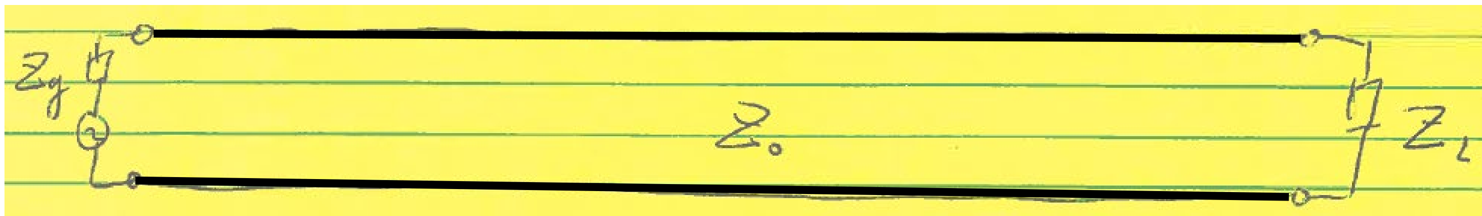
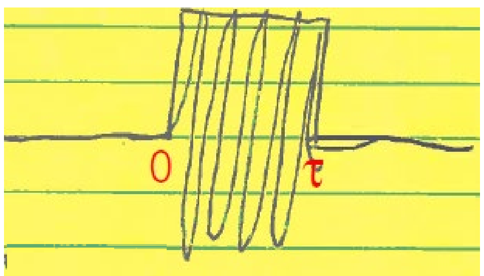
Propagation of Pulses on Transmission Lines

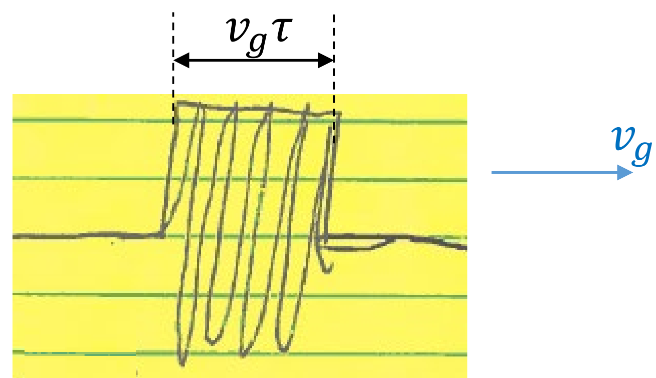
So far we have been talking about single-frequency (harmonic) “signals”, which carry no information.

To transmit information, we must modulate the signal.

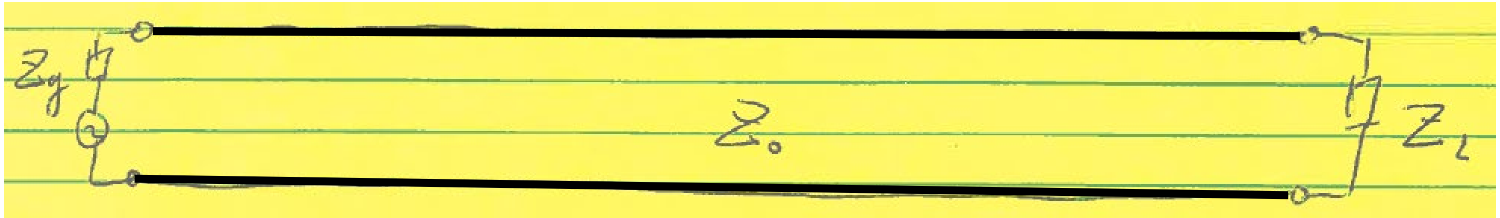


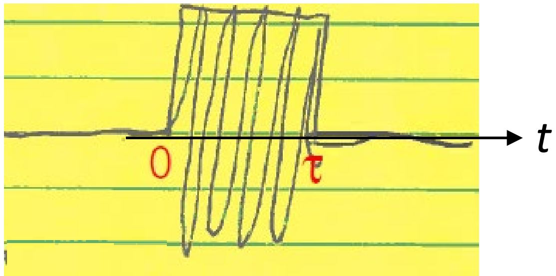


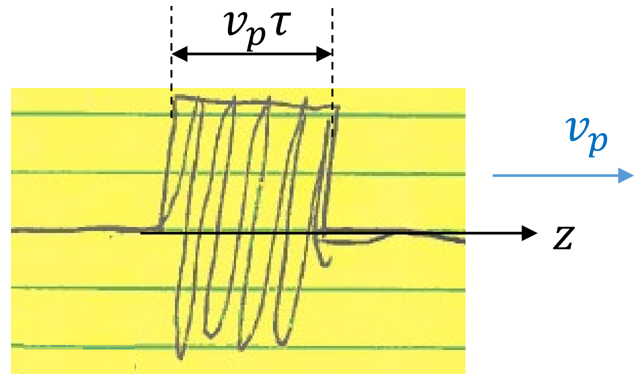
For example, we modulate a microwave carrier to send a bit down the line.



This envelope travels along the line at speed v_g, the “group velocity,” which is usually a little different from v_p, due to dispersion.





