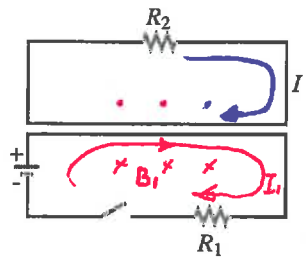
Chapter 6

Sections 6-1 to 6-6: Faraday's Law and its Applications

Problem 6.1 The switch in the bottom loop of Fig. 6-17 (P6.1) is closed at $t = 0$ and then opened at a later time t_1. What is the direction of the current I in the top loop (clockwise or counterclockwise) at each of these two times?

$$\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$



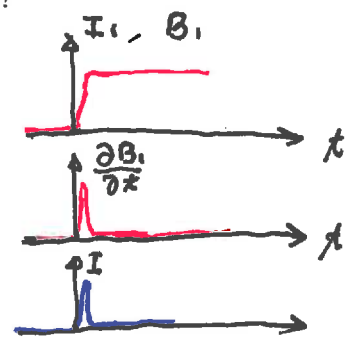


Figure P6.1: Loops of Problem 6.1.

Solution: The magnetic coupling will be strongest at the point where the wires of the two loops come closest. When the switch is closed the current in the bottom loop will start to flow clockwise, which is from left to right in the top portion of the bottom loop. To oppose this change, a current will momentarily flow in the bottom of the top loop from right to left. Thus the current in the top loop is momentarily clockwise when the switch is closed. Similarly, when the switch is opened, the current in the top loop is momentarily counterclockwise.

HW11:P2

Problem 6.2 The loop in Fig. 6-18 (P6.2) is in the x–y plane and $\mathbf{B} = \hat{\mathbf{z}}B_0 \sin \omega t$ with B_0 positive. What is the direction of I ($\hat{\boldsymbol{\phi}}$ or $-\hat{\boldsymbol{\phi}}$) at (a) $t = 0$, (b) $\omega t = \pi/4$, and (c) $\omega t = \pi/2$?

Solution: $I = V_{\text{emf}}/R$. Since the single-turn loop is not moving or changing shape with time, $V_{\text{emf}}^{\text{m}} = 0$ V and $V_{\text{emf}} = V_{\text{emf}}^{\text{tr}}$. Therefore, from Eq. (6.8),

$$I = V_{\text{emf}}^{\text{tr}}/R = \frac{-1}{R}\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}.$$

If we take the surface normal to be $+\hat{\mathbf{z}}$, then the right hand rule gives positive flowing current to be in the $+\hat{\boldsymbol{\phi}}$ direction.

$$I = \frac{-A}{R}\frac{\partial}{\partial t}B_0 \sin \omega t = \frac{-AB_0\omega}{R}\cos \omega t \quad \text{(A)},$$

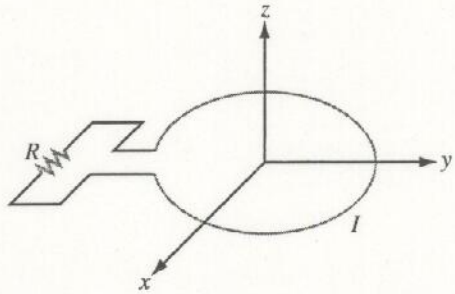


Figure P6.2: Loop of Problem 6.2.

HM11:P2

where A is the area of the loop.

(a) A, ω and R are positive quantities. At $t = 0$, $\cos\omega t = 1$ so $I < 0$ and the current is flowing in the $-\hat{\boldsymbol{\phi}}$ direction (so as to produce an induced magnetic field that opposes $\mathbf{B}$).

(b) At $\omega t = \pi/4$, $\cos\omega t = \sqrt{2}/2$ so $I < 0$ and the current is still flowing in the $-\hat{\boldsymbol{\phi}}$ direction.

(c) At $\omega t = \pi/2$, $\cos\omega t = 0$ so $I = 0$. There is no current flowing in either direction.

Problem 6.3 A coil consists of 100 turns of wire wrapped around a square frame of sides 0.25 m. The coil is centered at the origin with each of its sides parallel to the x- or y-axis. Find the induced emf across the open-circuited ends of the coil if the magnetic field is given by

(a) $\mathbf{B} = \hat{\mathbf{z}}20e^{-3t}$ (T),

(b) $\mathbf{B} = \hat{\mathbf{z}}20\cos x\,\cos 10^3 t$ (T),

(c) $\mathbf{B} = \hat{\mathbf{z}}20\cos x\,\sin 2y\,\cos 10^3 t$ (T).

