Propagation of Pulses on Transmission Lines

So far we have been talking about single-frequency (harmonic) “signals”, which carry no information.

To transmit information, we must modulate the signal.

For example, we modulate a microwave carrier to send a bit down the line.

This envelope travels along the line at speed $v_g$, the “group velocity,” which is usually a little different from $v_p$, due to dispersion.
For simplicity, we ignore dispersion and assume $v_g = v_p$.

If $Z_L = Z_0$, this pulse is totally absorbed upon arrival at the load. This is what we want.

For a lossless line, $Z_0$ is real.

If $Z_L$ is purely resistive, this match is (assumed to be) frequency-independent.
If $Z_L \neq Z_0 \neq Z_g$, things become complicated.

We first look at the case, $\tau < l/v_p$:

Lots of echoes. Echoes die off.
First case, $\tau < l/v_p$:

The bit is distorted and broadened.

Second case, $\tau > l/v_p$, i.e., $l < v_p\tau$:

Lots of echoes. Echoes die off. May corrupt other bits
Recall that we always have multiple reflections inside any matching network.

Does impedance matching really help us? Why?

\[ y(d) = 1 + 1.58j \]

\[ d = 0.063\lambda \]

\[ z_L = 0.5 - j \]

\[ y_L = 0.4 + 0.8j \]

\[ l = 0.09\lambda \]

Single stub matching example

Class ended here Thu 9/24/2020.
Recall that we always have multiple reflections inside any matching network.

Does impedance matching really help us? Why?

Notice that we can always choose to have $d < \lambda/2$ and $l < \lambda/2$.

The time to travel $\lambda/2$ is $1/(2f)$. Thus the bit is broadened only by several $1/(2f)$, at most.

Without matching, we have echoes.

The modulated case is quite complicated. We now look into a simple case quantitatively.
First, let’s list the basic assumptions to be used:

1. Lossless line. $Z_0$ is purely real.
3. Therefore, $\Gamma$ is frequency-independent.
   
   (If $R_L = Z_0$, impedance matched for all frequencies)
4. Dispersionless: $\nu_g = \nu_p$ for all frequencies.

Know the simplifying assumptions. Know the limitations.
Propagation of a voltage step on a transmission line

For $0 < t < T = l/v_p$,

Time of a single trip

The “turn-on” event has not reached the load yet. It does not know about $R_L$.

The transmission line feels like infinitely long. In other words, no reflection yet.

What is the equivalent input impedance at $z = 0$?

Notice change in convention. Generator at $z = 0$, load at $z = l$. 
Propagation of a voltage step on a transmission line

For $0 < t < T = l/v_p$, 

The "turn-on" event has not reached the load yet. It does not know about $R_L$. 

The transmission line feels like infinitely long. In other words, no reflection yet.

The equivalent input impedance at $z = 0$ is $Z_0$. Not $Z_{in}$!

Subscript "1" means the first round trip.

Superscript "+" means the incident direction.

$$V^+_i = \frac{V_g Z_0}{R_g + Z_0}$$

$$I^+_i = \frac{V_g}{R_g + Z_0}$$
Propagation of a voltage step on a transmission line

The equivalent input impedance at \( z = 0 \) is \( Z_0 \).

Not \( Z_{in} \)!

Snapshots at \( t = T/2 \)

Edge/front moving at \( v_p \) (actually \( v_g \))

The leading edge reaches the load at \( t = T \). Reflection.

\[
\begin{align*}
V_i^- &= \Gamma_l V_i^+ \\
I_i^- &= \Gamma_l I_i^+
\end{align*}
\]

\[ \Gamma_l = \frac{R_L - Z_0}{R_L + Z_0} \]

What is the voltage at the load at \( t = T \)?
What is the voltage at the load at $t = T$?

At $t = 2T$, the front hits the source. Reflection.

\[ V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+ \]
\[ I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+ \]

Assuming
\[ \Gamma_L > 0 \]
\[ \Gamma_g > 0 \]

\[ \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} \]
At $t = 3T$, the front hits the load again.

\[ V_2^- = \Gamma_L V_2^+ \]
\[ I_2^- = -\Gamma_L I_2^+ \]  
Again, notice the sign.

