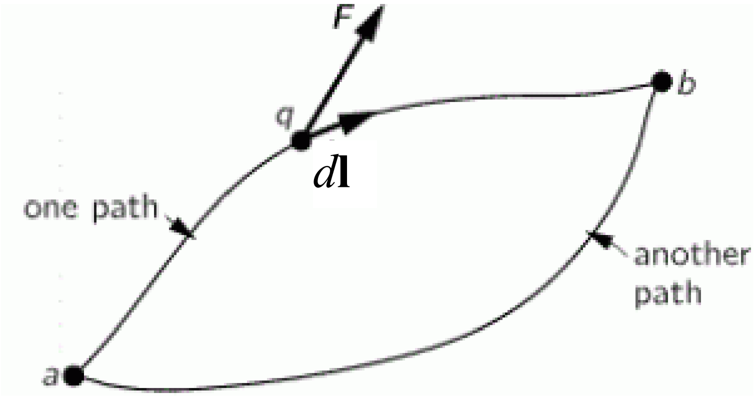
Potential

The electrostatic field Is conservative (just like gravity)



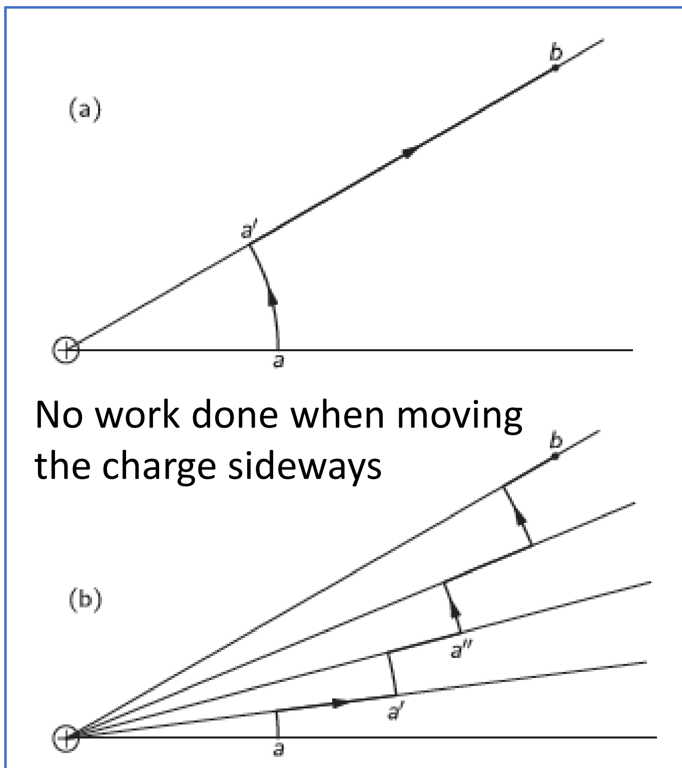
The (minimum) work done to move q from a to b:

$$W = -\int_a^b \mathbf{F} \cdot d\mathbf{l}$$

$$= -q\int_a^b \mathbf{E} \cdot d\mathbf{l}$$

The meaning of the negative sign: An external force $-\mathbf{F}$ is exerted to overcome the electrostatic force $\mathbf{F}$. Here W is the work done by the external (non-electrostatic) force.

W is independent of the path. Therefore, $\oint \mathbf{E} \cdot d\mathbf{l} = 0$



But why is W independent of the path?
Let's first consider the field of a point charge.

Because the field is radial, $\mathbf{E} = E(r)\hat{\mathbf{r}}$ (Coulomb's law) for a point charge.

By superposition, for any electrostatic field,

$$\boxed{\oint \mathbf{E} \cdot d\mathbf{l} = 0}$$

Such a field is said to be conservative.

We should point out an important fact. For any radial force the work done is independent of the path, and there exists a potential. If you think about it, the entire argument we made above to show that the work integral was independent of the path depended only on the fact that the force from a single charge was radial and spherically symmetric. It did not depend on the fact that the dependence on distance was as $1/r^2$—there could have been any r dependence.

--Richard Feynman

Side note: A non-conservative field does not violate the conservation of energy.

