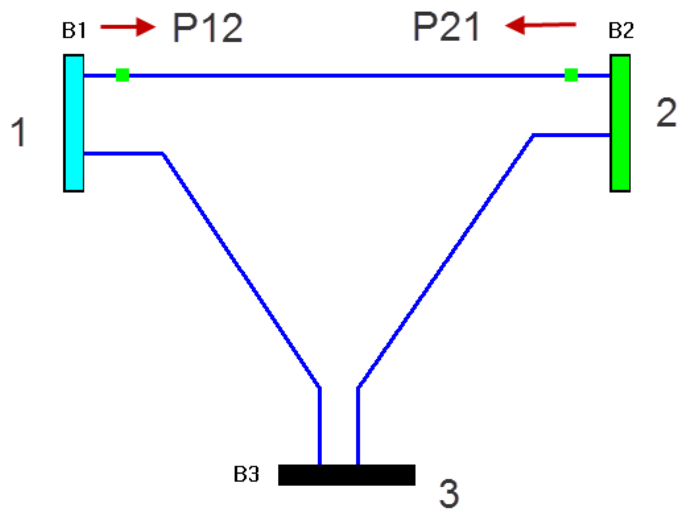
σ_i^2 Assumed error variance of measurement "*i*".

Fully observable network:

A power system is said to be ***fully observable*** if voltage phasors at all system buses can be uniquely estimated using the available measurements.

Network Observability

Necessary and Sufficient Conditions



$$Z_p = H \cdot \theta$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ H_{21} & \cdots & H_{2n} \\ H_{31} & \cdots & \vdots \\ \vdots & \cdots & \vdots \\ H_{m1} & \cdots & H_{mn} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$$

$$\textbf{State Vector} = [\theta_2 \quad \theta_3]$$

$$\hat{\theta} = [G^{-1}] \cdot [H^T] \cdot [W] \cdot [Z_P]$$

Singular Matrix !
Cannot be inverted.

m ≥ n ➔ NECESSARY BUT “NOT” SUFFICIENT

EXAMPLE: m = 2, n = 2, UNOBSERVABLE SYSTEM

Rank(H) = n → SUFFICIENT

Measurement Classification

Types of Measurements

1. CRITICAL MEASUREMENTS

   WHEN REMOVED, THE SYSTEM BECOMES UNOBSERVABLE

2. *REDUNDANT MEASUREMENTS*

   CAN BE REMOVED WITHOUT AFFECTING NETWORK OBSERVABILITY

Types of Measurements

Critical Measurements

CRITICAL MEASUREMENTS

- *If they have gross errors, such errors can not be detected*
- *Measurement residuals will always be equal to zero, i.e. critical measurements will be perfectly satisfied by the estimated state*
- *If they are lost or temporarily unavailable, the system will no longer be observable, thus state estimation can not be executed*

Unobservable branch:

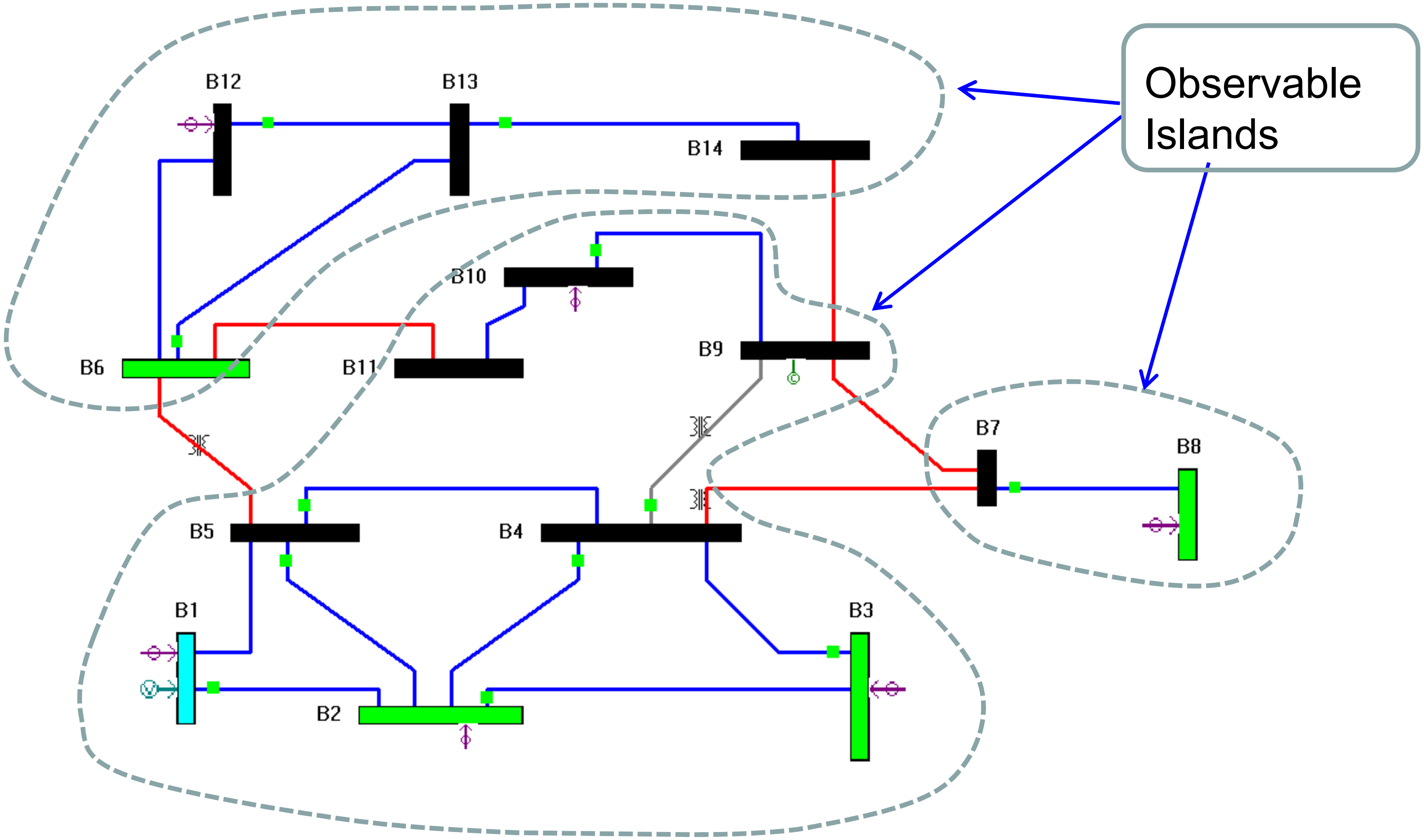
- If the system is found ***not*** to be observable, it will imply that there are ***unobservable*** branches whose power flows can not be determined.

Observable island:

- ***Unobservable*** branches connect ***observable*** islands of an ***unobservable*** system. State of each observable island can be estimated using any one of the buses in that island as the reference bus.

Network Observability

Definitions



RED LINES: Unobservable Branches

Merging Observable Islands

Pseudo-measurements

If the system is found unobservable, use pseudo-measurements in order to merge observable islands.

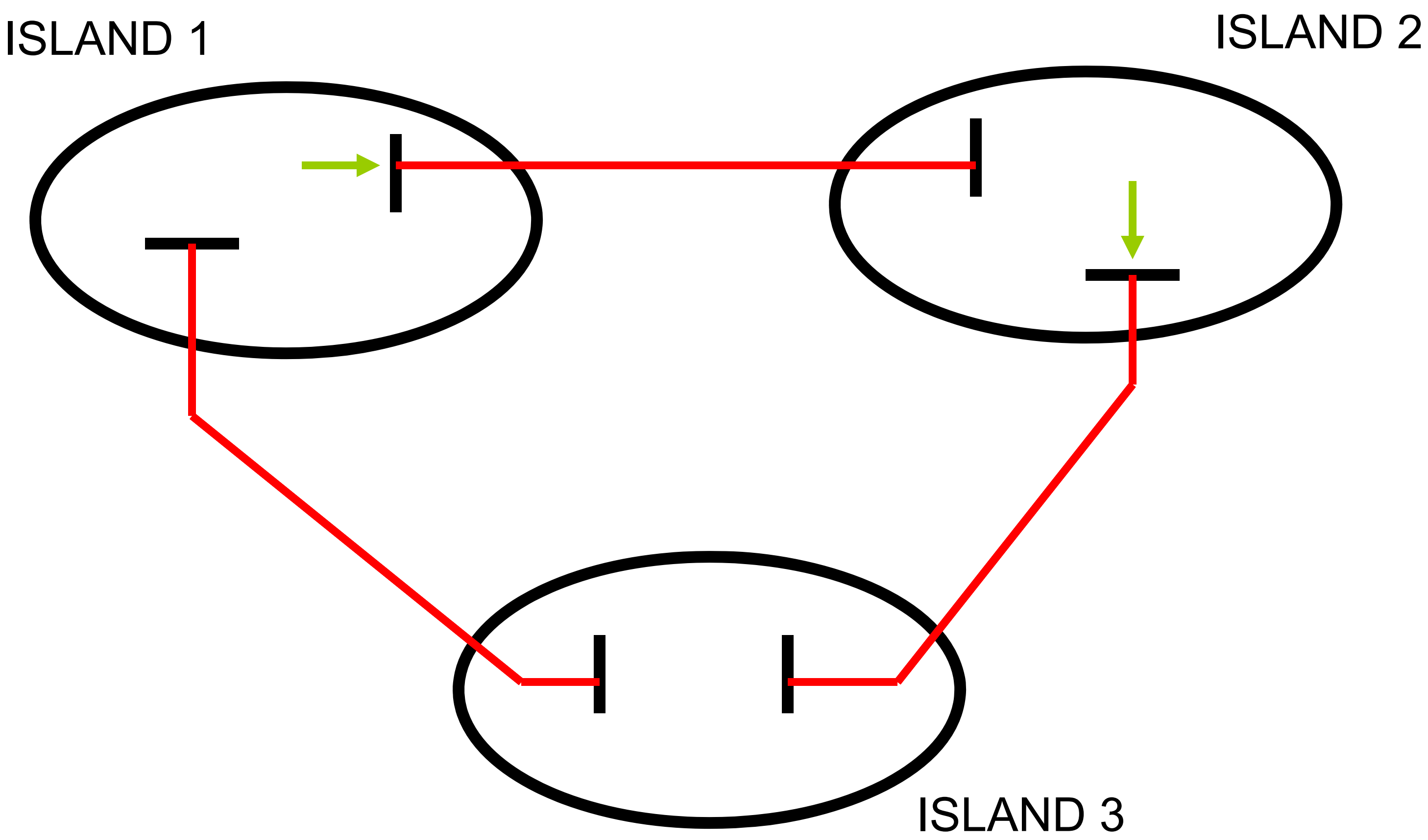
Pseudo-measurements:
- Forecasted bus loads
- Scheduled generation