Solution: Since the coil is not moving or changing shape, $V_{\text{emf}}^{\text{m}} = 0$ V and $V_{\text{emf}} = V_{\text{emf}}^{\text{tr}}$. From Eq. (6.6),

$$V_{\text{emf}} = -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} = -N\frac{d}{dt}\int_{-0.125}^{0.125}\int_{-0.125}^{0.125}\mathbf{B}\cdot(\hat{\mathbf{z}}\,dx\,dy),$$

where $N = 100$ and the surface normal was chosen to be in the $+\hat{\mathbf{z}}$ direction.

(a) For $\mathbf{B} = \hat{\mathbf{z}}20e^{-3t}$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}(20e^{-3t}(0.25)^2) = 375e^{-3t} \quad \text{(V)}.$$

(b) For $\mathbf{B} = \hat{\mathbf{z}}20\cos x \cos 10^3 t$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}\left(20\cos 10^3 t \int_{x=-0.125}^{0.125}\int_{y=-0.125}^{0.125} \cos x\, dx\, dy\right) = 124.6 \sin 10^3 t \quad \text{(kV)}.$$

(c) For $\mathbf{B} = \hat{\mathbf{z}}20\cos x \sin 2y \cos 10^3 t$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}\left(20\cos 10^3 t \int_{x=-0.125}^{0.125}\int_{y=-0.125}^{0.125} \cos x \sin 2y\, dx\, dy\right) = 0.$$

Problem 6.4 A stationary conducting loop with internal resistance of 0.5 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 5 A flows through it. What will the current be if the loop is opened to create a small gap and a 2-Ω resistor is connected across its open ends?

Solution: V_{emf} is independent of the resistance which is in the loop. Therefore, when the loop is intact and the internal resistance is only 0.5 Ω,

$$V_{\text{emf}} = 5\text{ A} \times 0.5\ \Omega = 2.5\text{ V}.$$

When the small gap is created, the total resistance in the loop is infinite and the current flow is zero. With a 2-Ω resistor in the gap,

$$I = V_{\text{emf}}/(2\ \Omega + 0.5\ \Omega) = 2.5\text{ V}/2.5\ \Omega = 1 \quad \text{(A)}.$$

HW11:P3

Problem 6.5 A circular-loop TV antenna with 0.02 m^2 area is in the presence of a uniform-amplitude 300-MHz signal. When oriented for maximum response, the loop develops an emf with a peak value of 30 (mV). What is the peak magnitude of **B** of the incident wave?

Solution: TV loop antennas have one turn. At maximum orientation, Eq. (6.5) evaluates to $\Phi = \int \mathbf{B}\cdot d\mathbf{s} = \pm BA$ for a loop of area A and a uniform magnetic field with magnitude $B = |\mathbf{B}|$. Since we know the frequency of the field is $f = 300$ MHz, we can express B as $B = B_0 \cos(\omega t + \alpha_0)$ with $\omega = 2\pi \times 300 \times 10^6$ rad/s and α_0 an arbitrary reference phase. From Eq. (6.6),

$$V_{\text{emf}} = -N\frac{d\Phi}{dt} = -A\frac{d}{dt}[B_0\cos(\omega t + \alpha_0)] = AB_0\omega\sin(\omega t + \alpha_0).$$

V_{emf} is maximum when $\sin(\omega t + \alpha_0) = 1$. Hence,

$$30 \times 10^{-3} = AB_0\omega = 0.02 \times B_0 \times 6\pi \times 10^8,$$

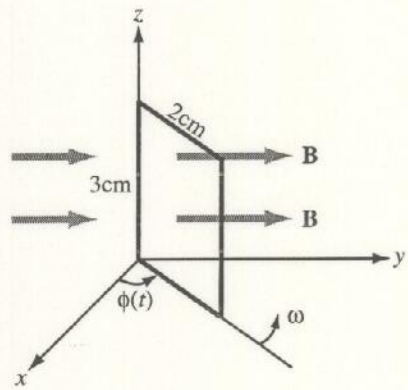


Figure P6.7: Rotating loop in a magnetic field (Problem 6.7).

The direction of the current is CW (if looking at it along $-\hat{\mathbf{x}}$-direction) when the loop is in the first quadrant ($0 \le \phi \le \pi/2$). The current reverses direction in the second quadrant, and reverses again every quadrant.