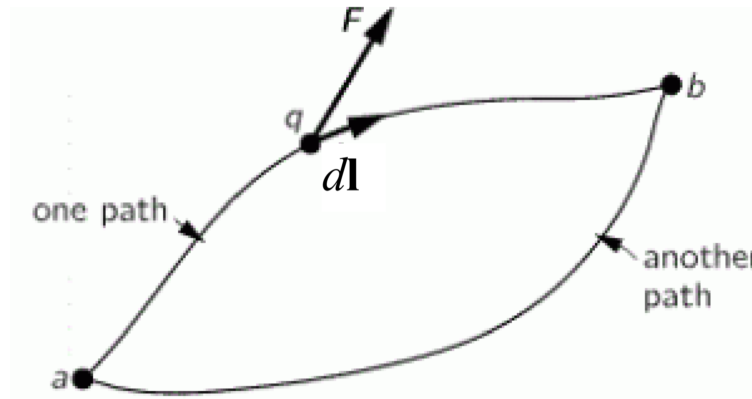
Similarly, the gravitational field is also conservative, due to the similarity between Newton's law of universal gravity and Coulomb's law.

Exceptions are found only in abstract art:

Waterfall by M.C. Escher

In the physical world, you need a pump, just as you need a battery (or the likes) for a circuit.

Since the work to be done to move a charge q from a to b is independent of the path and proportional to q, we can define a quantity called potential, just like height in the gravitational field.



$$qV_a - q\int \vec{E}\cdot d\vec{\ell} = qV_b$$

Potential at a

Work to be done

Potential at b

$$V_b - V_a = -\int \vec{E}\cdot d\vec{\ell} \quad \Leftrightarrow \quad dV = -\vec{E}\cdot d\vec{\ell}$$

$-\mathbf{E}$ is sort of the derivative of V in 3D space. In 1D, we have $dV = -E\,d\ell \Leftrightarrow E = -\dfrac{dV}{d\ell}$

Think about the infinitely large parallel plate capacitor.

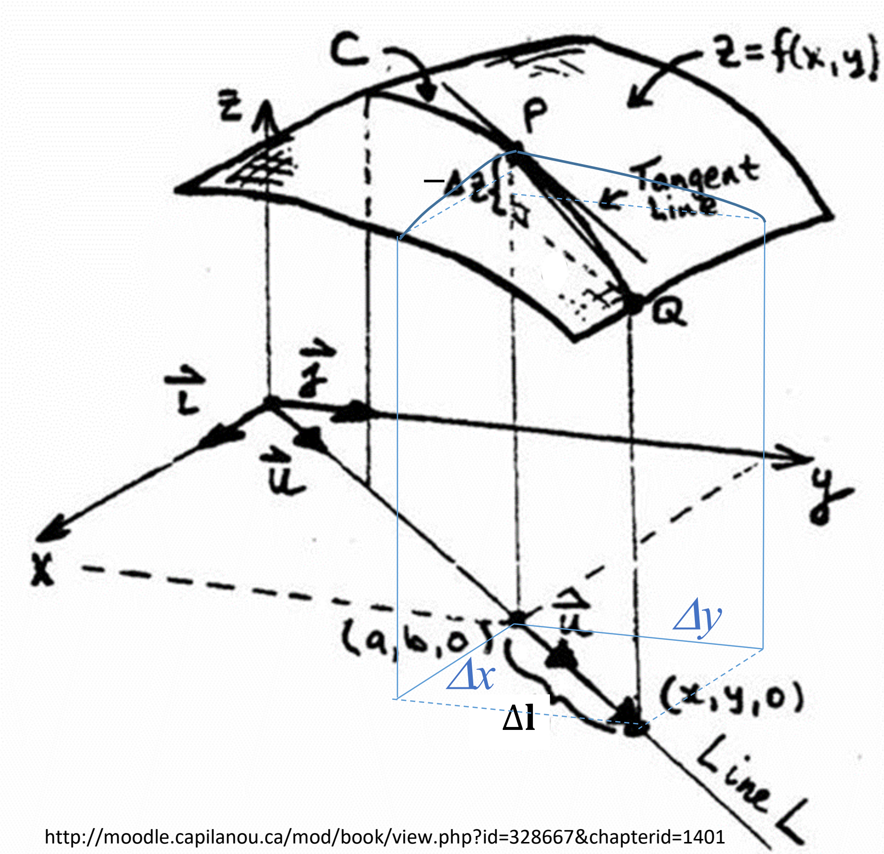
This "vector derivative" is called the gradient.