Select pseudo-measurements such that they are critical.

Errors in critical measurements do not propagate to the residuals of the other (redundant) measurements.

Observable Islands

Unobservable Branches



Robust (resilient) Estimation

If an estimator remains insensitive to a finite number of errors in the measurements, then it is considered to be ***robust***.

Example: Given z = { 0.9, 0.95, 1.05, 1.07, 1.09 }, estimate z using the following estimators:

1. $\hat{X}_a = mean\{z_i\} = \frac{1}{5}\sum_{i=1}^{5} z_i$

2. $\hat{X}_b = median\{z_i\}, \quad i = 1,\dots,5$

Solution:

Replace z_5=1.09 by an infinitely large number $z'_5 = \infty$.

The new estimate will then be: $\hat{X}'_a = \frac{1}{5}\sum_{i=1}^{5} z_i = \infty$

This estimator is NOT robust.

Replace both z_5 and z_4 by infinity.

The new estimate will then be: $\hat{X}'_b = 1.05 \quad \text{(finite)}$

This is a more robust estimator than the one above.

Robust Estimation

M-Estimators

M-Estimators (Huber 1964)

Consider the problem:

$$\text{Minimize } \sum_{i=1}^{m} \rho(r_i)$$

$$\text{Subject to } z = h(x) + r$$

Where $\rho(r_i)$ is a chosen function of the measurement residual

In the special case of the WLS state estimation:

$$\rho(r_i) = \frac{r_i^2}{\sigma_i^2}$$

Robust Estimation

M-Estimators

Some Examples of M-Estimators

Quadratic-Constant

$$\rho(r_i) = \begin{cases} \dfrac{r_i^2}{\sigma_i^2} & \left|\dfrac{r_i}{\sigma_i}\right| \le a \\ \dfrac{a_i^2}{\sigma_i^2} & \text{otherwise} \end{cases}$$

Quadratic-Linear

$$\rho(r_i) = \begin{cases} \dfrac{r_i^2}{\sigma_i^2} & \left|\dfrac{r_i}{\sigma_i}\right| \le a \\ 2a\sigma_i \, |r_i| - a^2\sigma_i^2 & \text{otherwise} \end{cases}$$

Least Absolute Value (LAV)

$$\rho(r_i) = |r_i|$$

Robust Estimation

LAV Estimator Example

Measurement Model: $z_i = A_{i1}x_1 + A_{i2}x_2 + e_i \quad i = 1,\ldots,5$

Measurements:

i	Z_i	A_{i1}	A_{i2}
1	-3.01	1.0	1.5
2	3.52	0.5	-0.5
3	-5.49	-1.5	0.25
4	4.03	0.0	-1.0
5	5.01	1.0	-0.5

LAV estimate for x
and measurement residuals:

$$x^T = [3.005; -4.010]$$

$$r^T = [\mathbf{0.0};\ 0.0125;\ 0.02;\ 0.02;\ \mathbf{0.0}]$$

CHANGE measurement 5 from 5.01 to 15.01 (Simulated Bad Datum):

