Dynamic Fields, Maxwell's Equations (Chapter 6)

So far, we have studied static electric and magnetic fields. In the real world, however, nothing is static. Static fields are only approximations when the fields change very slowly, and "slow" is in a relative sense here.

To really understand electromagnetic fields, we need to study the dynamic fields. You will see the electric & magnetic fields are coupled to each other.

Four visual pictures to help you understand the four Maxwell's equations

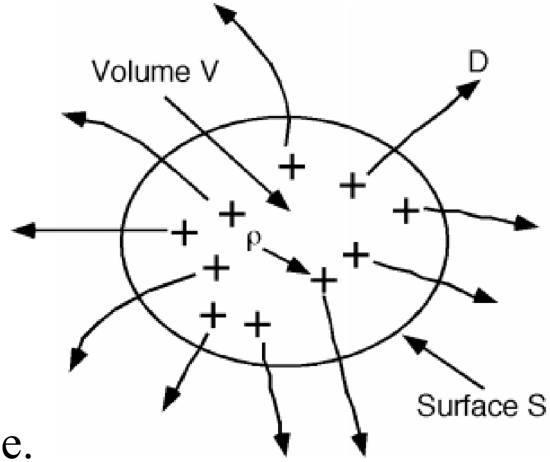
Two remain the same for dynamic and static fields. Two are different.

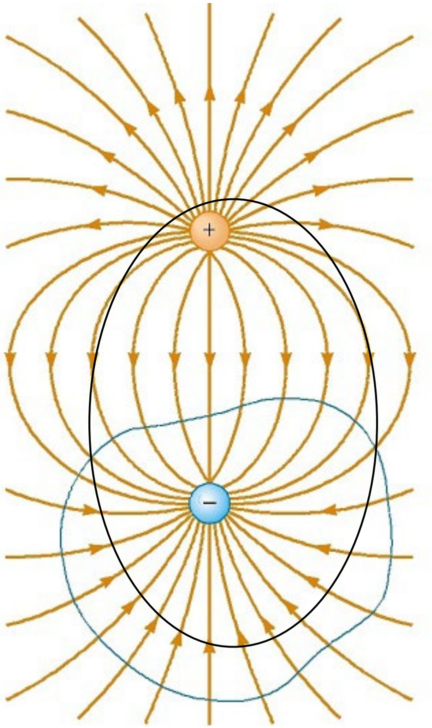
(1)

$$\oint \mathbf{E} \cdot d\mathbf{s} = \int \frac{\rho}{\varepsilon} dV = \frac{Q}{\varepsilon}$$

$$\oint \mathbf{D} \cdot d\mathbf{s} = \int \rho dV = Q$$

$$\varepsilon \nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{D} = \rho$$





ρ and Q represent external charge.

This holds for dynamic fields even when ρ changes with time.

Question: how can ρ change with time?

FYI

$$\oint \mathbf{E}\cdot d\mathbf{s} = \int \frac{\rho}{\varepsilon} dV = \frac{Q}{\varepsilon}$$

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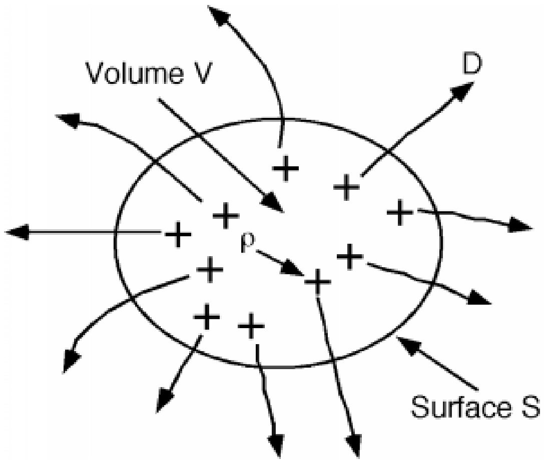
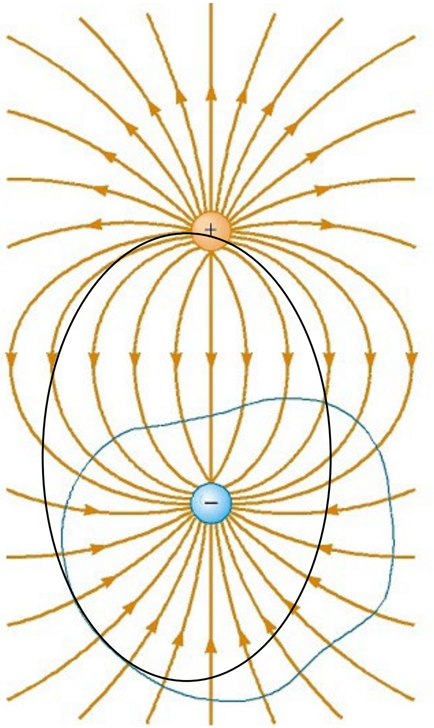
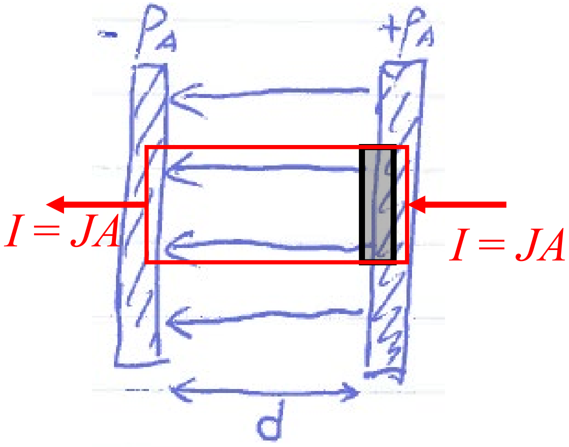
This holds for dynamic fields even when ρ changes with time.

Question: how can ρ change with time?





Consider a charging capacitor. The current density in the plates is **J**. Imagine a cylinder spanning both plates of top/bottom surface area A.



For this cylinder, $\oint_S \mathbf{J}\cdot d\mathbf{s} = 0$

But what about the **black cylinder**?

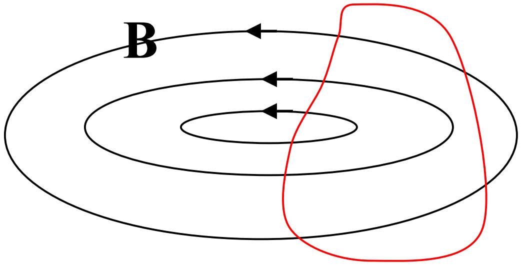
$$\oint \mathbf{E}\cdot d\mathbf{s} = \int \frac{\rho}{\varepsilon} dV = \frac{Q}{\varepsilon} \Rightarrow \varepsilon \frac{\partial}{\partial t} \oint \mathbf{E}\cdot d\mathbf{s} = \frac{dQ}{dt} \Rightarrow$$

$$\boxed{\varepsilon A \frac{dE}{dt}} = \frac{dQ}{dt} = I$$

surface inside the dielectric

Define displacement current density $\frac{\partial \mathbf{D}}{\partial t} = \frac{\partial}{\partial t}(\varepsilon \mathbf{E})$, then $\oint_S \mathbf{J}\cdot d\mathbf{s} = 0$ holds for all closed surfaces by including displacement currents.

(2) $\oint \mathbf{B}\cdot d\mathbf{s} = 0$ $\nabla\cdot\mathbf{B} = 0$



What goes in must come out: no such thing as a magnetic charge. Always true, static or dynamic.

