Answers to HW #8 Problem 1

(b)

$$\frac{\partial V}{\partial x} = y^2 z^4, \quad \frac{\partial V}{\partial y} = 2xyz^4, \quad \frac{\partial V}{\partial z} = 4xy^2 z^3$$

$$\therefore \nabla V = \hat{x}(y^2 z^4) + \hat{y}(2xyz^4) + \hat{z}(4xy^2 z^3)$$

(a)

$$\frac{\partial T}{\partial x} = -\frac{6x}{(x^2+z^2)^2}, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial T}{\partial z} = -\frac{6z}{(x^2+z^2)^2}$$

$$\therefore \nabla T = -\frac{6}{(x^2+z^2)^2}(\hat{x}x + \hat{z}z)$$

(c)

$$\frac{\partial U}{\partial r} = -\frac{2rz\cos\phi}{(1+r^2)^2}, \quad \frac{\partial U}{\partial \phi} = -\frac{z\sin\phi}{1+r^2}$$

$$\frac{\partial U}{\partial z} = \frac{\cos\phi}{(1+r^2)}$$

$$\nabla U = \hat{r}\frac{\partial U}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial U}{\partial \phi} + \hat{z}\frac{\partial U}{\partial z}$$

$$= -\hat{r}\frac{2rz\cos\phi}{(1+r^2)^2} - \hat{\phi}\frac{z\sin\phi}{r(1+r^2)} + \hat{z}\frac{\cos\phi}{1+r^2}$$