LAV estimate for x
and measurement residuals:

$$x^T = [3.02; -4.02]$$

$$r^T = [\mathbf{0.0};\ \mathbf{0.0};\ 0.045;\ 0.01;\ 9.98\ ]$$

Robust Estimation

LAV Estimator Example

Measurement Model: $z_i = A_{i1} x_1 + A_{i2} x_2 + e_i \quad i = 1,\ldots,5$

Measurements:

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LAV estimate for x
and measurement residuals:

$$x^T = [3.02; -4.02]$$

$$r^T = [\mathbf{0.0};\ \mathbf{0.0};\ 0.045;\ 0.01;\ 9.98\ ]$$

Bad Data Detection

Chi-squares χ^2 Test

Consider X_1, X_2, … X_N, a set of N independent random variables where:

$$X_i \sim N(0,1)$$

Then, a new random variable Y will have a χ^2 distribution with N degrees of freedom, i.e.:

$$\sum_{i=1}^{N} X_i^2 = Y \sim \chi_N^2$$

Bad Data Detection

Now, consider the function

$$f(x) = \sum_{i=1}^{m} R_{ii}^{-1} e_i^2 = \sum_{i=1}^{m} \left(\frac{e_i^2}{R_{ii}} \right) = \sum_{i=1}^{m} \left(e_i^N \right)^2$$

and assuming:

$$e_i^N \sim N(0,1)$$

f(x) will have a χ^2 distribution with at most **(m-n)** degrees of freedom.

In a power system, since at least **n** measurements will have to satisfy the power balance equations, at most **(m-n)** of the measurement errors will be linearly independent.

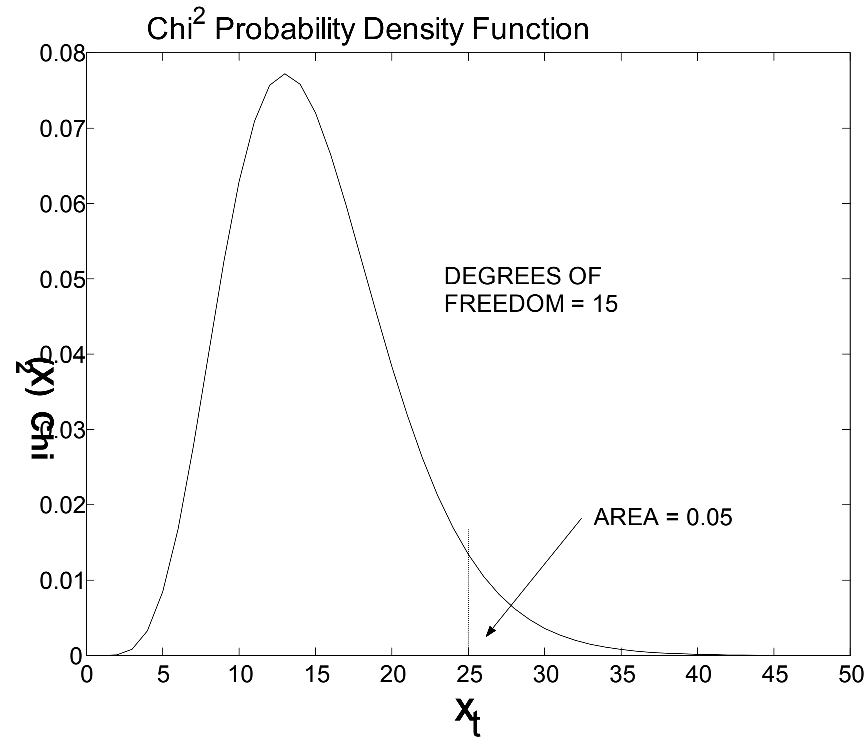
Bad Data Detection

Chi-squares Distribution:

$$\Pr\{X \geq x_t\} = \int_{x_t}^{\infty} \chi^2(u) \cdot du$$

Choose** x_t **such that:

$$\Pr\{X \geq x_t\} = \alpha = 0.05$$



Test:

If the measured** $X \geq x_t$ **, then with 0.95 probability, bad data will be suspected.

Bad Data Detection

Detection Algorithm χ^2 *--Test*

Solve the WLS estimation problem and compute the objective function:

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{\sigma_i^2}$$

Look up the value corresponding to ***p*** (e.g. 95 %) probability and ***(m-n)*** degrees of freedom, from the Chi-squares distribution table.

Let this value be $\chi^2_{(m-n),p}$ Here: $p = \Pr\{ J(x) \le \chi^2_{(m-n),p} \}$

Test if

$$J(x) \ge \chi^2_{(m-n),p}$$

If yes, then bad data are detected.

Else, the measurements are not suspected to contain bad data.