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(3) The electrostatic field is conservative

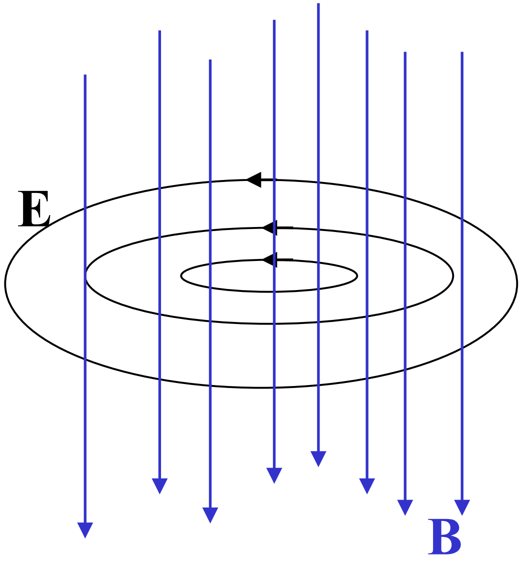
$$\oint \mathbf{E}\cdot d\mathbf{l} = 0 \qquad \nabla\times\mathbf{E} = 0$$

This is why we can define "potential."

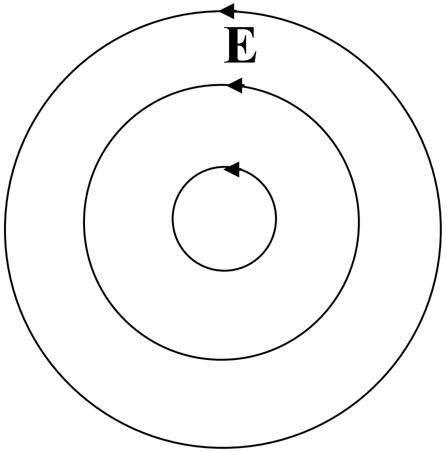
Faraday's law:

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} \qquad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Pay attention to this negative sign.



This electric field induced by a changing magnetic field is not conservative! It's **not** an "electrostatic field" even when $\frac{\partial \mathbf{B}}{\partial t}$ is a constant. DC is not necessarily electrostatic.

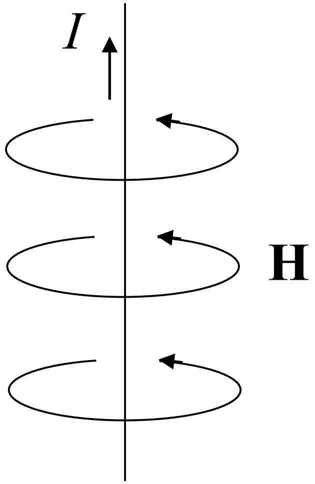


Cannot define a potential!

(4) Ampere’s law (static)

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int \mathbf{J} \cdot d\mathbf{S} = I$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$



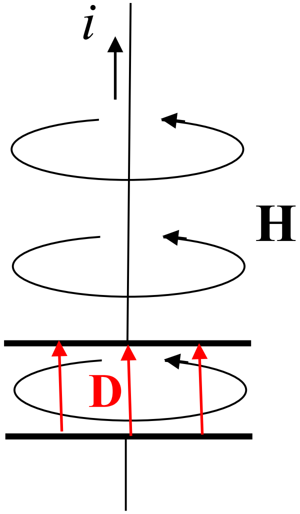
Ampere’s law (dynamic)

Displacement current density

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) \cdot d\mathbf{S} = I + \boxed{\int \boxed{\frac{\partial \mathbf{D}}{\partial t}} \cdot d\mathbf{S}}$$

Displacement current

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$



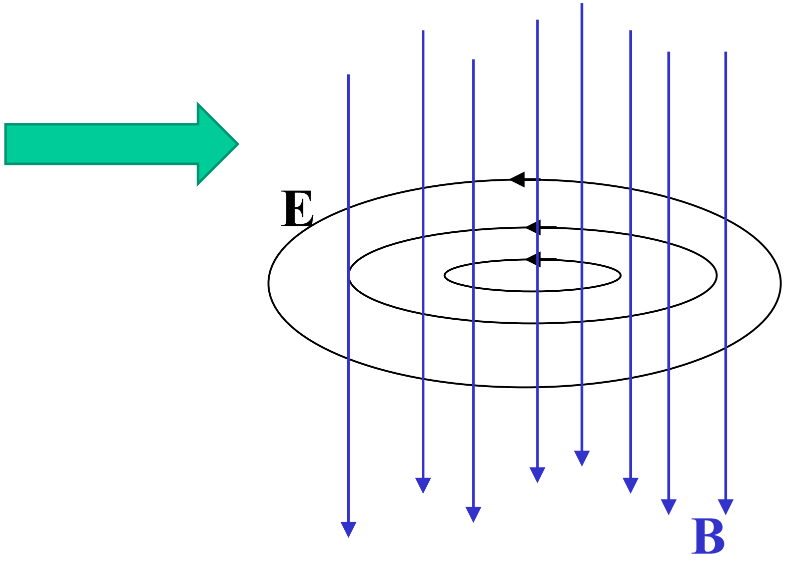
We have covered the static case in pretty much detail. Here, in the dynamic case the current could include the displacement current.

(3) and (4) are about the coupling between E & M fileds. They are the foundations of electromagnetic waves, to be discussed in Ch. 7.

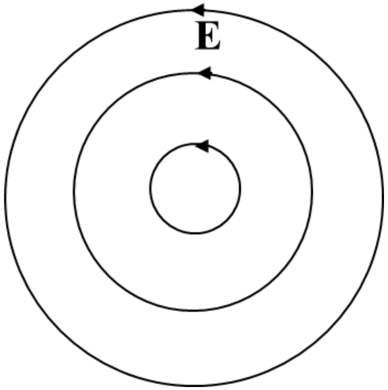
In Chapter 6, we focus on Eq. (3), Faraday's law:

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} \quad \Longleftrightarrow \quad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Pay attention to this negative sign.



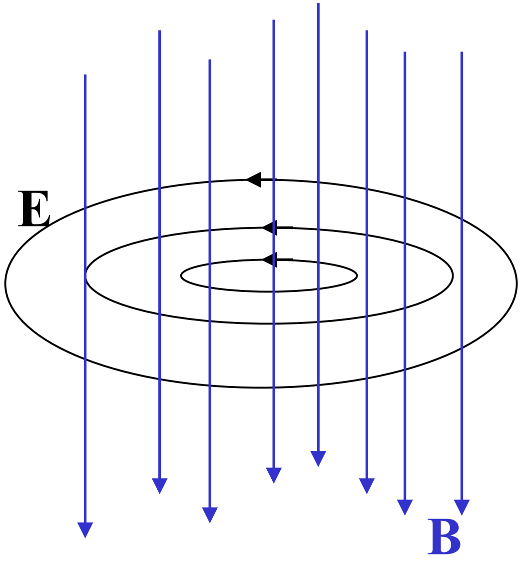
Plan view:



In Chapter 6, we focus on Eq. (3), Faraday's law:

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} \iff \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Pay attention to this negative sign.



