

State Space Analysis of a Time-Invariant SRC using MATLAB

The series resonant converter of Fig. 2 is commonly employed in high power dc-dc conversion. With appropriate impedances, this converter can also model certain wireless power transfer converters. A complete state space model of the converter exhibits A and B matrices which vary depending on the switching subinterval, resulting in a time-varying system model

If sufficiently large output capacitance is employed, and the transistors are sufficiently ideal, the circuit behavior can be modeled by the equivalent circuit of Fig. 2. In this model, A and B matrices are constant, and only the inputs $\mathbf{u}(t) = [v_p(t) \ v_s(t)]^T$ vary in time.

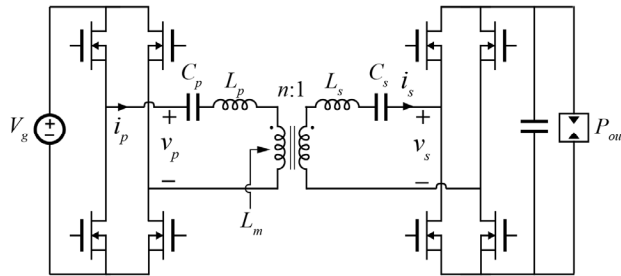


Fig. 1: Series Resonant Converter

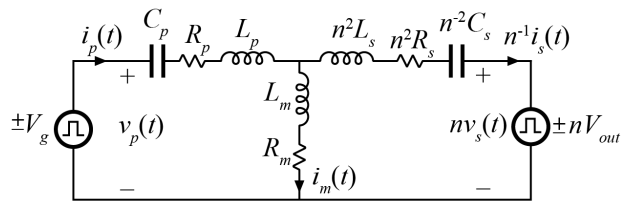


Fig. 2: Time-Invariant Circuit Model of SRC

- What is the minimum number of states that can completely characterize the model of Fig. 2?
- Select a state vector $\mathbf{x}(t)$. Choose any appropriate vector which contains a minimum number of states as determined in (a) and contains only real capacitor voltages and inductor currents which are measurable in the full system of Fig. 1.
- Solve for symbolic expressions for A and B for the $\mathbf{x}(t)$ in (b) and $\mathbf{u}(t) = [v_p(t) \ v_s(t)]^T$. Show all steps in your derivation.
- Derive symbolic expressions for C and D matrices for the output vector $\mathbf{y}(t) = [i_p(t) \ i_m(t) \ i_s(t)]^T$

With the complete state space model, we can use numerical integration to solve the waveforms of the converter. Additional MATLAB example code is available on the website.

- Using MATLAB, plot the steady-state output waveforms of the state-space model developed in (a)-(d) over one switching period for the following implementation

C_p	R_p	L_p	L_m	R_m	n^2L_s	n^2R_s	n^2C_s	V_g	nV_{out}	f_s	t_ϕ^\dagger	n
30 nF	.2 Ω	10 μ H	10 μ H	.1 Ω	10 μ H	.2 Ω	30 nF	20 V	20 V	200 kHz	$T_s/4$	4

[†] See Fig. 3 for waveform timing definitions

- From your MATLAB waveforms, solve numerical values for the output power and efficiency of the converter at this operating point

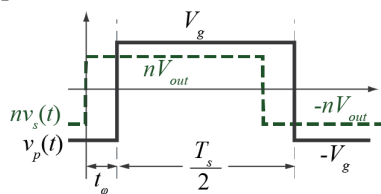


Fig. 3: Input waveform timing definitions