Bad Data Identification

Properties of Measurement Residuals

Linear measurement model: $\Delta\hat{\boldsymbol{x}} = (\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}\Delta\mathbf{z}$

$$\Delta\hat{\boldsymbol{z}} = \boldsymbol{H}\Delta\hat{\boldsymbol{x}} = \boldsymbol{K}\Delta\mathbf{z}, \qquad \boldsymbol{K} = \boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}$$

***K** is called the hat matrix. Now, the measurement residuals can be expressed as follows:*

$$\begin{aligned}\boldsymbol{r} &= \Delta\mathbf{z} - \Delta\hat{\boldsymbol{z}} \\ &= (\boldsymbol{I} - \boldsymbol{K})\Delta\mathbf{z} \\ &= (\boldsymbol{I} - \boldsymbol{K})(\boldsymbol{H}\Delta\boldsymbol{x} + \boldsymbol{e}) \\ &= (\boldsymbol{I} - \boldsymbol{K})\boldsymbol{e} \;\; [\text{Note that KH} = \text{H}] \\ &= \boldsymbol{S}\boldsymbol{e}\end{aligned}$$

*where **S** is called the **residual sensitivity matrix**.*

Bad Data Identification

Distribution of Measurement Residuals

The residual covariance matrix Ω can be written as:

$$E[rr^T] = \Omega = S \cdot E[e \cdot e^T] \cdot S^T$$

$$= S \cdot R \cdot S^T = S \cdot R$$

Hence, the normalized value of the residual for measurement ***i*** will be given by:

$$r_i^N = \frac{r_i}{\sqrt{\Omega_{ii}}} = \frac{r_i}{\sqrt{R_{ii} S_{ii}}}$$

Bad Data Identification

Classification of Measurements

Measurements can be classified as ***critical*** and ***redundant(or non-critical)*** with the following properties:

- A *critical measurement* is the one whose elimination from the measurement set will result in an *unobservable system.*
- The row/column of *S* corresponding to a critical measurement will be *zero*.
- The *residuals of critical measurements* will always be *zero,* and therefore errors in critical measurements can not be detected.

It can be shown that if there is a single bad data in the measurement set (provided that it is not a critical measurement) the largest normalized residual will correspond to bad datum.

Bad Data Identification / Elimination

Two commonly used approaches:

1. Post-processing of measurement residuals – Largest normalized residuals

2. Modifying measurement weights during iterative solution of WLS estimation

Bad Data Identification

Largest Normalized Residual Test

Steps of the largest normalized residual test for identification of single and non-interacting multiple bad data:

Compute the elements of the measurement residual vector :

Compute the normalized residuals

Find k such that r_k^N is the largest among all $r_i^N, i = 1, \ldots, m$.

If r_k^N > c, then the k-th measurement will be suspected as bad data.

Else, stop, no bad data will be suspected. Here, c is a chosen identification threshold, e.g. 3.0.

Eliminate the k-th measurement from the measurement set and go to step 1.

Use of Synchrophasor Measurements

- Given enough phasor measurements, state estimation problem will become LINEAR, thus can be solved directly without iterations