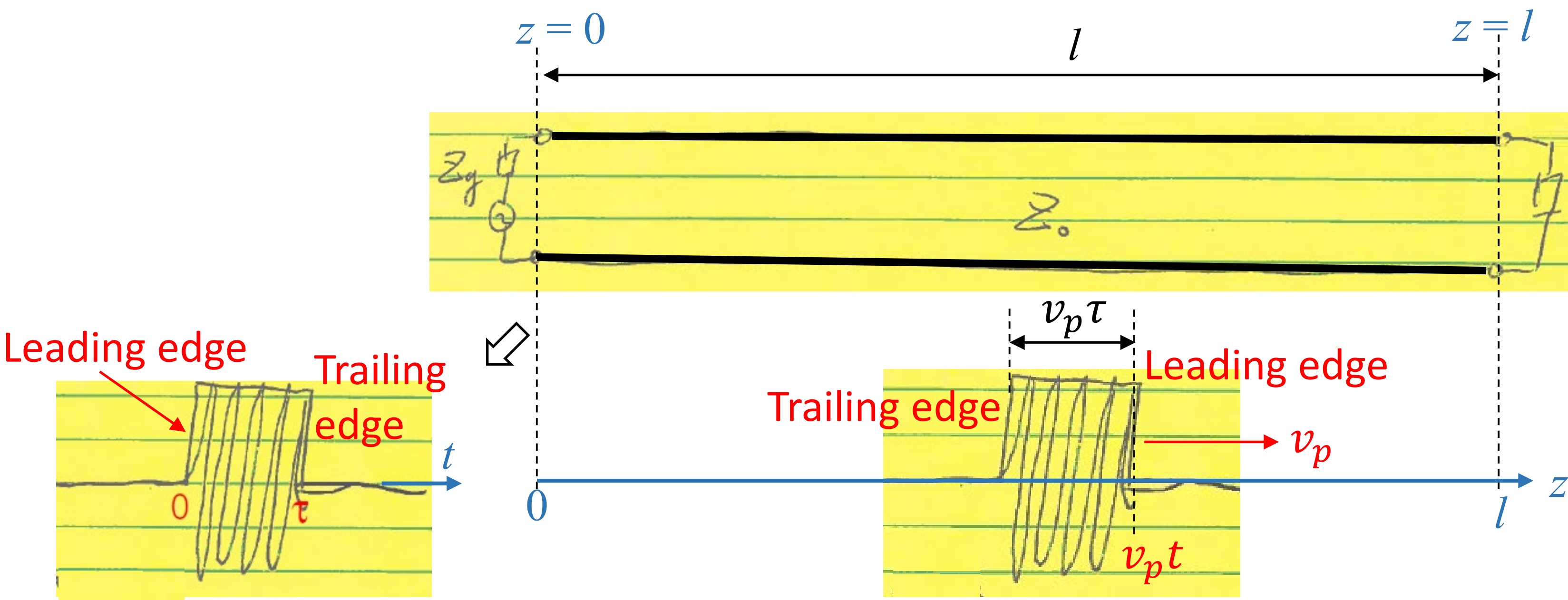


For simplicity, we ignore dispersion and assume $v_g = v_p$.

If $Z_L = Z_0$, this pulse is totally absorbed upon arrival at the load.
This is what we want.

For a lossless line, Z_0 is real.

If Z_L is purely resistive, this match is (assumed to be) frequency-independent.

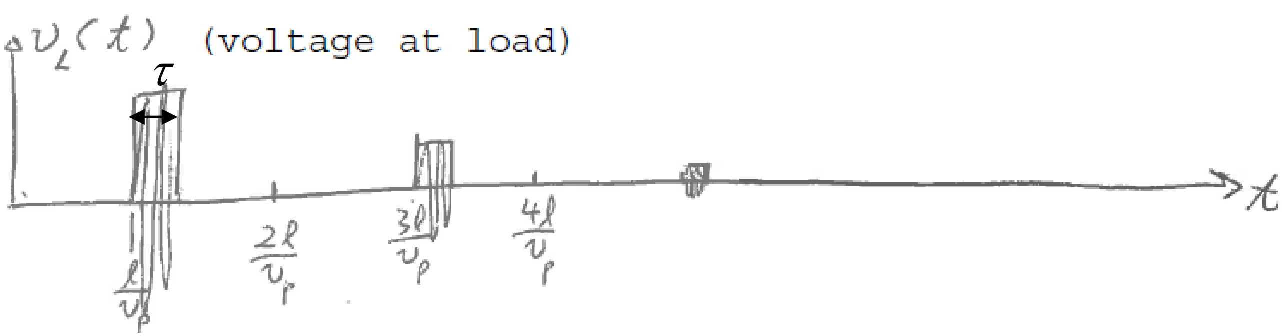


Waveform at z = 0 (generator)

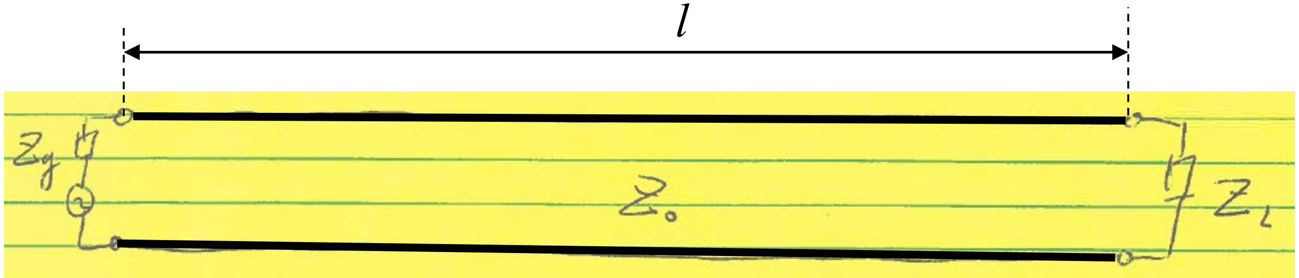
Snapshot at $t < l/v_p$

If $Z_L \neq Z_0 \neq Z_g$, things become complicated.