Problem 6.8 A rectangular conducting loop 5 cm $\times$10 cm with a small air gap in one of its sides is spinning at 7200 revolutions per minute. If the field $\mathbf{B}$ is normal to the loop axis and its magnitude is 6×10^{-6} T, what is the peak voltage induced across the air gap?

Solution:

$$\omega = \frac{2\pi \text{ rad/cycle} \times 7200 \text{ cycles/min}}{60 \text{ s/min}} = 240\pi \text{ rad/s},$$

$$A = 5 \text{ cm} \times 10 \text{ cm}/(100 \text{ cm/m})^2 = 5.0 \times 10^{-3} \text{ m}^2.$$

From Eqs. (6.36) or (6.38), $V_{\text{emf}} = A\omega B_0 \sin \omega t$; it can be seen that the peak voltage is

$$V_{\text{emf}}^{\text{peak}} = A\omega B_0 = 5.0 \times 10^{-3} \times 240\pi \times 6 \times 10^{-6} = 22.62 \quad (\mu\text{V}).$$

HW11:P4

Problem 6.9 A 50-cm-long metal rod rotates about the z-axis at 90 revolutions per minute, with end 1 fixed at the origin as shown in Fig. 6-21 (P6.9). Determine the induced emf V_{12} if $\mathbf{B} = \hat{\mathbf{z}}2 \times 10^{-4}$ T.

Solution: Since $\mathbf{B}$ is constant, $V_{\text{emf}} = V_{\text{emf}}^{\text{m}}$. The velocity $\mathbf{u}$ for any point on the bar is given by $\mathbf{u} = \hat{\boldsymbol{\phi}} r\omega$, where

$$\omega = \frac{2\pi \text{ rad/cycle} \times (90 \text{ cycles/min})}{(60 \text{ s/min})} = 3\pi \text{ rad/s}.$$

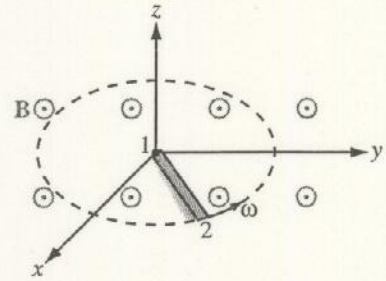


Figure P6.9: Rotating rod of Problem 6.9.

From Eq. (6.24),

$$
\begin{aligned}
V_{12} = V_{\text{emf}}^{\text{m}} = \int_2^1 (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} &= \int_{r=0.5}^{0} (\hat{\boldsymbol{\phi}} 3\pi r \times \hat{\mathbf{z}} 2 \times 10^{-4}) \cdot \hat{\mathbf{r}}\, dr \\
&= 6\pi \times 10^{-4} \int_{r=0.5}^{0} r\, dr \\
&= 3\pi \times 10^{-4} r^2 \Big|_{0.5}^{0} \\
&= -3\pi \times 10^{-4} \times 0.25 = -236 \quad (\mu\text{V}).
\end{aligned}
$$

HW11:P5

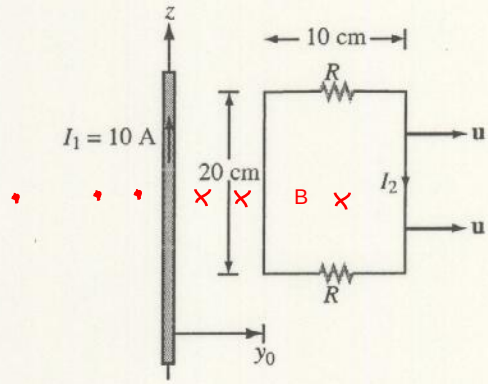
Problem 6.10 The loop shown in Fig. 6-22 (P6.10) moves away from a wire carrying a current $I_1 = 10$ (A) at a constant velocity $\mathbf{u} = \hat{\mathbf{y}} 7.5$ (m/s). If $R = 10\ \Omega$ and the direction of I_2 is as defined in the figure, find I_2 as a function of y_0, the distance between the wire and the loop. Ignore the internal resistance of the loop.