$$Conventional\ Measurements$$

$$Z = h(X) + e$$

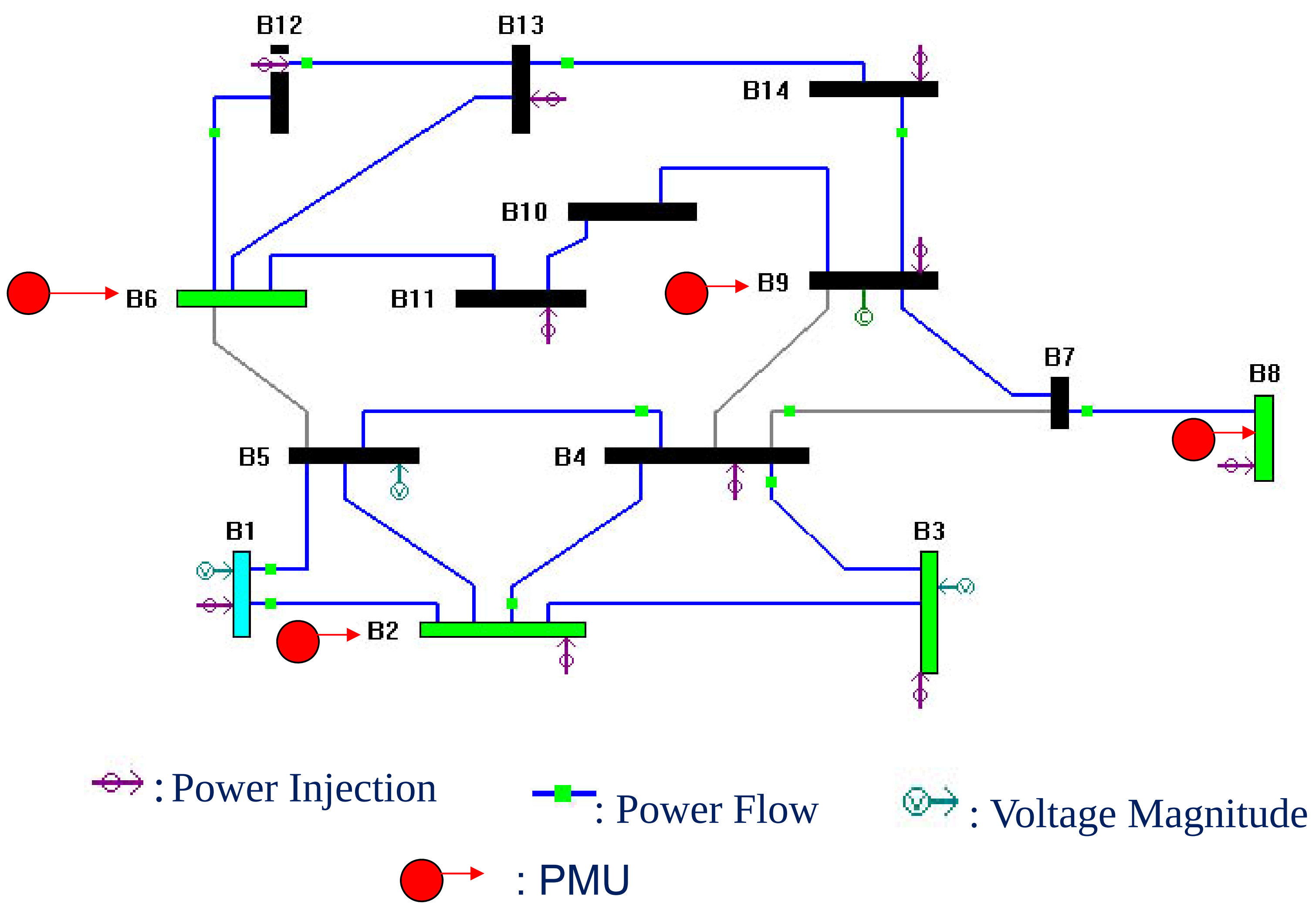
$$\Delta\hat{X} = (H^T R^{-1} H)^{-1} R^{-1} \Delta Z \quad Iterative$$