The gradient of a scalar function, or, as we call it here, a scalar field $V(x,y,z)$, is

$$\hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

The **vector sum** of the three derivatives in respective directions

Visualize the gradient in 2D



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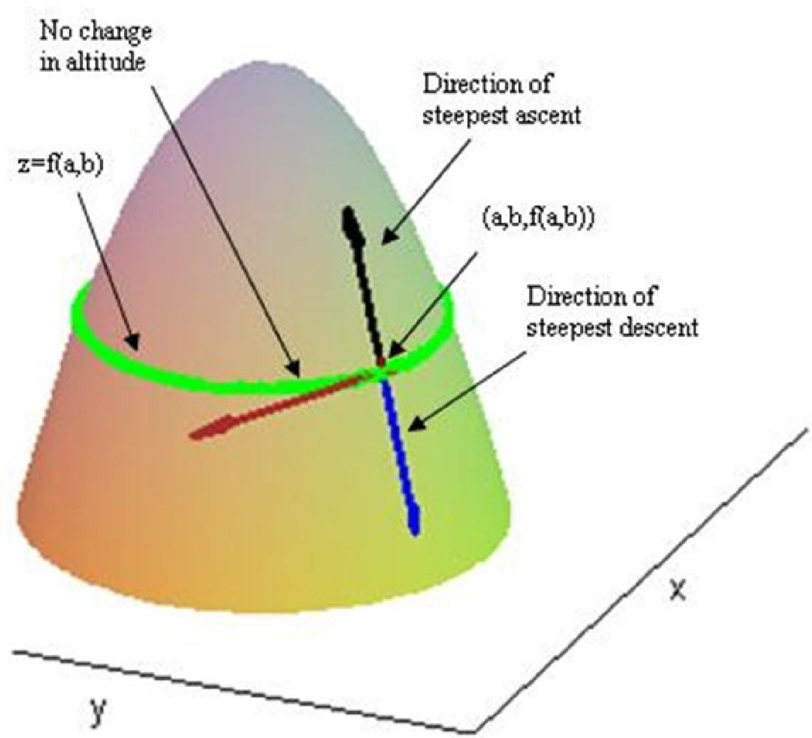

The gradient is the steepest slope.
In the direction perpendicular to the gradient, the slope is zero.

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$$\hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

The vector sum of the three derivatives in respective directions

Recall that we defined

$$\nabla = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)$$

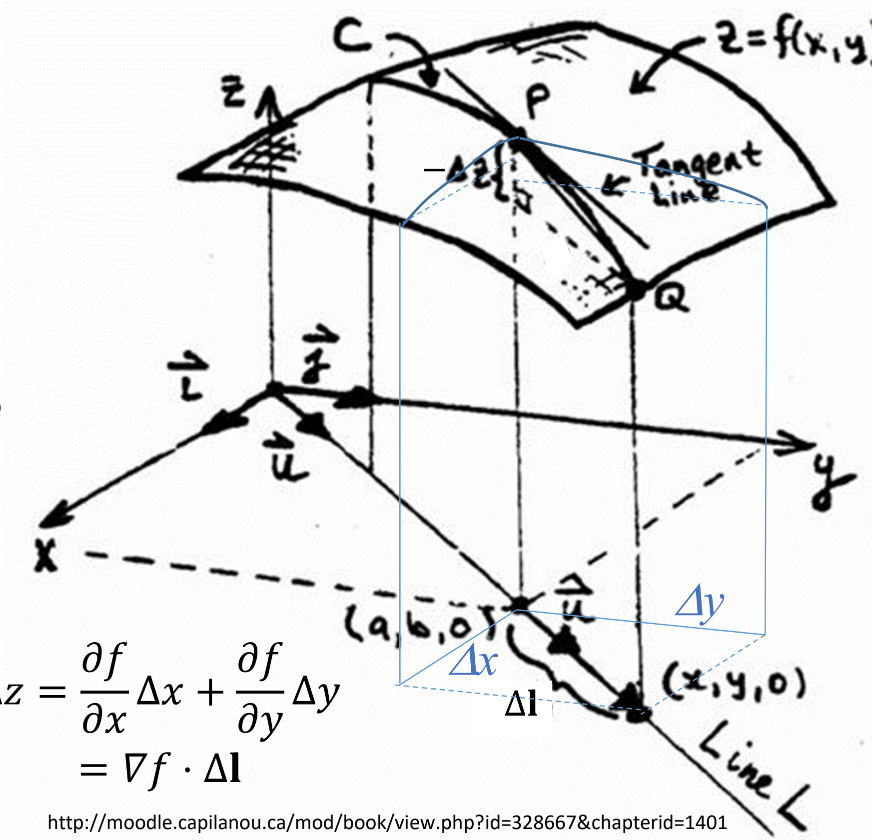
$$\therefore \nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