Solution: Assume that the wire carrying current I_1 is in the same plane as the loop. The two identical resistors are in series, so $I_2 = V_{\text{emf}}/2R$, where the induced voltage is due to motion of the loop and is given by Eq. (6.26):

$$V_{\text{emf}} = V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$

The magnetic field $\mathbf{B}$ is created by the wire carrying I_1. Choosing $\hat{\mathbf{z}}$ to coincide with the direction of I_1, Eq. (5.30) gives the external magnetic field of a long wire to be

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r}.$$



Get the direction right:
B points into paper as shown.
B decreases with increasing distance from wire, therefore flux decreases as loop moves away from wire, i.e., flux change rate opposite to flux itself.
Right-hand rule defines loop direction counterclockwise, given the above flux change rate.
Considering the negative sign in the Flux Rule, current direction is as shown.

Figure P6.10: Moving loop of Problem 6.10.

For positive values of y_0 in the y-z plane, $\hat{\mathbf{y}} = \hat{\mathbf{r}}$, so

$$\mathbf{u} \times \mathbf{B} = \hat{\mathbf{y}}|\mathbf{u}| \times \mathbf{B} = \hat{\mathbf{r}}|\mathbf{u}| \times \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r} = \hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r}.$$

Integrating around the four sides of the loop with $d\mathbf{l} = \hat{\mathbf{z}}\, dz$ and the limits of integration chosen in accordance with the assumed direction of I_2, and recognizing that only the two sides without the resistors contribute to $V^{\mathrm{m}}_{\mathrm{emf}}$, we have

$$\begin{aligned} V^{\mathrm{m}}_{\mathrm{emf}} &= \int_0^{0.2} \left(\hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r} \right)\bigg|_{r=y_0} \cdot (\hat{\mathbf{z}}\, dz) + \int_{0.2}^{0} \left(\hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r} \right)\bigg|_{r=y_0+0.1} \cdot (\hat{\mathbf{z}}\, dz) \\ &= \frac{4\pi \times 10^{-7} \times 10 \times 7.5 \times 0.2}{2\pi} \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \\ &= 3 \times 10^{-6} \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \quad \text{(V)}, \end{aligned}$$

and therefore

$$I_2 = \frac{V^{\mathrm{m}}_{\mathrm{emf}}}{2R} = 150 \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \quad \text{(nA)}.$$

Problem 6.11 The conducting cylinder shown in Fig. 6-23 (P6.11) rotates about its axis at 1,200 revolutions per minute in a radial field given by

$$\mathbf{B} = \hat{\mathbf{r}}\, 6 \quad \text{(T)}.$$

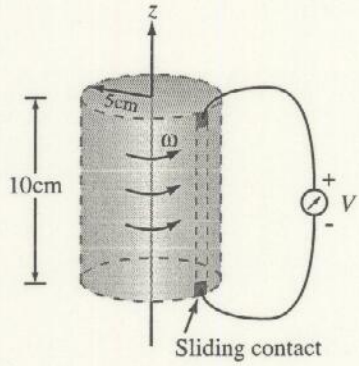


Figure P6.11: Rotating cylinder in a magnetic field (Problem 6.11).

The cylinder, whose radius is 5 cm and height 10 cm, has sliding contacts at its top and bottom connected to a voltmeter. Determine the induced voltage.

Solution: The surface of the cylinder has velocity **u** given by

$$\mathbf{u} = \hat{\boldsymbol{\phi}}\omega r = \hat{\boldsymbol{\phi}} 2\pi \times \frac{1,200}{60} \times 5 \times 10^{-2} = \hat{\boldsymbol{\phi}} 2\pi \quad \text{(m/s)},$$

$$V_{12} = \int_0^L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} = \int_0^{0.1} (\hat{\boldsymbol{\phi}} 2\pi \times \hat{\mathbf{r}} 6) \cdot \hat{\mathbf{z}}\, dz = -3.77 \quad \text{(V)}.$$

Problem 6.12 The electromagnetic generator shown in Fig. 6-12 is connected to an electric bulb with a resistance of 150 Ω. If the loop area is 0.1 m^2 and it rotates at 3,600 revolutions per minute in a uniform magnetic flux density $B_0 = 0.4$ T, determine the amplitude of the current generated in the light bulb.