We first look at the case, $\tau < l/v_p$:

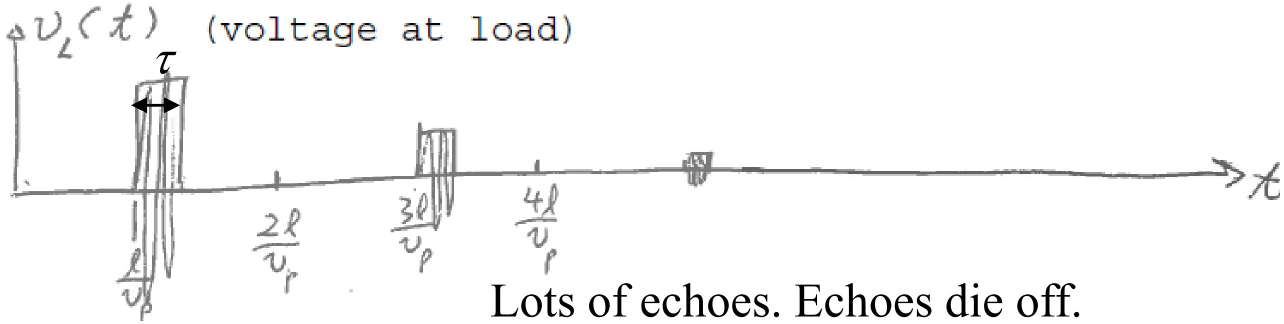


Lots of echoes. Echoes die off.



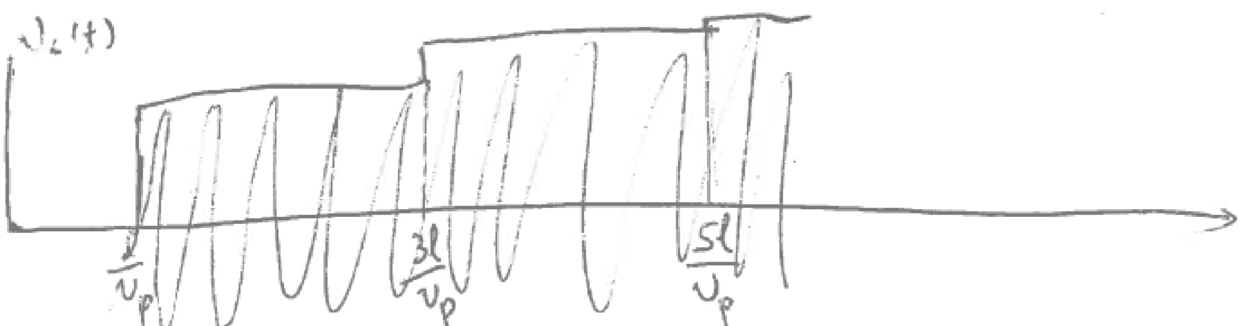
$Z_L \neq Z_0 \neq Z_g$

First case, $\tau < l/v_p$:



Lots of echoes. Echoes die off.
May corrupt other bits

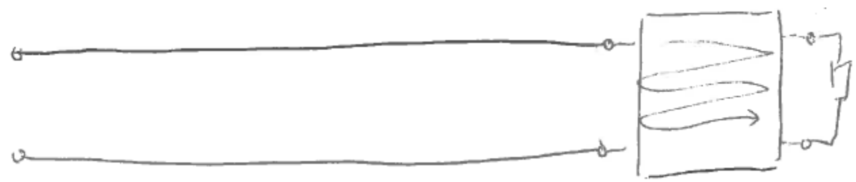
Second case, $\tau > l/v_p$:

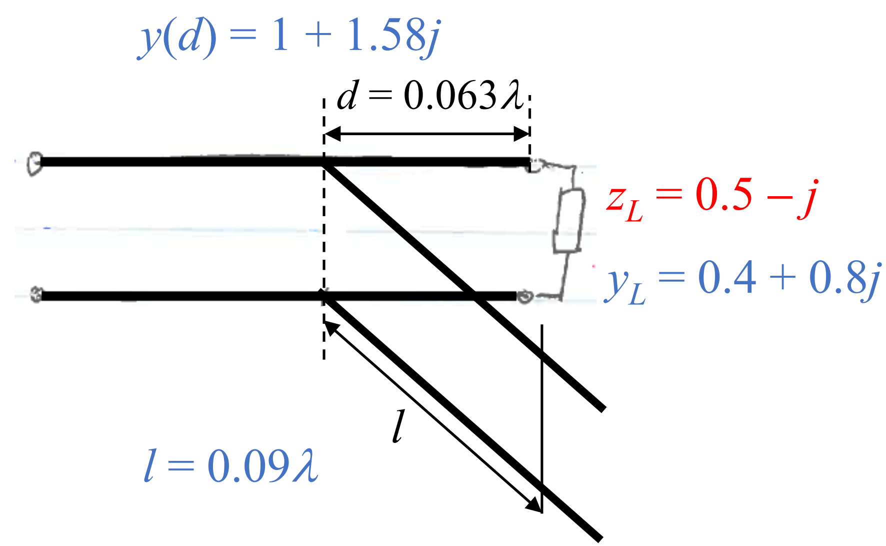


The bit is distorted and broadened.

Recall that we always have multiple reflections inside any matching network.

Does impedance matching really help us? Why?





Single stub matching example

Recall that we always have multiple reflections inside any matching network.

Does impedance matching really help us? Why?