Thus we can write the gradient of $V(x,y,z)$ as ∇V

Thus $dV = -\vec{E}\cdot d\vec{\ell} = \nabla V \cdot d\vec{\ell}$

$$\boxed{\nabla V = -\vec{E}}$$

Visualize the gradient in 2D



$$\Delta z = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y$$
$$= \nabla f \cdot \Delta \mathbf{l}$$

The electric field is the negative gradient of the potential.

Example: potential distribution due to a point charge

$$\vec{E} = \frac{q}{4\pi\varepsilon} \frac{\hat{r}}{r^2}$$

Take a reference $V(\infty) = 0.$

Recall that no work is done if we move a probe charge (not the charge q) sideways. So move it right towards q.

Pay attention to the sign: a force $-q'$**E** need to push a probe charge q'

$$V(R) = V(\infty) - \int_{\infty}^{R} \vec{E} \cdot d\vec{r}$$

$$= \frac{-q}{4\pi\varepsilon} \int_{\infty}^{R} \frac{dr}{r^2}$$

$$= \frac{q}{4\pi\varepsilon} \frac{1}{r} \Big|_{\infty}^{R} = \frac{q}{4\pi\varepsilon R}$$

Why is the negative sign gone?

If in free space, $\varepsilon = \varepsilon_0$

Poisson's Equation

$$\left.\begin{aligned} \nabla \cdot \vec{E} &= \frac{\rho}{\varepsilon} \\ \vec{E} &= -\nabla V \end{aligned}\right\} \Rightarrow \quad \nabla \cdot (\nabla V) = -\frac{\rho}{\varepsilon}$$

Recall that $\nabla \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla^2 V \equiv \nabla \cdot (\nabla V) = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot \left(\hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}\right)$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Notice that this is a scalar

∇^2 is called the Laplacian operator.

Poisson's Equation

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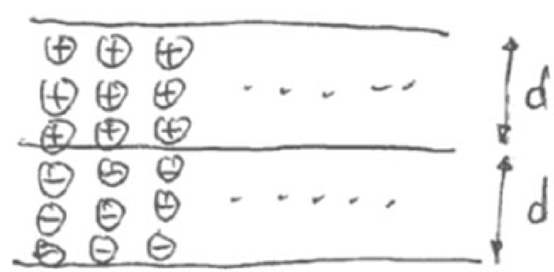
$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}.$$

$$\therefore\ \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon}$$

Recall this problem.
One way to solve it is to use the 1D Poisson's Equation. Here, 1D means there is no variation in the other two dimensions – the slabs are assumed to be infinitely large in lateral dimensions.



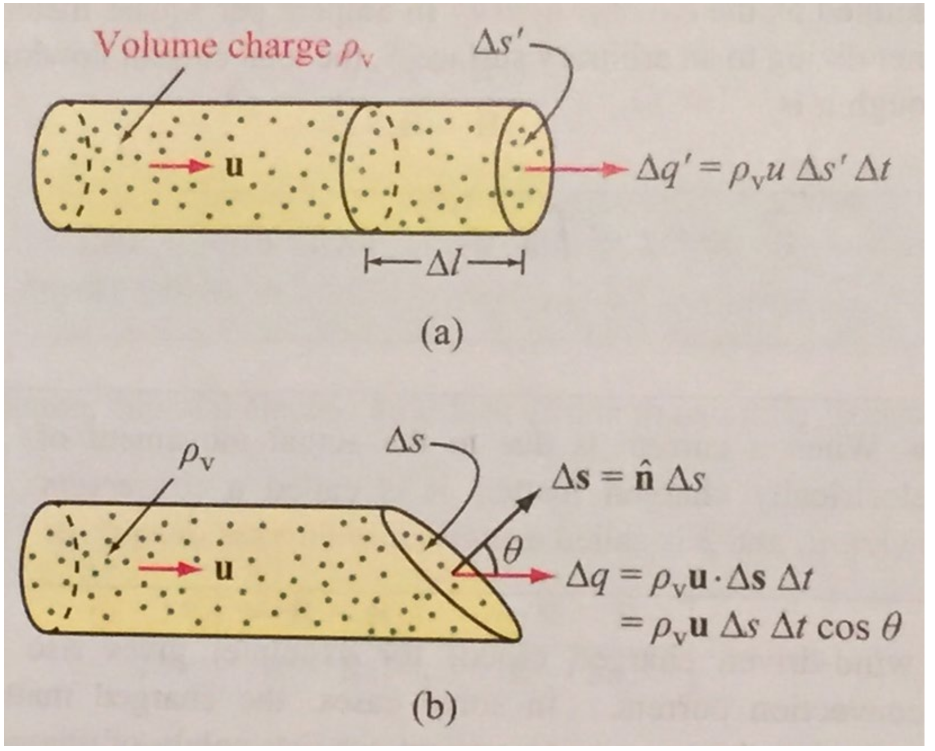
Do Problems 1, 2 of Homework 9. Read Sections 4-5 and 3-4 of textbook. Continue working Chapter 3.