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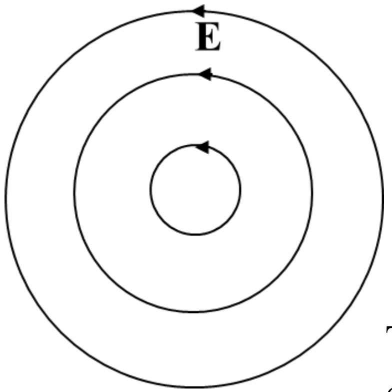
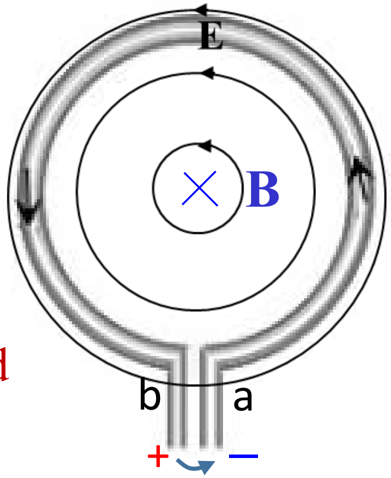


Place a wire loop in this electric field. It will drive a current.



The non-electrostatic field establishes an electrostatic potential difference $V_b - V_a$

$\mathbf{E}_{\text{stat}}$: external electrostatic field

Recall the following:

A potential energy difference QV results from a non-electrostatic force $\mathbf{F}_{\text{nes}} = -Q\mathbf{E}_{\text{stat}}$ doing work to charge Q against the electrostatic force $Q\mathbf{E}_{\text{stat}}$:

$$QV = \int_a^b \mathbf{F}_{\text{nes}}\cdot \text{d}\mathbf{l} = -Q\int_a^b \mathbf{E}_{\text{stat}}\cdot \text{d}\mathbf{l}$$

Here, $\mathbf{F}_{\text{nes}} = Q\mathbf{E}$ — Non-electrostatic!

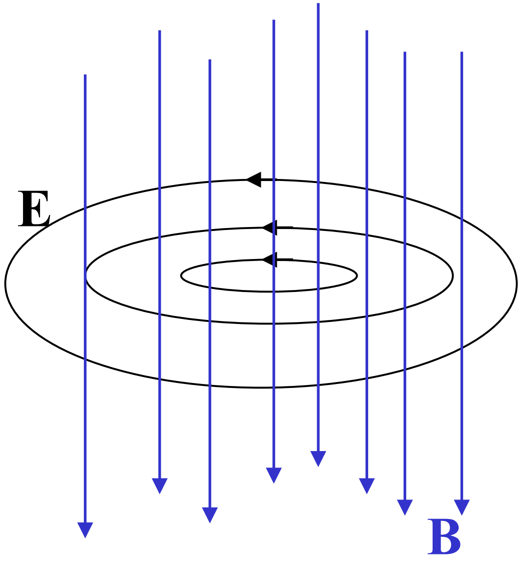
$$V = \int_a^b \frac{1}{Q}\mathbf{F}_{\text{nes}}\cdot \text{d}\mathbf{l} = \int_a^b \mathbf{E}\cdot \text{d}\mathbf{l}$$

$$= -\int_a^b \mathbf{E}_{\text{stat}}\cdot \text{d}\mathbf{l}$$

In Chapter 6, we focus on Eq. (3), Faraday's law:

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} \quad \Longleftrightarrow \quad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Pay attention to this negative sign.



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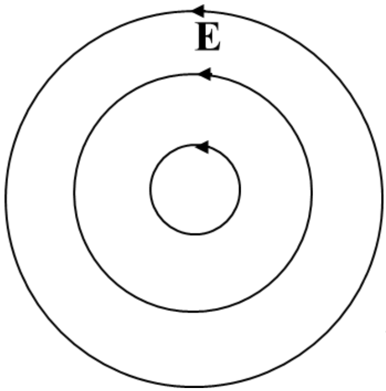
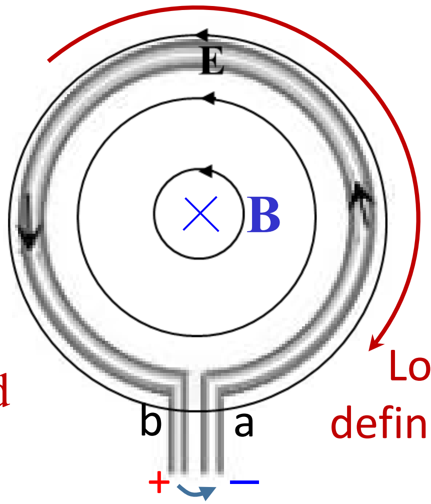


Place a wire loop in this electric field. It will drive a current.

The non-electrostatic field establishes an electrostatic potential difference $V_b - V_a$



Loop integral direction defined by direction of d**B**/dt

$\mathbf{E}_{stat}$: external electrostatic field

Recall the following:

A potential energy difference QV results from a non-electrostatic force $\mathbf{F}_{nes} = -Q\mathbf{E}_{stat}$ doing work to charge Q against the electrostatic force $Q\mathbf{E}_{stat}$:

$$QV = \int_a^b \mathbf{F}_{nes}\cdot d\mathbf{l} = -Q\int_a^b \mathbf{E}_{stat}\cdot d\mathbf{l}$$

This gap is so small that $\int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -\oint \mathbf{E}\cdot d\mathbf{l}$

This "voltage" is due to the non-electrostatic field. It is an "electromotive force." Just like that of a battery, which is due to chemistry.

$$V_b - V_a = V_{emf} = \text{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -\oint \mathbf{E}\cdot d\mathbf{l}$$

In Chapter 6, we focus on Eq. (3), Faraday's law:

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} \quad \Longleftrightarrow \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

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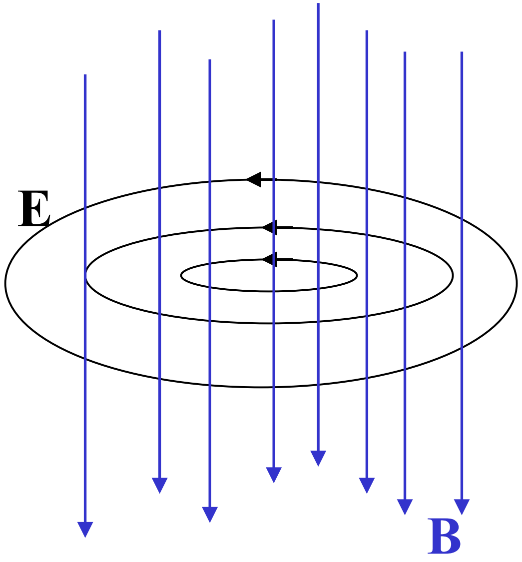

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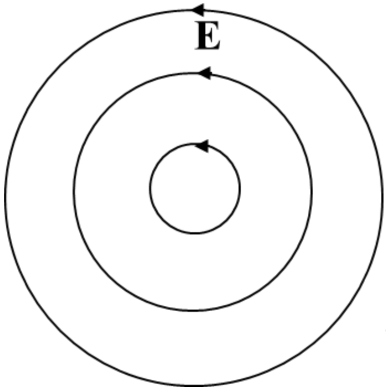