Solution: From Eq. (6.38), the sinusoidal voltage generated by the a-c generator is $V_{\text{emf}} = A\omega B_0 \sin(\omega t + C_0) = V_0 \sin(\omega t + C_0)$. Hence,

$$V_0 = A\omega B_0 = 0.1 \times \frac{2\pi \times 3,600}{60} \times 0.4 = 15.08 \quad \text{(V)},$$

$$I = \frac{V_0}{R} = \frac{15.08}{150} = 0.1 \quad \text{(A)}.$$

HW11:P6

Problem 6.13 The circular disk shown in Fig. 6-24 (P6.13) lies in the x–y plane and rotates with uniform angular velocity ω about the z-axis. The disk is of radius a and is present in a uniform magnetic flux density $\mathbf{B} = \hat{\mathbf{z}} B_0$. Obtain an expression for the emf induced at the rim relative to the center of the disk.

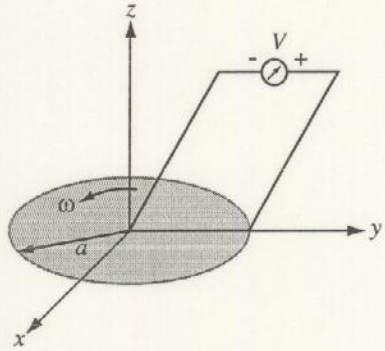


Figure P6.13: Rotating circular disk in a magnetic field (Problem 6.13).

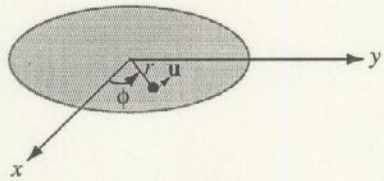


Figure P6.13: (a) Velocity vector **u**.

Solution: At a radial distance r, the velocity is

$$\mathbf{u} = \hat{\boldsymbol{\phi}}\,\omega r$$

where ϕ is the angle in the x–y plane shown in the figure. The induced voltage is

$$V = \int_0^a (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} = \int_0^a [(\hat{\boldsymbol{\phi}}\,\omega r) \times \hat{\mathbf{z}} B_0] \cdot \hat{\mathbf{r}}\, dr.$$

$\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}}$ is along $\hat{\mathbf{r}}$. Hence,

$$V = \omega B_0 \int_0^a r\, dr = \frac{\omega B_0 a^2}{2}.$$

Section 6-7: Displacement Current

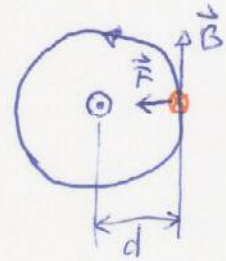
Problem 6.14 The plates of a parallel-plate capacitor have areas 10 cm^2 each and are separated by 2 cm. The capacitor is filled with a dielectric material with

HW11
Problem 7

Two infinitely long parallel wires (as in the parallel wire transmission line) of radius a carry currents of equal magnitude I but in opposite directions. The distance between the axes of the two wires is d.
(1) What is the force between the two wires, assuming $d >> a$ (meaning you can ignore the finite radii of the wires)? Derive the expression rather than copy it from the book. Do the two wires attract or repel each other?
(2) What is resultant magnetic field due to the two wires at a point mid way between the two wires?
(3) Find the total flux per unit length threading through the area between the two lines.

(1) Consider one wire in the field of the other.

$$2\pi d B = \mu I \Rightarrow B = \frac{\mu I}{2\pi d}$$

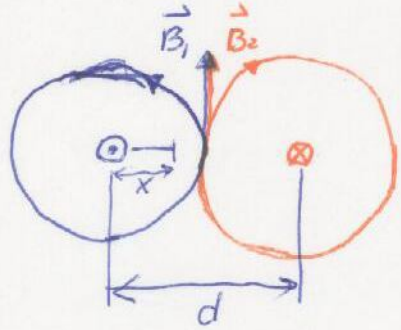


$$F' = \frac{F}{l} = IBl/l = IB = \frac{\mu I^2}{2\pi d}$$

The two wires, carrying opposite currents, repel each other.