$$Phasor\ Measurements$$

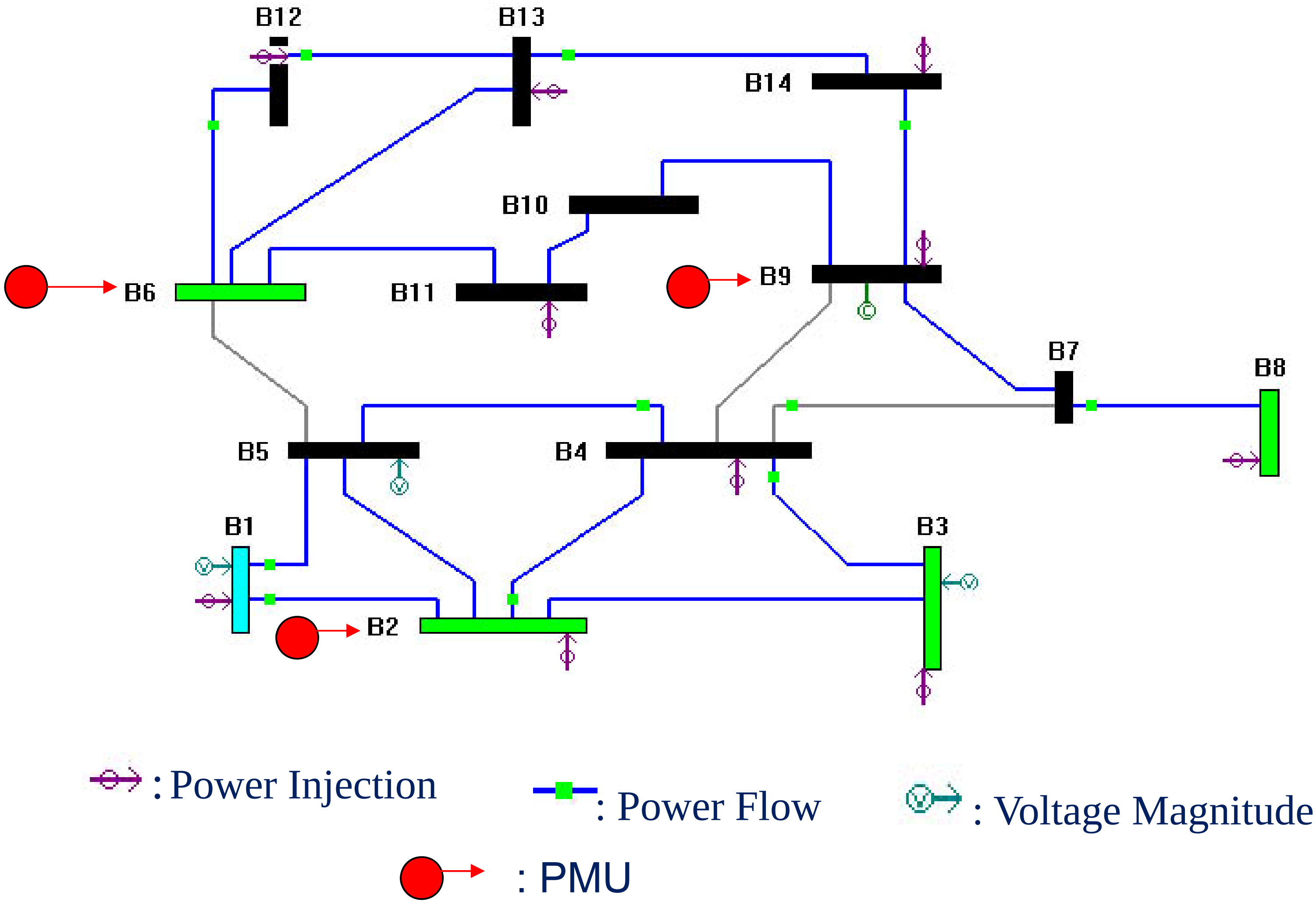
$$Z = H \cdot X + e$$

$$\hat{X} = (H^T R^{-1} H)^{-1} R^{-1} Z \quad Non-iterative$$

Placing PMUs:



Exploiting zero injections

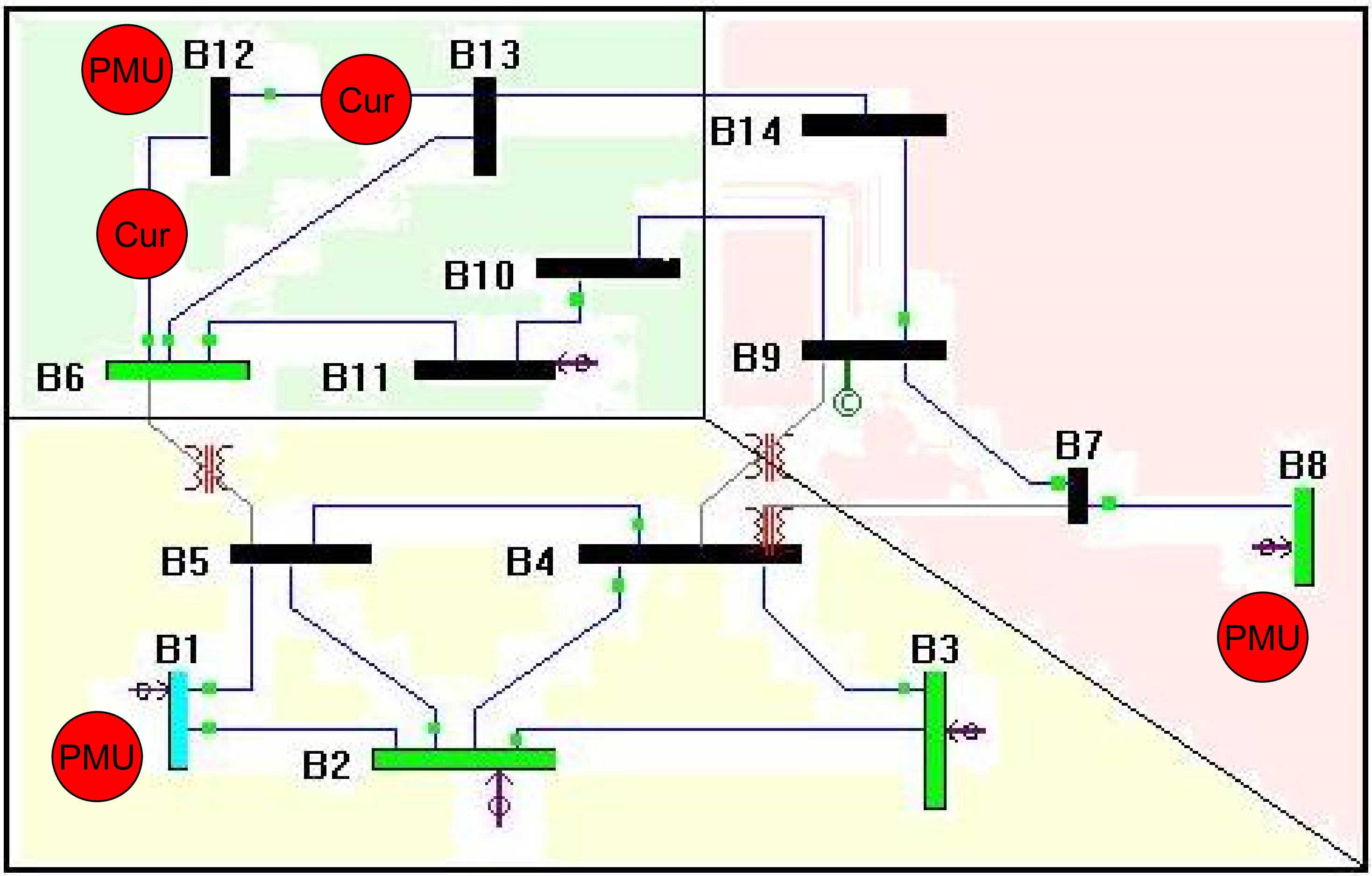


Use of Synchrophasor Measurements

- Given at least one phasor measurement, there will be no need to use a reference bus in the problem formulation
- Given unlimited number of available channels per PMU, it is sufficient to place PMUs at roughly 1/3rd of the system buses to make the entire system observable just by PMUs.

Systems	No. of zero injections	Number of PMUs	
		Ignoring zero Injections	Using zero injections
14-bus	1	4	3
57-bus	15	17	12
118-bus	10	32	29

Merging Observable Islands with PMUs



Performance Metrics

- State Estimation Solution

  - Accuracy:

    Variance of State = inverse of the gain matrix, $[G]^{-1}$
    $= E[\,(x - x^*)\,(x - x^*)'\,]$

  - Convergence:

    Condition Number = Ratio of the largest to smallest eigenvalue

    Large condition number implies an ill-conditioned problem.

Performance Metrics

- Measurement Design

  - Critical Measurements:

    Number of critical measurements and their types

  - Local Redundancy

    Number of measurements incident to a given bus

  - (N-1) Robustness

    Capability of the measurement configuration to render a fully observable system during single measurement and branch losses

Performance Metrics

- Measurement Quality
  - Performance Index (WLS objective function):

    Weighted sum of squares of residuals. Has a Chi-Squares distribution. Large numbers imply presence of bad data in the measurement set.

  - Largest Absolute Normalized Residual:

    If larger than 3.0, the measurement corresponding to the largest absolute value will be suspected of gross errors.

  - Sample variance (Based on historical data):

    Measurement weights are based on sample error variances calculated according to historical data and estimation results. They reflect the quality of individual measurements.

Summary

- State Estimation and its related functions are reviewed.
- Importance of measurement design is illustrated.
- Commonly used methods of identifying and eliminating bad data are described.
- Impact of incorporating phasor measurements on state estimation is briefly reviewed.
- Metrics for state estimation solution, measurement design and measurement quality are suggested.

Free software to:
Build one-line diagrams of power networks
Run power flow studies
Run state estimation

http://www.ece.neu.edu/~abur/pet.html

Thank You

Any Questions?

References

F.C. Schweppe and J. Wildes, ``Power System Static-State Estimation, Part I: Exact Model'', IEEE Transactions on Power Apparatus and Systems, Vol.PAS-89, January 1970, pp.120-125.

F.C. Schweppe and D.B. Rom, ``Power System Static-State Estimation, Part II: Approximate Model'', IEEE Transactions on Power Apparatus and Systems, Vol.PAS-89, January 1970, pp.125-130.

F.C. Schweppe, ``Power System Static-State Estimation, Part III: Implementation'', IEEE Transactions on Power Apparatus and Systems, Vol.PAS-89, January 1970, pp.130-135.

A. Monticelli and A. Garcia, "Fast Decoupled State Estimators", IEEE Transactions on Power Systems, Vol.5, No.2, pp.556-564, May 1990.

A. Monticelli and F.F. Wu, ``Network Observability: Theory'', IEEE Transactions on PAS, Vol.PAS-104, No.5, May 1985, pp.1042-1048.

References

A. Monticelli and F.F. Wu, ``Network Observability: Identification of Observable Islands and Measurement Placement'', IEEE Transactions on PAS, Vol.PAS-104, No.5, May 1985, pp.1035-1041.

G.R. Krumpholz, K.A. Clements and P.W. Davis, ``Power System Observability: A Practical Algorithm Using Network Topology'', IEEE Trans. on Power Apparatus and Systems, Vol. PAS-99, No.4, July/Aug. 1980, pp.1534-1542.

A. Garcia, A. Monticelli and P. Abreu, ``Fast Decoupled State Estimation and Bad Data Processing'', IEEE Trans. on Power Apparatus and Systems, Vol. PAS-98, pp. 1645-1652, September 1979.

Xu Bei, Yeojun Yoon and A. Abur, "Optimal Placement and Utilization of Phasor Measurements for State Estimation," 15th Power Systems Computation Conference Liège (Belgium), August 22-26, 2005.