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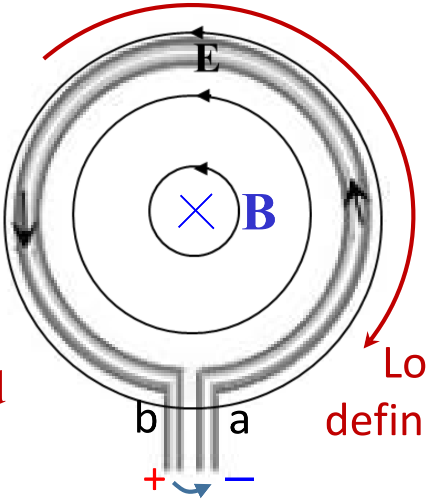
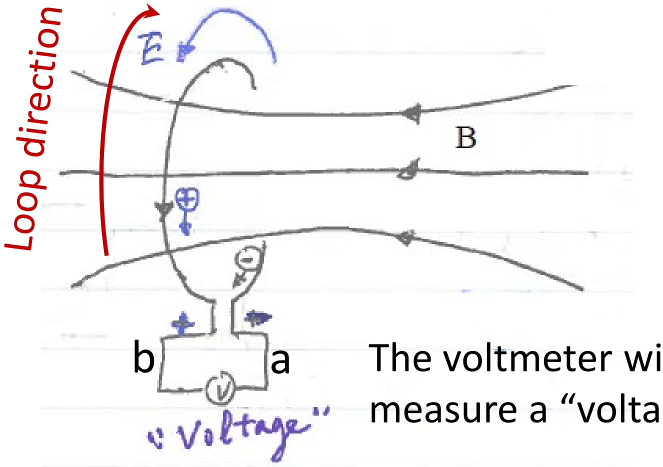


Loop integral direction defined by direction of d**B**/dt

The non-electrostatic field establishes an electrostatic potential difference $V_b - V_a$

$\mathbf{E}_{stat}$: external electrostatic field

Viewed from another perspective:



The voltmeter will measure a "voltage"

This gap is so small that $\int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -\oint \mathbf{E}\cdot d\mathbf{l}$

This "voltage" is due to the non-electrostatic field. It is an "electromotive force." Just like that of a battery, which is due to chemistry.

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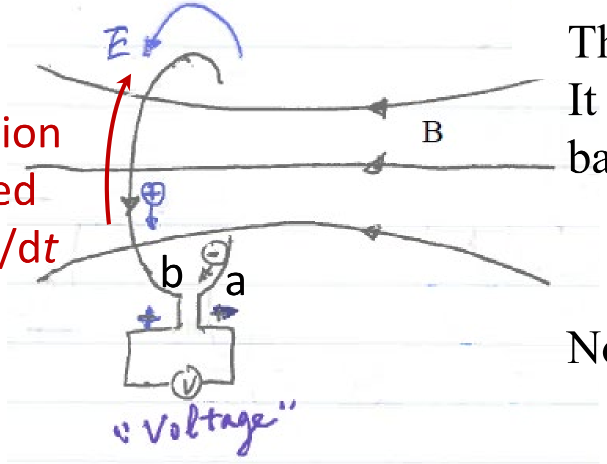


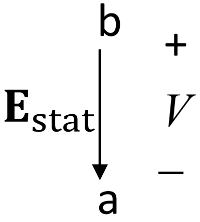
Loop direction defined by d**B**/dt

This "voltage" is due to the non-electrostatic field. It is a "electromotive force." Just like that of a battery, which is due to chemistry.

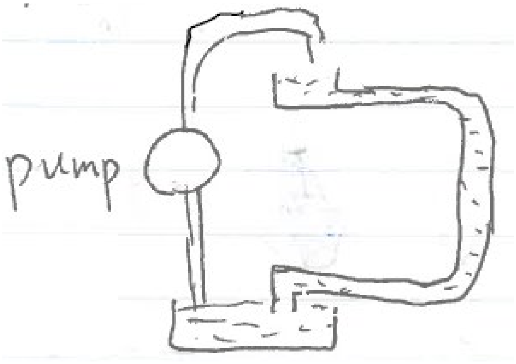
$$V_b - V_a = V_{emf} = \text{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -\oint \mathbf{E}\cdot d\mathbf{l}$$

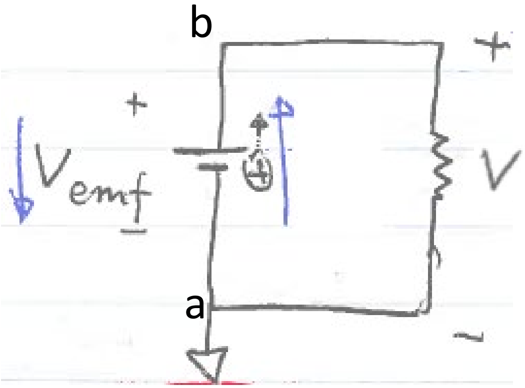
Notice that for an electrostatic field

$$V_b - V_a = V = -\int_a^b \mathbf{E}_{stat} d\mathbf{l} \Rightarrow$$



Let's use an analogy to explain the "subtle" difference between an emf and a voltage:





$$V_b - V_a = V$$
$$= V_{emf} = \text{emf}$$

The pump works against gravity.

The battery works against the electrostatic force.

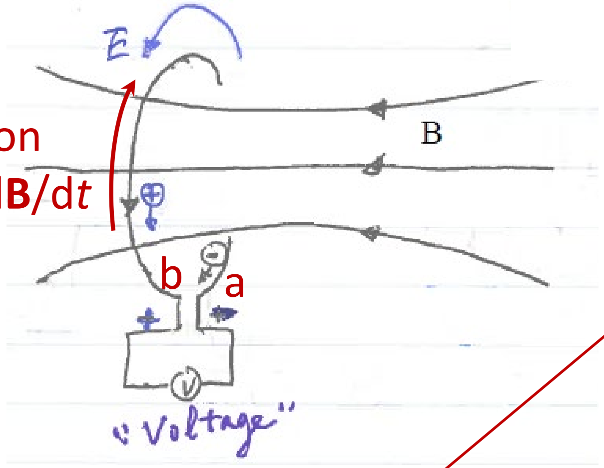
Generally, inside the source (e.g. battery), emf $= \frac{1}{Q}\int_a^b \mathbf{F}_{nes}\cdot d\mathbf{l}$, where $\mathbf{F}_{nes}$ is the non-electrostatic force acting on charge carrier Q, and $V = -\frac{1}{Q}\int_a^b \mathbf{F}_{stat}\cdot d\mathbf{l} = -\int_a^b \mathbf{E}_{stat}\cdot d\mathbf{l}$, where $\mathbf{F}_{stat}$ is the electrostatic force.

Lecture of Tue 11/15/2022 ends here.

$$V_b - V_a = V_{\text{emf}} = \text{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} = -\int_{ⓑ}^{ⓐ} \mathbf{E}\cdot d\mathbf{l} \approx \boxed{-\oint \mathbf{E}\cdot d\mathbf{l} = \int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}}$$

$$= \frac{d}{dt}\int B\left(\hat{\mathbf{B}}\cdot\hat{\mathbf{n}}\right)dS$$

$$= \frac{d\Phi}{dt} \approx \frac{d}{dt}\int B\,dS$$



Loop direction defined by d**B**/d*t*

Define magnetic flux $\Phi = \int \vec{B}\cdot d\vec{S}$

B is therefore called the "magnetic flux density."

Unit of Φ: Wb = T m^2
(weber)

Meaning of the negative sign:

Nominally positive direction of **B** defines direction of d**S**, such that Φ is positive when dB/dt > 0.

The negative sign signifies that the direction of **E** is **against the right hand rule**. This is **the key** to get the correct voltage polarity.