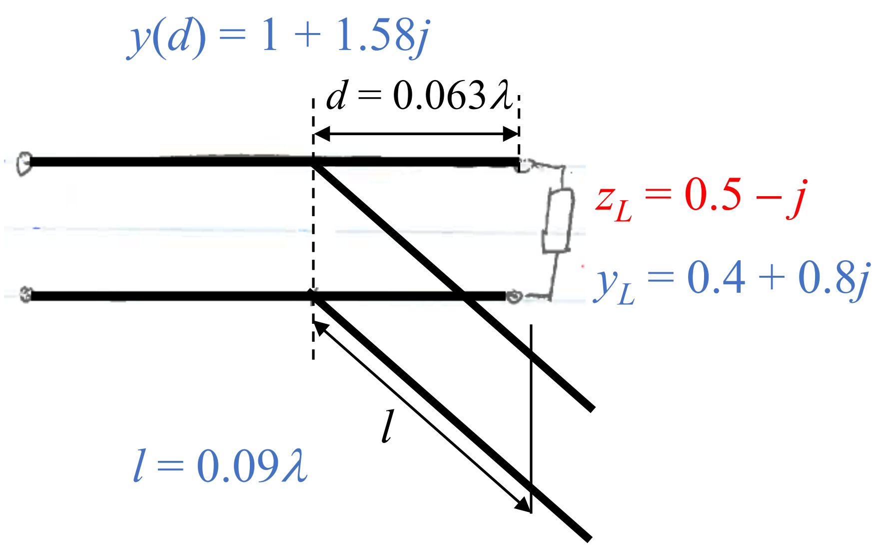
Notice that we can always choose to have $d < \lambda/2$ and $l < \lambda/2$.

The time to travel $\lambda/2$ is $1/(2f)$.

Thus the bit is broadened only by several $1/f$, at most.

Without matching, we have echoes.

Recall that we have two solutions for $d < \lambda/2$. We may want to choose the smaller d.



Single stub matching example

The modulated case is quite complicated. We now look into a simple case quantitatively.

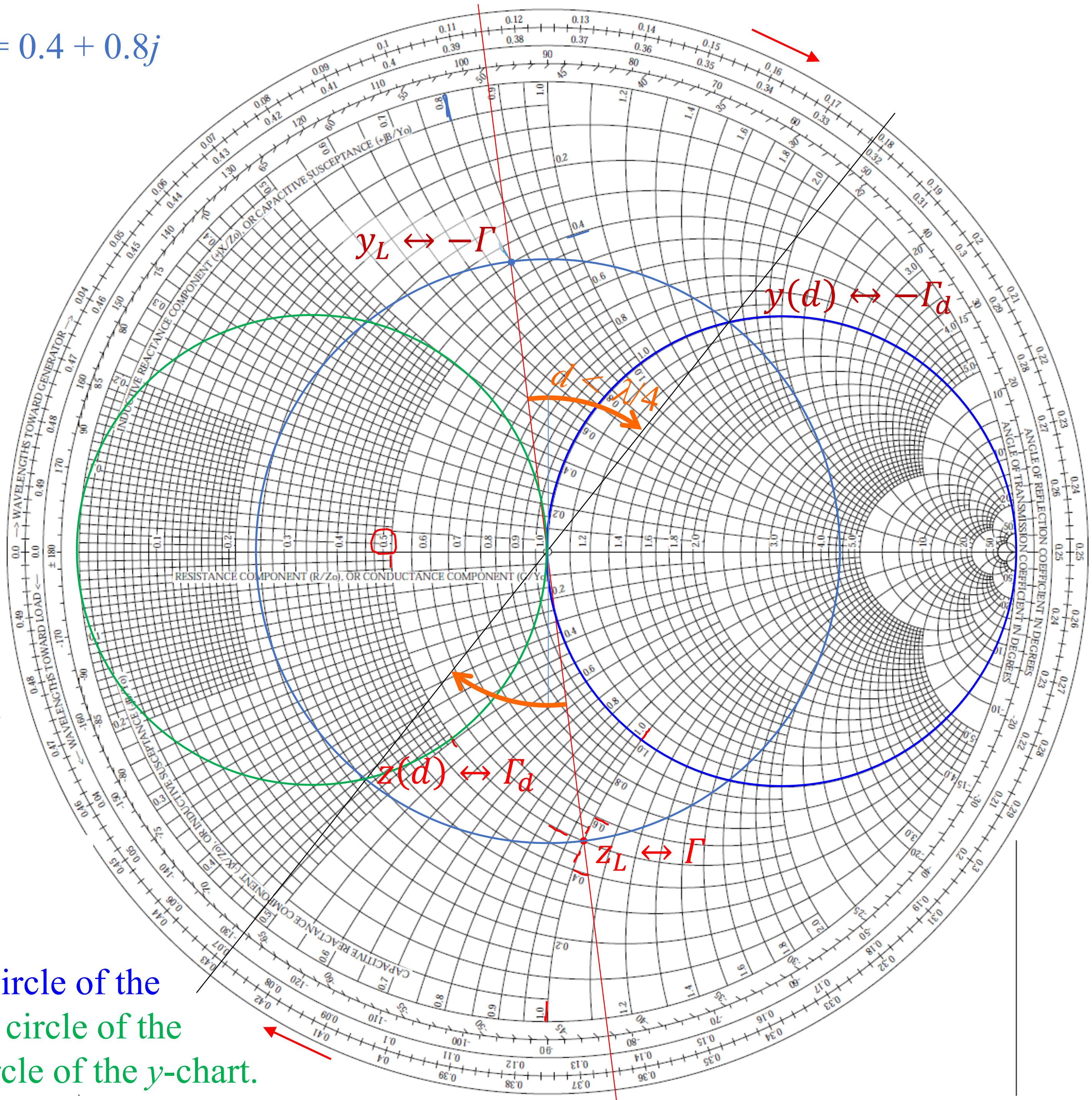
$z_L = 0.5 - j$ $\quad$ $y_L = 0.4 + 0.8j$

On this circle, $g = 1$
$y = 1 + jb$, i.e.,
$Y = Y_0 + jB$

$d = 0.178\lambda - 0.115\lambda$
$= 0.063\lambda$

$y(d) = 1 + 1.58j$

When working with $y(d)$, keep in mind that $y(d) \leftrightarrow -\Gamma_d$.



