Electrostatics

The Image Method

Now, let's talk about the last topic of Ch. 4.

— The image method.

We talked about how to calculate the field of a body of charge. Basically, we use the Coulomb law to add up the fields due to many many small volumes of charge dV.

\[
\vec{E} = \frac{1}{4\pi \varepsilon_0} \int \frac{P \, dV}{|\vec{R} - \vec{R}'|^2} \frac{\vec{R} - \vec{R}'}{|\vec{R} - \vec{R}'|}
\]

What if we place an infinitely large sheet of ideal conductor nearby? How do we calculate the field?

We know that this charged body will induce opposite charge in the conductor.

The total field is the sum of the field due to the original charged body and that of the induced charge.

But we don't know the distribution of the induced charge.
One could use the Coulomb's law and the boundary conditions to find the induced charge distribution.

But that's extremely mathematically involved. Nobody wants to do that.

There is a better way.

Let's first consider one point charge

\[ \mathbf{Q} \]

The conductor plane is infinitely big. We define \( V(\infty) = 0 \).

The potential of a conductor is equal everywhere.

The point charge \( Q \) induces a distribution of negative charge at the conductor surface, such that the entire plane is \( Q \) at potential 0.

There is something we didn't and will not discuss in this course. It's called the uniqueness of the solution to the Poisson Eq. It says that the solution to the Poisson Eq., i.e., the field distribution is uniquely determined by the charge distribution and boundary conditions.
Therefore, if we can construct a simpler situation that has the same charge distribution and boundary conditions for the upper half of the space, we have the solution.

Let's consider this situation.

a negative charge ...

This is the "image charge method."

Basically, this image charge is the mirror image of the original charge.

By linear superposition, if we have a body of charge ...

Do Homework 9 Problem 7.