(2) $$B_1 = B_2 = \frac{\mu I}{2\pi\left(\frac{d}{2}\right)}$$

$$= \frac{\mu I}{\pi d}$$



Total field due to the two wires is

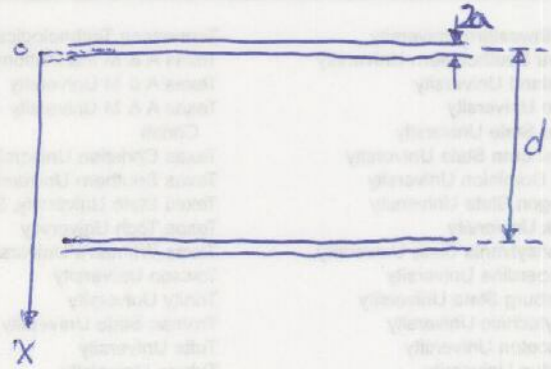
$$B = B_1 + B_2 = \frac{2\mu I}{\pi d}$$

(3) For any point x from wire 1.

$$B_1 = \frac{\mu I}{2\pi x}, \qquad B_2 = \frac{\mu I}{2\pi(d-x)}$$

$$B = B_1 + B_2 = \frac{\mu I}{2\pi}\left(\frac{1}{x} + \frac{1}{d-x}\right)$$

$$\Phi = l\int_a^{d-a} B\,dx$$



$$= l\int_a^{d-a}\left(\frac{1}{x}+\frac{1}{d-x}\right)dx\left(\frac{\mu I}{2\pi}\right)$$

$$= 2l\int_a^{d-a}\frac{1}{x}\,dx\left(\frac{\mu I}{2\pi}\right)$$

$$= 2l\frac{\mu I}{2\pi}\ln\frac{d-a}{a}$$

$$\frac{\Phi}{l} = \frac{\mu I}{\pi}\ln\frac{d-a}{a}$$

Extra:

$$L' = \frac{L}{l} = \frac{\Phi}{lI} = \frac{\mu}{\pi}\ln\frac{d-a}{a} \approx \frac{\mu}{\pi}\ln\frac{d}{a}$$

Notice: The above is just an approximation when $d >> 2a$ and thus the current distribution within the wire doesn't matter.

Actually, the current in the two wires interact w/ each other, therefore the distribution is not uniform.

Chapter 6

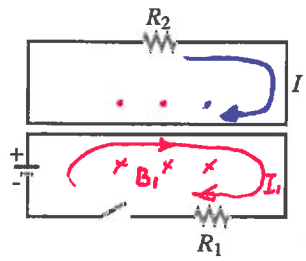
HW11

Problem 8

Sections 6-1 to 6-6: Faraday's Law and its Applications

Problem 6.1 The switch in the bottom loop of Fig. 6-17 (P6.1) is closed at $t = 0$ and then opened at a later time t_1. What is the direction of the current I in the top loop (clockwise or counterclockwise) at each of these two times?

$$\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$



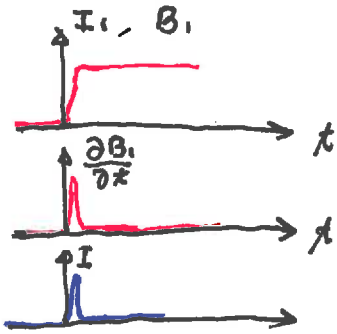


Figure P6.1: Loops of Problem 6.1.

Solution: The magnetic coupling will be strongest at the point where the wires of the two loops come closest. When the switch is closed the current in the bottom loop will start to flow clockwise, which is from left to right in the top portion of the bottom loop. To oppose this change, a current will momentarily flow in the bottom of the top loop from right to left. Thus the current in the top loop is momentarily clockwise when the switch is closed. Similarly, when the switch is opened, the current in the top loop is momentarily counterclockwise.

Problem 6.2 The loop in Fig. 6-18 (P6.2) is in the x–y plane and $\mathbf{B} = \hat{\mathbf{z}}B_0 \sin \omega t$ with B_0 positive. What is the direction of I ($\hat{\boldsymbol{\phi}}$ or $-\hat{\boldsymbol{\phi}}$) at (a) $t = 0$, (b) $\omega t = \pi/4$, and (c) $\omega t = \pi/2$?

Solution: $I = V_{\text{emf}}/R$. Since the single-turn loop is not moving or changing shape with time, $V_{\text{emf}}^{\text{m}} = 0$ V and $V_{\text{emf}} = V_{\text{emf}}^{\text{tr}}$. Therefore, from Eq. (6.8),

$$I = V_{\text{emf}}^{\text{tr}}/R = \frac{-1}{R}\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}.$$

If we take the surface normal to be $+\hat{\mathbf{z}}$, then the right hand rule gives positive flowing current to be in the $+\hat{\boldsymbol{\phi}}$ direction.

$$I = \frac{-A}{R}\frac{\partial}{\partial t}B_0 \sin \omega t = \frac{-AB_0\omega}{R}\cos \omega t \quad \text{(A)},$$