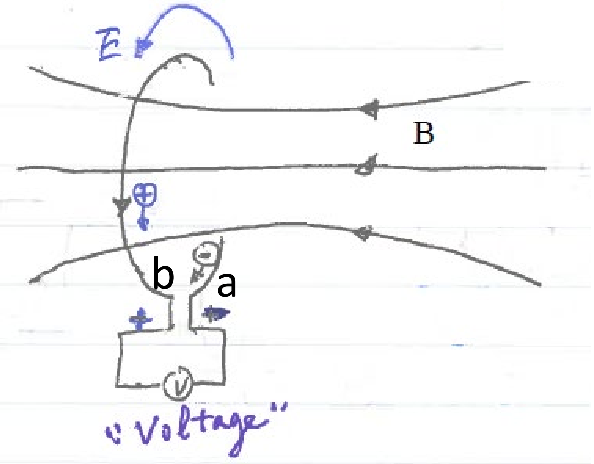
$$\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$$

The direction of d**S** (defined by **B**), defines the direction in which the loop integral is taken: from b to a along the loop (rather than across the gap).

But we define the voltage as $V_b - V_a$.

This sign convention is more consistent than that used in the book.
We do not need to carry the negative sign before $d\Phi/dt$, e.g. in Eq. (6.8).

$$V_b - V_a = V_{emf} = \mathrm{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -\oint \mathbf{E}\cdot d\mathbf{l} = \int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$$



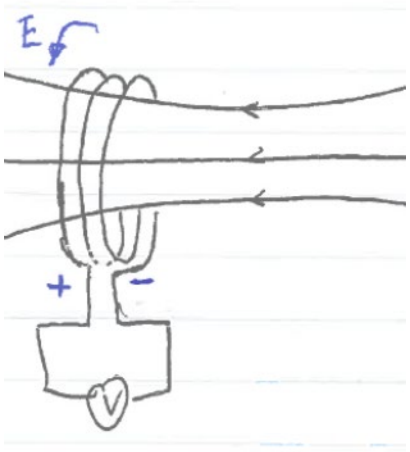
$$= \frac{d}{dt}\int B\left(\hat{\mathbf{B}}\cdot\hat{\mathbf{n}}\right)dS = \frac{d\Phi}{dt} \approx \frac{d}{dt}\int B\,dS$$

Define magnetic flux $\Phi = \int \vec{B}\cdot d\vec{S}$

B is therefore called the "magnetic flux density."

Unit of Φ: $\mathrm{Wb} \doteq \mathrm{T\,m^2}$
(weber)

We may have a coil of N turns :



$$V_{emf} = \mathrm{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -N\oint \mathbf{E}\cdot d\mathbf{l} = N\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$$

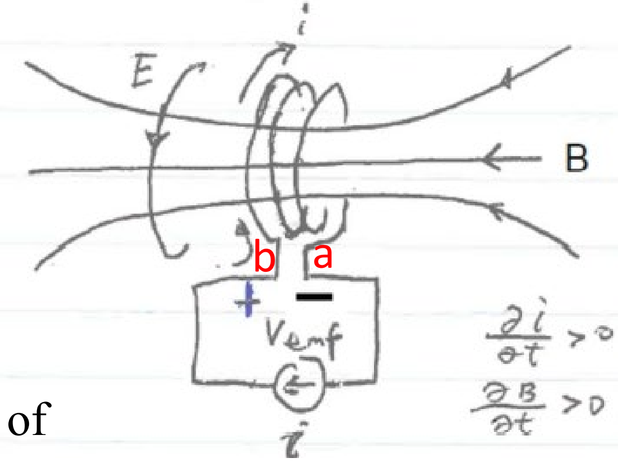
$$= N\frac{d}{dt}\int \vec{B}\cdot d\vec{S}$$

$$= N\frac{d\Phi}{dt} = \frac{d\Lambda}{dt}$$

Magnetic flux linkage $\Lambda = N\Phi$

What if you replace the voltmeter with a load resistor?

What if we feed a current to the coil, when there is no external magnetic field?



The current will induce magnetic field **B**.
This is true, regardless of the coil's shape or number of turns. For simplicity, we use the expression of **B** for a long solenoid

$$B = \mu\left(\frac{N}{l}\right)I$$

In the general case, $B \propto I$. The (nominal positive) directions of *I* and **B** follow the right hand rule.

If the current changes with time, so does **B**.

$$\frac{dB}{dt} = \mu\left(\frac{N}{l}\right)\frac{di}{dt}$$ for a long solenoid. $$\frac{dB}{dt} \propto \frac{di}{dt}$$ in general.

This changing **B** induces an emf (as we just discussed):

$$v = V_{\mathrm{emf}} = \mathrm{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -N\oint \mathbf{E}\cdot d\mathbf{l} = N\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} = N\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = N\frac{d\Phi}{dt} = \frac{d\Lambda}{dt}$$

$B \propto i \;\Rightarrow\; \Lambda \propto \Phi \propto B \propto i \;\Rightarrow$ Define proportional constant $L \equiv \frac{\Lambda}{i}$

What's this?

$$v = V_b - V_a = V_{\text{emf}} = \text{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -N\oint \mathbf{E}\cdot d\mathbf{l} = N\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = N\frac{d\Phi}{dt} = \frac{d\Lambda}{dt}$$

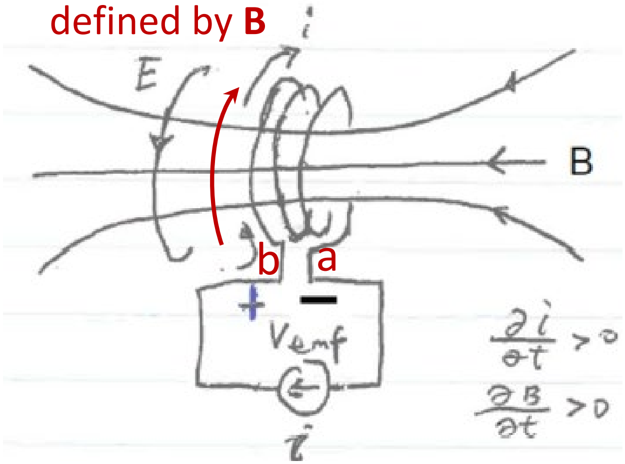
$$B \propto i \;\Rightarrow\; \Lambda \propto \Phi \propto B \propto i \;\Rightarrow$$

Define proportional constant $L \equiv \dfrac{\Lambda}{i}$

$$\Rightarrow\; \Lambda = Li \qquad \Rightarrow\; v = L\frac{di}{dt}$$

This is how the inductor works.

Loop direction defined by **B**



The induced electric field acts against the current change.

For the long solenoid,

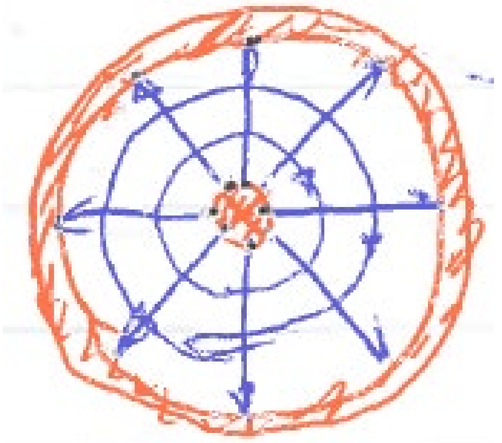
$$\left.\begin{aligned}\Lambda &= NB\overset{\text{area}}{S} = \mu\frac{N^{\overset{\text{squared}}{2}}}{\ell}\, i S \\ B &= \mu\left(\frac{N}{\ell}\right)I\end{aligned}\right\} \Rightarrow L = \mu\frac{N^2}{\ell}S$$

Our consistent sign conversion leads to the correct sign in the inductor equation in a straightforward way.

No need to go through the process every time.

Just keep in mind: The induced electric field acts against the current change.