When $y(d)$ is on $g = 1$ circle of the y-chart, $z(d)$ is on $g = 1$ circle of the z-chart, i.e. the $r = 1$ circle of the y-chart.

$z_L = 0.5 - j$ $y_L = 0.4 + 0.8j$

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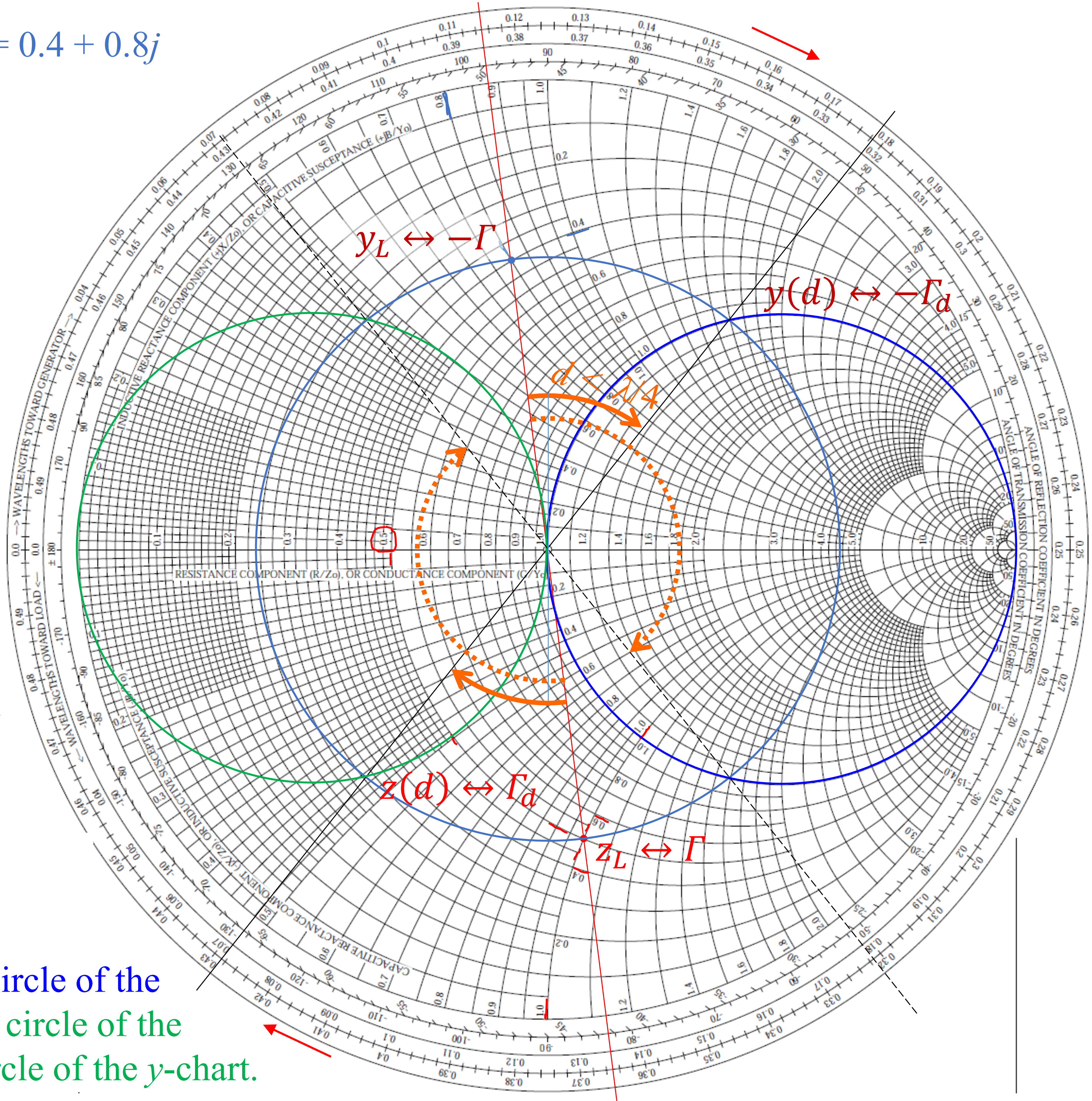


When working with $y(d)$, keep in mind that

$y(d) \leftrightarrow -\Gamma_d$.

When $y(d)$ is on $g = 1$ circle of the y-chart, $z(d)$ is on $g = 1$ circle of the z-chart, i.e. the $r = 1$ circle of the y-chart.

First, let's list the basic assumptions to be used:

1. Lossless line. Z_0 is purely real.
2. Purely resistive load $Z_L = R_L$.
3. Therefore, Γ is frequency-independent.

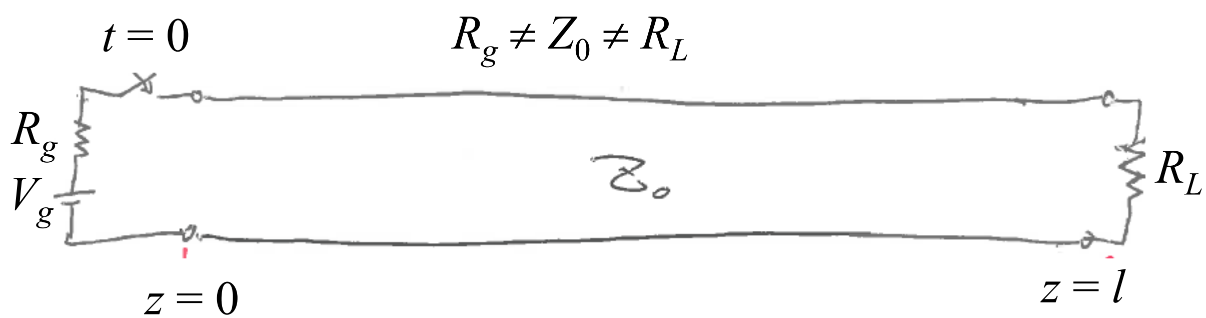
   (If $R_L = Z_0$, impedance matched for all frequencies)
4. Dispersionless: $v_g = v_p$ for all frequencies.