Current & Ohm's Law

Let's first consider the current w/o asking what drives it. (The kinematics of current)

Closely read these notes and Section 4-2.2 for details.

Conductor w/ mobile charge density ρ_V, or just ρ for short.



What are the mobile charge carriers in metals?

Usually the overall conductor is charge neutral.

Mobile charge carriers move at an average net velocity $\mathbf{u}$.

Special case in Fig. (a):

$$\Delta I = \frac{\Delta q'}{\Delta t} = \rho u \Delta s'$$

$$J = \frac{\Delta I}{\Delta s'} = \rho u = -neu$$

where n is the mobile electron density and e the electron charge.

General case in (b): $\Delta I = \dfrac{\Delta q}{\Delta t} = \rho \mathbf{u} \cdot \Delta \mathbf{s} = \mathbf{J} \cdot \Delta \mathbf{s}$

$$\mathbf{J} = \rho \mathbf{u}$$

The unit of J is....?

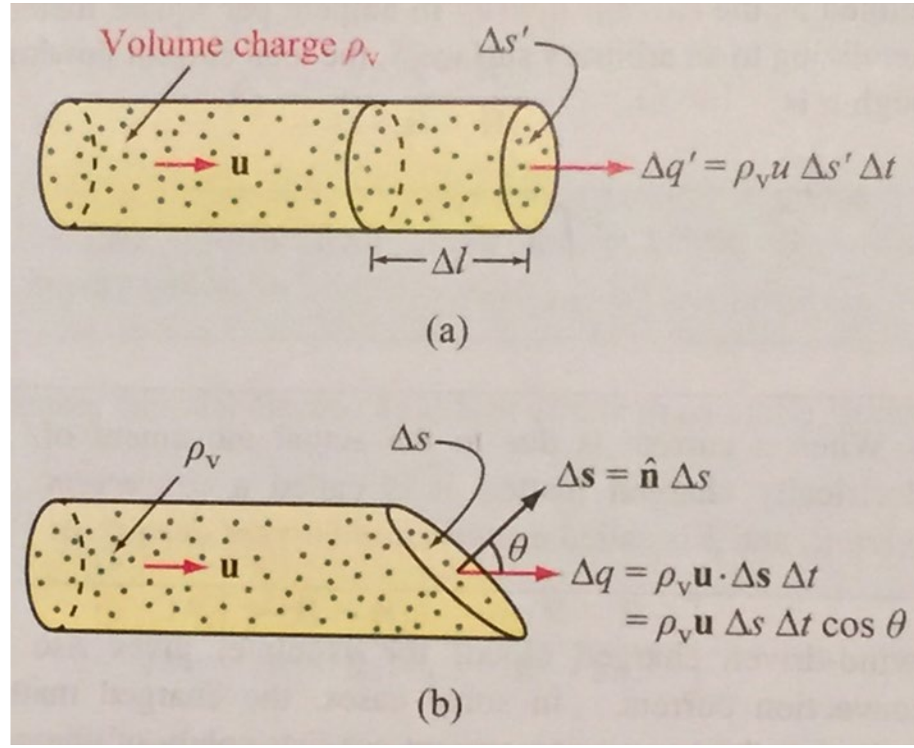
$$\text{A/m}^2 = (\text{C/m}^3)(\text{m/s}) = (\text{C/s})/\text{m}^2$$

$$J = \frac{\Delta I}{\Delta s'} = \rho u = -neu$$

More generally, $\mathbf{J} = \rho \mathbf{u}$

The unit of J is....