Again, notice that reflection happens instantaneously.

\[ V_2^+ + V_2^- = V_2^+ (1 + \Gamma_L) \]
\[ I_2^+ + I_2^- = I_2^- (1 - \Gamma_L) \]

It goes on and on. For the $i$th round trip,

\[ V_i^+ + V_i^- = V_i^+ (1 + \Gamma_L) \]
\[ I_i^+ + I_i^- = I_i^- (1 - \Gamma_L) \]

\[ u (t = \infty) = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \ldots \]
\[ = \sum_{i=1}^{\infty} (V_i^+ + V_i^-) \]
\[ = V_1^+ (1 + \Gamma_L) + V_2^+ (1 + \Gamma_L) + \ldots = (1 + \Gamma_L) \left[ V_1^+ + V_2^+ + \ldots \right] \]
\[ = (1 + \Gamma_L) \sum_{i=1}^{\infty} V_i^+ \]

Note: At the steady state, $v$ is the same at all $z$, therefore we do not specify $z$. 
\[ U(x = \infty) = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \ldots \]
\[ = \sum_{i=1}^{\infty} (V_i^+ + V_i^-) \]
\[ = V_1^+ (1 + \Gamma_L) + V_2^+ (1 + \Gamma_L) + \ldots = (1 + \Gamma_L) \left[ V_1^+ + V_2^+ + \ldots \right] \]
\[ = (1 + \Gamma_L) \sum_{i=1}^{\infty} V_i^+ \]

\[ V_2^+ = \Gamma_g \Gamma_L V_1^+ \quad \text{and} \quad V_i^+ = \Gamma_g \Gamma_L V_i^+ \]

\[ U(x = \infty) = V_1^+ (1 + \Gamma_L) \left[ 1 + \Gamma_g \Gamma_L + (\Gamma_g \Gamma_L)^2 + \ldots \right] \]
\[ = V_1^+ (1 + \Gamma_L) \sum_{i=0}^{\infty} (\Gamma_g \Gamma_L)^i \]

Use \[ l + x + x^2 + \ldots = \sum_{i=0}^{\infty} x^i = \frac{1}{1 - x} \] with \[ x = \Gamma_g \Gamma_L \]

We get:
\[ U(x = \infty) = V_1^+ \left( 1 + \Gamma_L \right) \frac{1}{1 - \Gamma_g \Gamma_L} \]
Similarly,

\[ U(t = \infty) = V_i^+(1 + \Gamma_L) \cdot \frac{1}{1 - \Gamma_g \Gamma_L} \]

\[ V_i^+ = \frac{V_g Z_0}{R_g + Z_0} \]

\[ \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} \]

\[ \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \]

\[ \Rightarrow \quad U(t = \infty) = \frac{V_g R_L}{R_g + R_L} \]

We have traced \( v(t) \) and \( i(t) \) all the way to \( t = \infty \).

That’s quite tedious.

We have a graphical tool to trace this bouncing back and forth.

It’s called the bounce diagram.

Review textbook Section 2-12 overview, Section 2-12.1
Several Things to Talk about

Homework
Previous student comments:
I prefer HW assignments to be mandatory. That way I'm forced to do it and learn. I was good about doing the optional HW at the beginning of the semester, but lost steam towards the end.

There is no homework assignment after some class meetings (e.g. this one), what should you do after the class?

Test 1
Chapters 1 & 2.
New date: Thu 10/15

Reading to prepare for the Field Theory
Chapter 3: vector analysis.
Homework 8.
**Project**

A circuit simulation project to transition you from *lumped component*-based circuit theory

In Part 1 and Part 2, you built an LC network:

In Part 3, you built a cascade of 10 instances of this LC network.

And, you did transient simulations of the following circuits (1-unit and 10-unit networks) with the generator signal being voltage steps with different rise times:

**Part 4:** Now, create a new network that is a cascade of 10 instances of the 10-unit network, so that this new network contains 100 units. You may create a symbol for this new network for convenience. Using the same inductance and capacitance values, do the same simulations you have done for the above 1- and 10-unit networks. (Same generators with same internal impedance. Simulate for both open circuit and 50-ohm loads, the two rise times for each case, as done for the single LC.)

Ongoing project. Stay tuned for next steps.
Project

In Part 3, you built a cascade of 10 instances of this LC network. In Part 4, you built a cascade of ten such 10-unit networks, which is 100-unit.

And, you did transient simulations of the following circuits (with the 1-unit and 10-unit networks) with the generator signal being voltage steps with different rise times:

Part 5: Now, create a new network that is a cascade of 10 instances of the 100-unit network, so that this new network contains 1000 units. Using the same inductance and capacitance values, do the same simulations you have done for the above 1-, 10-, and 100-unit networks.

Note: You want to see the entire transient process (until it essentially settles). You may also want to see the details of the transients, e.g. the details of the rises and falls. To see the entire thing and the details, you may want to plot on different scales. If you are curious about the transients at the internal nodes of the cascades, plot those and see. Need help with the tool to do that, ask the TA.

Ongoing project. Stay tuned for next steps.