Know the simplifying assumptions. Know the limitations.

Propagation of a voltage step on a transmission line

For $0 < t < T = l/v_p$,

Time of a single trip

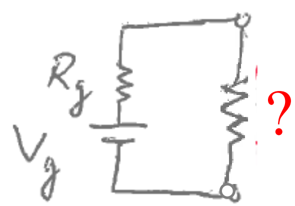


Notice change in convention. Generator at $z = 0$, load at $z = l$.

The "turn-on" event has not reached the load yet. It does not know about R_L.

The transmission line feels like infinitely long. In other words, no reflection yet.

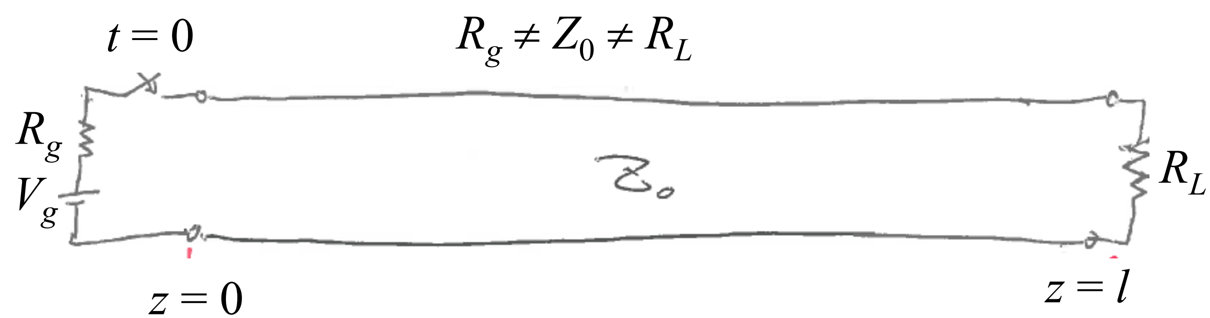
What is the equivalent input impedance seen by the incident voltage step at $z = 0$?



Propagation of a voltage step on a transmission line

For $0 < t < T = l/v_p$,

Time of a single trip



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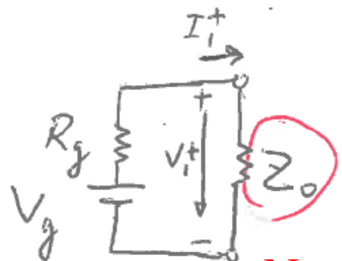


Not Z_{in}!

The equivalent input impedance seen by the incident voltage step at $z = 0$ is Z_0.

$$V_1^+ = \frac{V_g Z_o}{R_g + Z_o}$$

$$I_1^+ = \frac{V_g}{R_g + Z_o}$$

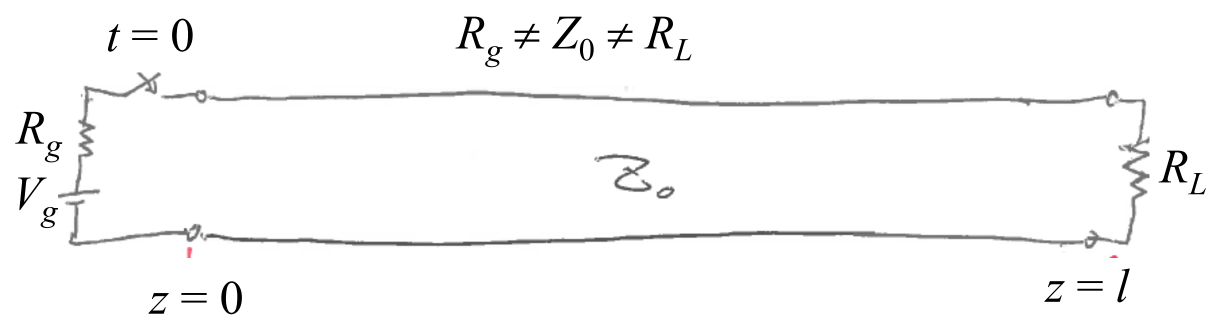
Substript "1" means the first round trip.

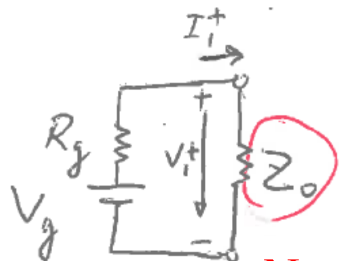
Superscript "+" means the incident direction.

Propagation of a voltage step on a transmission line

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}$$

$$I_1^+ = \frac{V_g}{R_g + Z_0}$$

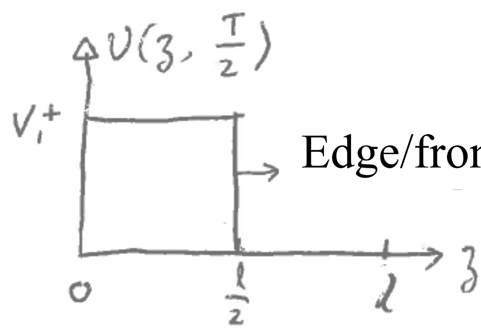




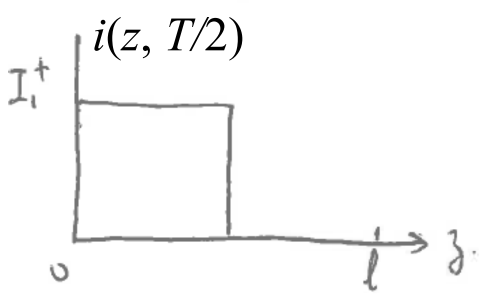
The equivalent input impedance seen by the incident pulse at $z = 0$ is Z_0.