Example 1: Inductance of the co-ax cable

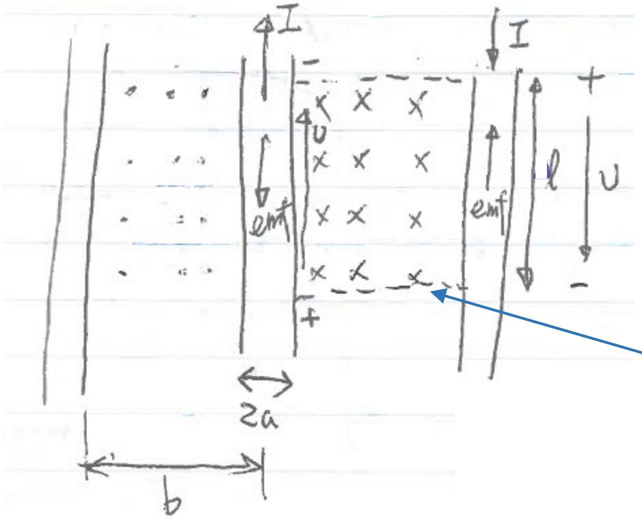


Important to understand what's really going on.

(This is why we discuss inductance after dynamic fields)

We assume current flows only at the outer surface of the core and the inner surface of the shield.

What's the magnetic field inside the core ($r < a$)?
What's the magnetic field outside the shield inner surface ($r > b$)?



Parameter of the filling dielectric

For $a < r < b$, $B = \frac{\mu I}{2\pi r}$

Make sure you get the directions/polarities of the quantities correctly.

Consider this rectangle.

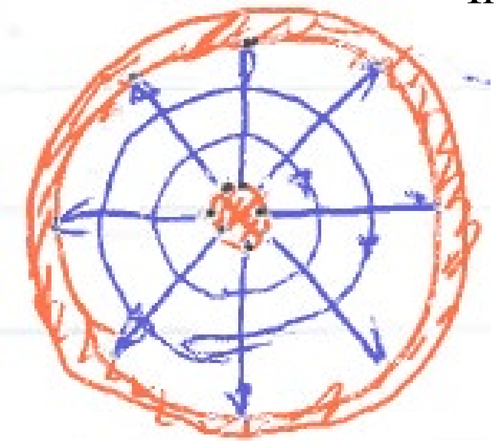
$$\Lambda = \Phi = l\int_a^b B\,dr = \frac{\mu I l}{2\pi}\int_a^b \frac{1}{r}\,dr$$

$$= \frac{\mu I l}{2\pi} \ln \frac{b}{a}$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$$

$$L = \frac{\mu l}{2\pi} \ln \frac{b}{a} \qquad l' = \frac{L}{l}$$

Important to understand what’s **really** going on.

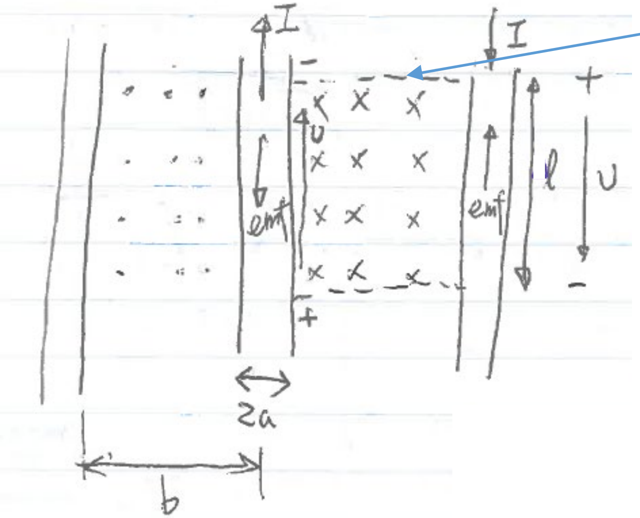


We assume current flows only at the outer surface of the core and the inner surface of the shield.

(Not so true for low frequencies. What is the consequence?)

Parameter of the filling dielectric

For $a < r < b$, $$B = \frac{\mu I}{2\pi r}$$



Consider this rectangle.

$$\Lambda = \Phi = l\int_a^b B\,dr = \frac{\mu I l}{2\pi}\int_a^b \frac{1}{r}\,dr$$
$$= \frac{\mu I l}{2\pi}\ln\frac{b}{a}$$

$$L = \frac{\mu l}{2\pi}\ln\frac{b}{a} \qquad L' = \frac{L}{l}$$

$$\boxed{\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}}$$

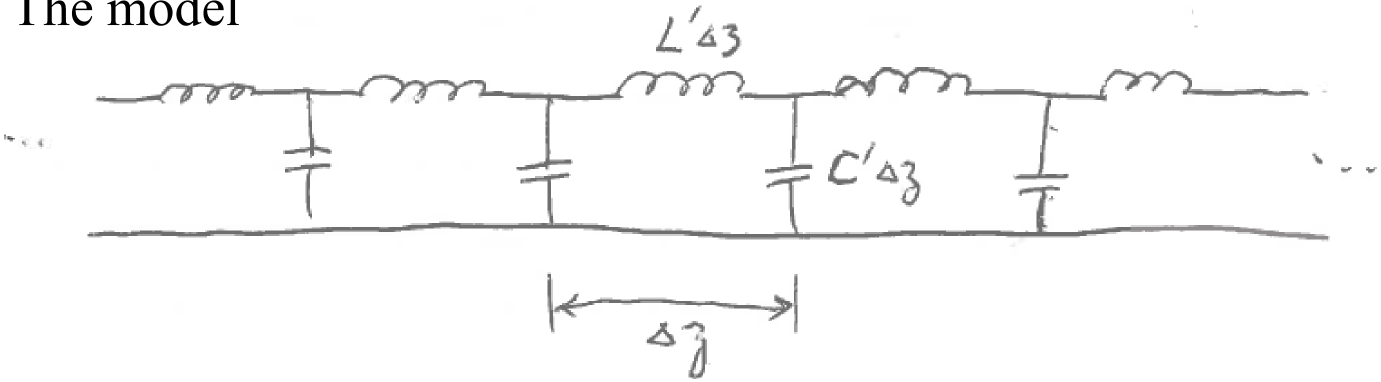
Make sure you get the directions/polarities of the quantities correctly.

Where is the induced electric field and the “voltage”?
Think about the distributed circuit model of the transmission line.

Review textbook Section 5-7 up to 5-7.2. Pay attention to the parallel-wire line geometry.
We explain how the inductor works after presenting Faraday’s law for true understanding.

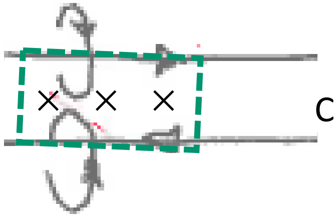
An old slide

The model



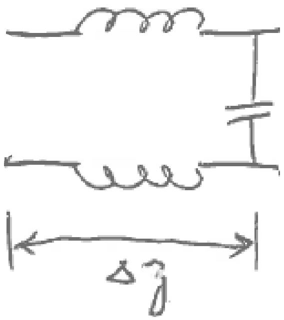
The inductors (and resistors in lossy lines) are on only one side.
Which side is which wire???

A pair of coupled wires

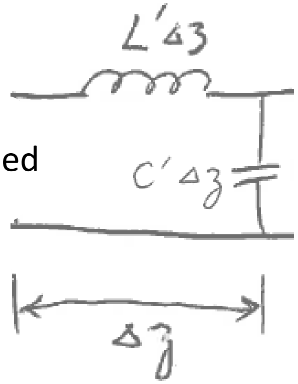


Voltage v around the loop!