A/m^2 = (C/m^3)(m/s) = (C/s)/m^2



For an arbitrary surface S (not necessarily planar),

$$I = \int_S \mathbf{J} \cdot d\mathbf{s}$$

What about a closed surface? $\oint_S \mathbf{J} \cdot d\mathbf{s} = ???$

$$\oint_S \mathbf{J} \cdot d\mathbf{s} = 0$$ for an arbitrary closed surface.

Kirchhoff's current law (KCL)!

Closely review these notes and textbook Section 4-2.2.

Recall that $\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$ for any arbitrary closed contour C.

What does this correspond to in circuit theory?

Kirchhoff's voltage law (KVL)

(Not exactly, if we define voltage just as electrostatic potential difference between two points. A circuit needs things like batteries. We will talk about this later.)

Now, back to the current – the dynamics of it, in semiconductors.

The average net velocity $\mathbf{u}$ is often called the drift velocity ($\mathbf{v}_\mathrm{d}$), as it's driven by the field $\mathbf{E}$: $\mathbf{v}_\mathrm{d} = \mu\mathbf{E}$

μ: a proportional constant (material property) called "mobility"

But, think about it. $\mathbf{E} = \mathbf{F}/q$

Charge of the carrier. $-e$ for the electron.
In semiconductor and circuit books, q stands for e.

$$\mathbf{v}_\mathrm{d} \propto \mathbf{F}$$

Does this contradict Newton's second law?

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Does this contradict Newton's second law?

Here is roughly what happens:

On average, a charge carrier collides with something every time interval τ. It loses/forgets its $\mathbf{v}_d$, or "randomizes". Then it starts over. Therefore, $\mathbf{v}_d = \mathbf{a}\tau$. By Newton's second law, $\mathbf{a} \propto \mathbf{F} \propto \mathbf{E}$ thus $\mathbf{v}_d \propto \mathbf{E}$.

$$\mathbf{J} = \rho \mathbf{u} = nq\mathbf{v}_d = nq\mu \mathbf{E} = \sigma \mathbf{E}$$

charge carrier density (unit?)

charge density, not resistivity here

Conductivity $\sigma = nq\mu$

In a semiconductor, you may have both electrons and holes, carrying $-e$ and $+e$ each, respectively.

$$\mathbf{J} = -ne\mathbf{v}_{\mathrm{e}} + pe\mathbf{v}_{\mathrm{h}} = -ne(-\mu_e\mathbf{E}) + pe\mu_h\mathbf{E} = (n\mu_e + p\mu_h)e\mathbf{E} \equiv \sigma\mathbf{E}$$

hole density

electron density

or, in the simple scalar form

$$J = nev_{\mathrm{e}} + pev_{\mathrm{h}} = (n\mu_e + p\mu_h)eE \equiv \sigma E$$

For metals, the concept of mobility is not very useful.
There are so many free electrons that only the highest-energy ones contribute to conduction. (Detailed physics way beyond this course)
But Ohm's law holds.

For a metal wire or a semiconductor channel of length l and cross section area A,

resistance $R = (1/\sigma)(l/A)$

Read textbook: Section 4-6

(More) Boundary Conditions

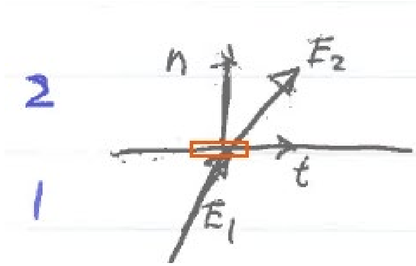
Boundaries between different media/materials

Let's look at the boundary between two materials in general.

In each material, we decompose the electric field into normal (n) and tangential (t) components.

Imagine a tiny rectangular loop, Δl long and with a zero width, but with the two Δl long sides on opposite sides of the boundary.

The boundary is not necessarily planar.



Recall that the electric field is conservative: $\oint \vec{E} \cdot d\vec{l} = 0$

Therefore, $E_{2t} \Delta l - E_{1t} \Delta l = 0 \Rightarrow \boxed{E_{2t} = E_{1t}}$

Special case: medium 1 is a perfect conductor:

$E_1 = 0 \Rightarrow E_{1t} = 0 = E_{2t} = 0$

The electric field at a perfect conductor surface must be perpendicular to the surface.