Not Z_{in}!

Snapshots at $t = T/2$



Edge/front moving at v_p (actually v_g)



The leading edge reaches the load at $t = T$. Reflection.

$$V_1^- = \Gamma_L V_1^+$$

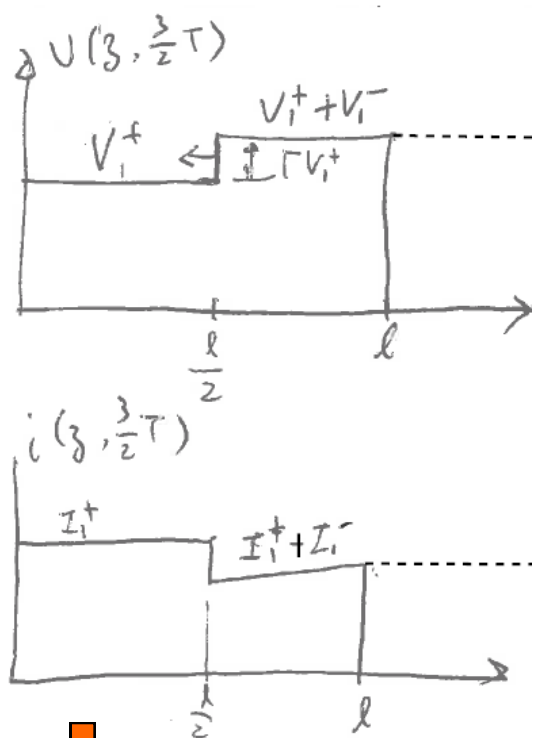
$$I_1^- = -\Gamma_L I_1^+$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

What is the voltage at the load at $t = T$?

What is the voltage at the load at $t = T$? $V_1^+ + V_1^-$

Snapshots at $t = 3T/2$



Assuming

$\Gamma_L > 0$

$\Gamma_g > 0$

At $t = 2T$, the front hits the source. Reflection.

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

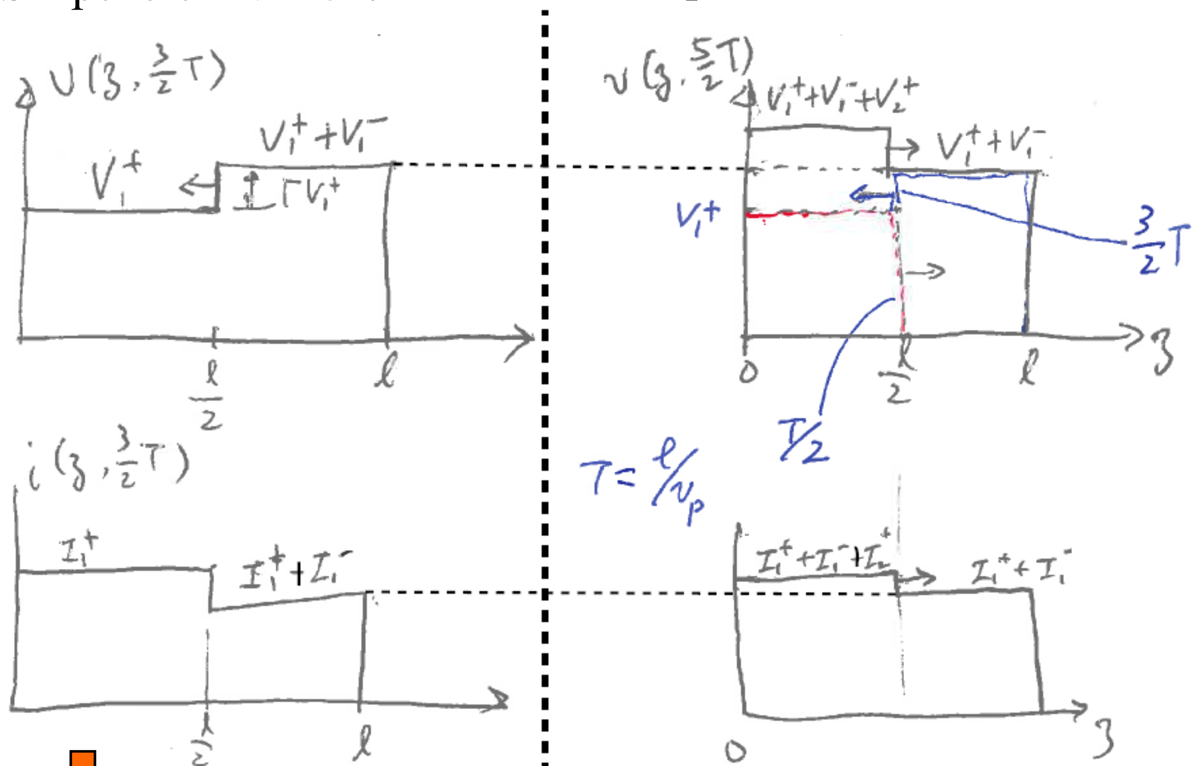
$$I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+$$

What is the voltage at the load at $t = 2T$?

We paused here on Tue 10/11/2022.

What is the voltage at the load at $t = T$? $V_1^+ + V_1^-$

Snapshots at $t = 3T/2$ | Snapshots at $t = 5T/2$



Assuming

$\Gamma_L > 0$

$\Gamma_g > 0$

At $t = 2T$, the front hits the source. Reflection.

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

$$I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+$$

What is the voltage at the load at $t = 2T$?