Capacitance also considered



Inductance lumped to one side

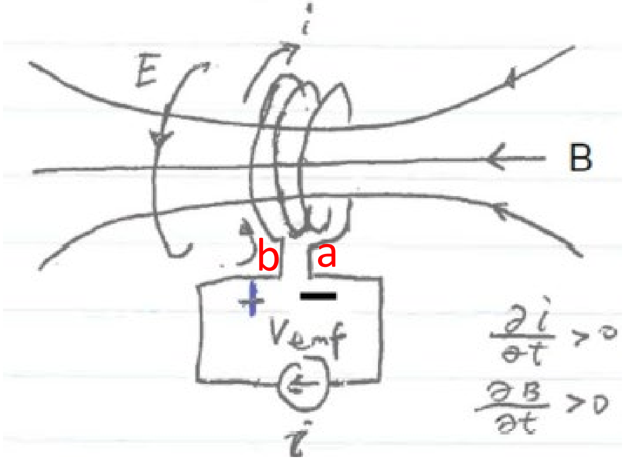


$$\boxed{L = L'\Delta z}$$

L' is inductance per length

Energy stored in an inductor

Say, we increase the current *i* from 0 to *I*.
The current induces a magnetic field, which increases as *i* increases.
The increasing magnetic field induces an electric field, which is against the current *i*.
The current source therefore has to push the current against this non-electrostatic electric field, which establish a voltage *v*. Thus the current source does some work.



$$v = V_{\text{emf}} = \text{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -N\oint \mathbf{E}\cdot d\mathbf{l} = N\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = N\frac{d\Phi}{dt} = \frac{d\Lambda}{dt}$$

$$\Lambda = Li \quad \Rightarrow \quad v = L\frac{di}{dt}$$

The work done by the current source becomes energy stored in the magnetic field, or equivalently, magnetic energy in the inductor:

$$W_m = \int i v\, dt = \int i L\frac{di}{dt}\, dt = L\int_0^I i\, di = \frac{1}{2}LI^2$$

Using the long solenoid as the archetypical inductor, we get energy density $\frac{W_m}{V} = \frac{1}{2}\mu H^2$
Just as the parallel plates as the archetypical capacitor.
And, the conclusion is also general here.

volume

Compare energy storage by capacitors & inductors

	Capacitor	Inductor
	$i = C\frac{dv}{dt}$	$v = L\frac{di}{dt}$
Stored energy	$W_E = \frac{1}{2}CV^2$	$W_m = \frac{1}{2}LI^2$
Energy density	$\frac{W_e}{V} = \frac{1}{2}\varepsilon E^2$	$\frac{W_m}{V} = \frac{1}{2}\mu H^2$
Archetypical geometry	Infinitely large parallel-plate	Infinitely long solenoid
Limited by	Breakdown	???

voltage

volume

Lastly, the unit of inductance

$$L \equiv \frac{\Lambda}{i}$$

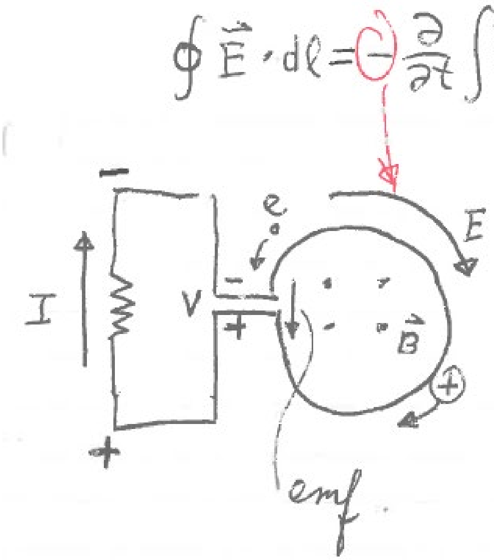
$$Wb \doteq Tm^2$$

(weber)

$$H = \frac{Wb}{A} \doteq \frac{Tm^2}{A}$$

Henry

Example 2: emf induced by a time-varying magnetic field



$$\oint \vec{E}\cdot d\ell = - \frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{s}$$

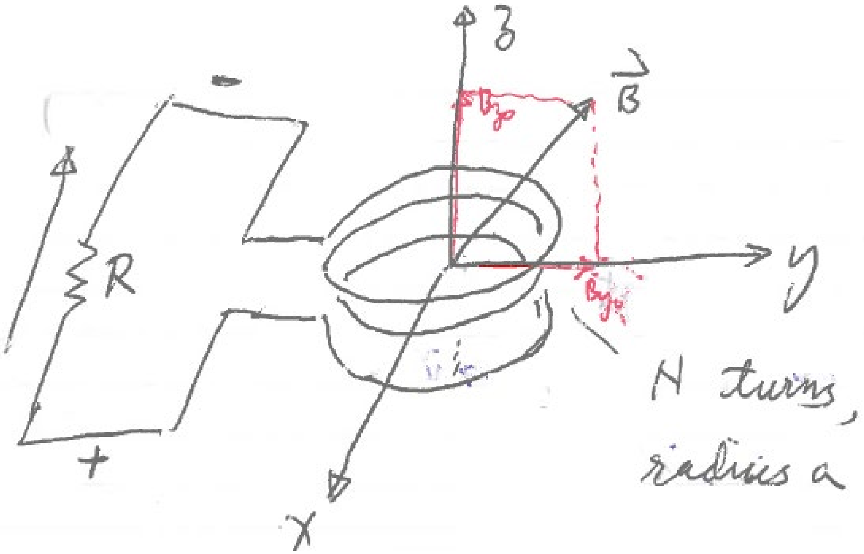
$$\frac{\partial B}{\partial t} > 0$$

Important to get the directions right from the very basic principle;
Different sign conventions may be adopted, but eventually the directions/polarity of measurable quantities must be correct.
The key to remember is the negative sign in Faraday's law. What does it mean?

Compare this with Figure 6-2 in textbook.

Notice that the induced electric field is non-electrostatic.

Example 3: emf induced by a time-varying magnetic field



$$\vec{B} = \left(\hat{y}\, B_{y0} + \hat{z}\, B_{z0}\right)\sin\omega t$$

Find the current through the resistor *R*.

$$\Phi = \int \vec{B}\cdot d\vec{s} = B_{z0}\sin\omega t\,(\pi a^2)$$

$$\Lambda = N\Phi$$

$$V = \frac{d\Lambda}{dt} = \omega N B_{z0}\pi a^2 \cos\omega t$$

$$I = \frac{V}{R}$$

Review textbook:

- Sections 1-3.3, 1-3.4, 4-1,
- Chapter 5 Overview,
- Chapter 6 overview: Dynamic Fields,
- Sections 6-1, 6-2,
- Section 5-7 overview, subsections 5-7.1, 5-7.2, Section 5-8