$V_1^+ + V_1^- + V_2^+$

At $t = 3T$, the front hits the load again.

$$V_2^- = \Gamma_L V_2^+$$

$$I_2^- = -\Gamma_L I_2^+$$

Again, notice the sign.

$$V_2^+ + V_2^- = V_2^+ (1 + \Gamma_L)$$

$$I_2^+ + I_2^- = I_2^- (1 - \Gamma_L)$$

Again, notice that reflection happens instantaneously.

It goes on and on. For the ith round trip,

$$V_i^+ + V_i^- = V_i^+ (1 + \Gamma_L)$$

$$I_i^+ + I_i^- = I_i^+ (1 - \Gamma_L)$$

$$
\begin{aligned}
v(t=\infty) &= V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots \\
&= \sum_{i=1}^{\infty} (V_i^+ + V_i^-) \\
&= V_1^+(1+\Gamma_L) + V_2^+(1+\Gamma_L) + \cdots = (1+\Gamma_L)\left[V_1^+ + V_2^+ + \cdots\right] \\
&= (1+\Gamma_L) \sum_{i=1}^{\infty} V_i^+
\end{aligned}
$$

Note: At the steady state, v is the same at all z, therefore we do not specify z.

$$v(t=\infty) = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots$$

$$= \sum_{i=1}^{\infty} (V_i^+ + V_i^-)$$

$V_i^+ + V_i^- = V_i^+ (1+\Gamma_L)$

$$= V_1^+(1+\Gamma_L) + V_2^+(1+\Gamma_L) + \cdots = (1+\Gamma_L)\left[V_1^+ + V_2^+ + \cdots\right]$$

$$= (1+\Gamma_L) \sum_{i=1}^{\infty} V_i^+$$

$$V_2^+ = \Gamma_g \Gamma_L V_1^+ \qquad V_{i+1}^+ = \Gamma_g \Gamma_L V_i^+$$

$$v(t=\infty) = V_1^+(1+\Gamma_L)\left[1 + \Gamma_g \Gamma_L + (\Gamma_g \Gamma_L)^2 + \cdots\right]$$

$$= V_1^+(1+\Gamma_L) \sum_{i=0}^{\infty} (\Gamma_g \Gamma_L)^i$$

$$v(t=\infty) = V_1^+(1+\Gamma_L) \times ?$$

$$v(t=\infty) = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots$$

$$= \sum_{i=1}^{\infty} (V_i^+ + V_i^-)$$

$V_i^+ + V_i^- = V_i^+ (1+\Gamma_L)$

$$= V_1^+(1+\Gamma_L) + V_2^+(1+\Gamma_L) + \cdots = (1+\Gamma_L)\left[V_1^+ + V_2^+ + \cdots\right]$$

$$= (1+\Gamma_L)\sum_{i=1}^{\infty} V_i^+$$

$$V_2^+ = \Gamma_g \Gamma_L V_1^+ \qquad V_{i+1}^+ = \Gamma_g \Gamma_L V_i^+$$

$$v(t=\infty) = V_1^+(1+\Gamma_L)\left[1 + \Gamma_g\Gamma_L + (\Gamma_g\Gamma_L)^2 + \cdots\right]$$

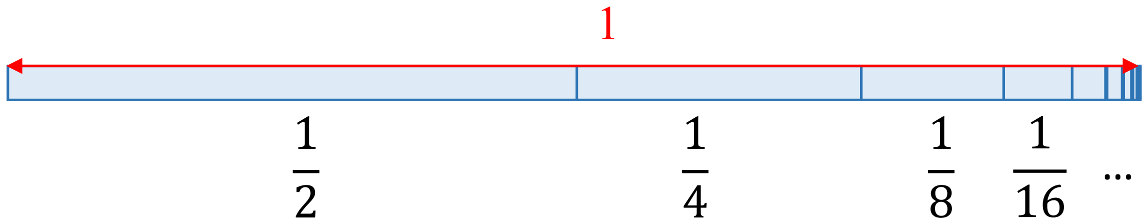
$$= V_1^+(1+\Gamma_L)\sum_{i=0}^{\infty}(\Gamma_g\Gamma_L)^i$$

Use $1 + x + x^2 + \cdots = \sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$ with $x = \Gamma_g \Gamma_L$

We get: $$v(t=\infty) = V_1^+(1+\Gamma_L)\cdot\frac{1}{1-\Gamma_g\Gamma_L}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^i} + \cdots = \sum_{i=1}^{\infty} \frac{1}{2^i} = ?$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^i}+\cdots=\sum_{i=1}^{\infty}\frac{1}{2^i}=?$$



$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^i}+\cdots=1+\sum_{i=1}^{\infty}\frac{1}{2^i}=2$$

$$=\sum_{i=0}^{\infty}\frac{1}{2^i}=\frac{1}{1-{}^{1}\!/_{2}}=2$$

$$v(t=\infty) = V_1^+ (1+\Gamma_L) \cdot \frac{1}{1-\Gamma_g \Gamma_L}$$

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

$$\Rightarrow \quad v(t=\infty) = \frac{V_g R_L}{R_g + R_L}$$

Surprising?

Similarly,

$$i(t=\infty) = I_1^+ (1-\Gamma_L) \sum_{i=0}^{\infty} (\Gamma_g \Gamma_L)^i$$

$$= I_1^+ \frac{1-\Gamma_L}{1-\Gamma_g \Gamma_L}$$

$$= \frac{V_g}{R_g + R_L}$$

We have traced $v(t)$ and $i(t)$ all the way to $t = \infty$.

That's quite tedious.

We have a graphical tool to trace this bouncing back and forth.

It's called the bounce diagram.

Review textbook Section 2-12 overview, Section 2-12.1

We went through this slide set and moved on to the next one on Thu 10/13/2022.