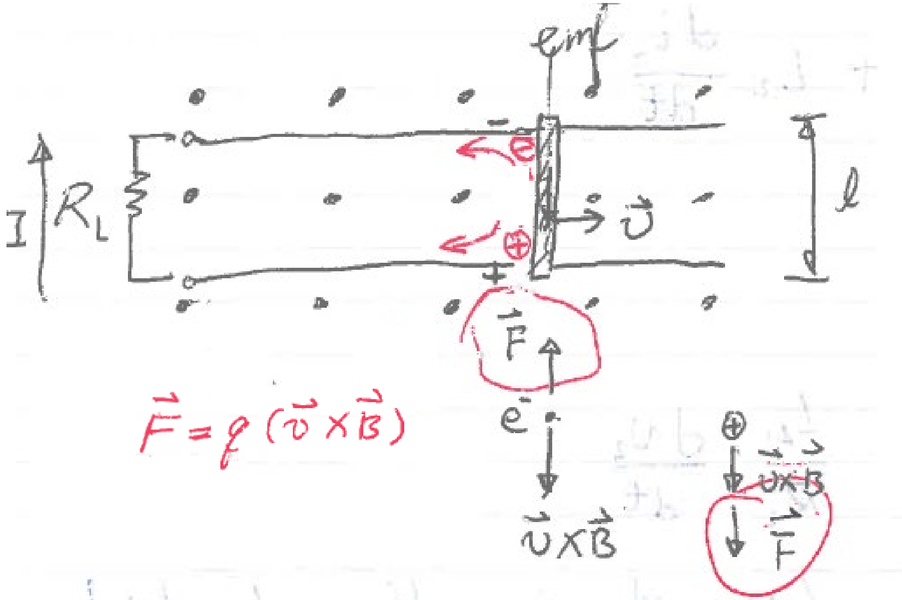
Do Homework 11: Problems 2 through 4, and 7.

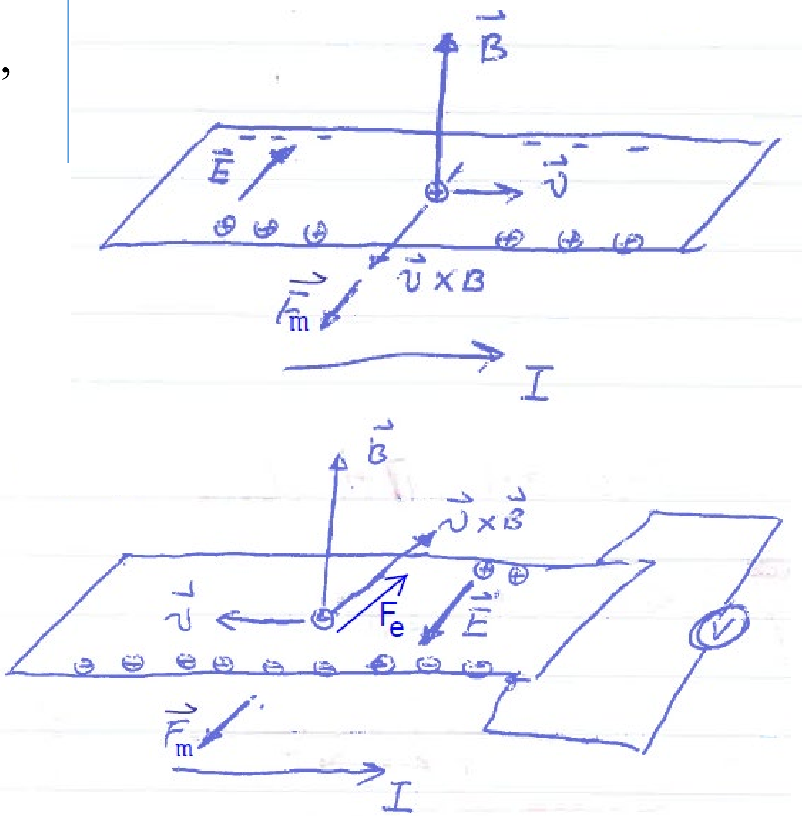
emf due to motion

Recall the Hall effect and the force on a current-carrying wire in a magnetic field. See figures to the right.

If a conductor mechanically moves in a magnetic field, its charge carriers move along and the magnetic force gives rise to an emf:

We discussed Hall effect earlier



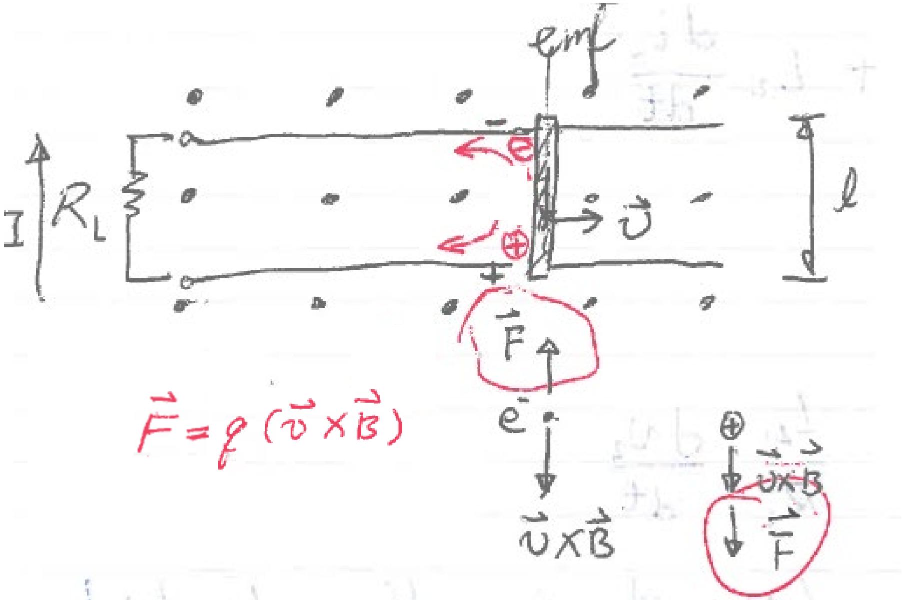


The magnetic force is non-electrostatic.

Generally, inside the source (e.g. battery), emf $= \frac{1}{Q}\int_a^b \mathbf{F}_{\text{nes}} \cdot d\mathbf{l}$, where $\mathbf{F}_{\text{nes}}$ is the non-electrostatic force acting on charge carrier Q, and $V = -\frac{1}{Q}\int_a^b \mathbf{F}_{\text{stat}} \cdot d\mathbf{l} = -\int_a^b \mathbf{E}_{\text{stat}} \cdot d\mathbf{l}$, where $\mathbf{F}_{\text{stat}}$ is the electrostatic force.

Here, emf $= \frac{1}{q}\int_a^b \mathbf{F} \cdot d\mathbf{l} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l} = vBl$ (if B is uniform)

Recall the Hall effect and the force on a current-carrying wire in a magnetic field. See figures to the right.
If a conductor mechanically moves in a magnetic field, its charge carriers move along and the magnetic force gives rise to an emf:



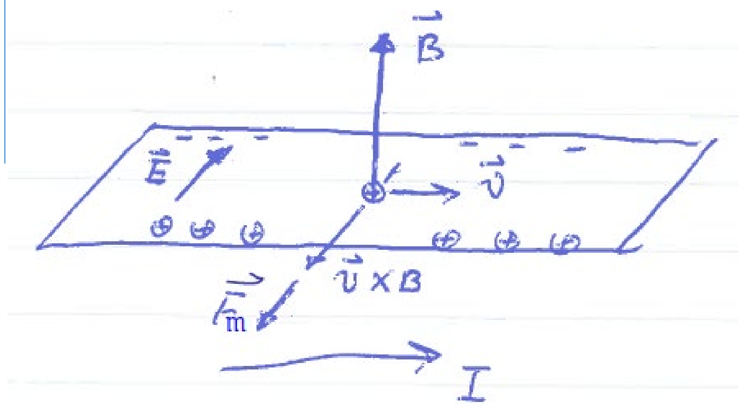
The magnetic force is non-electrostatic.

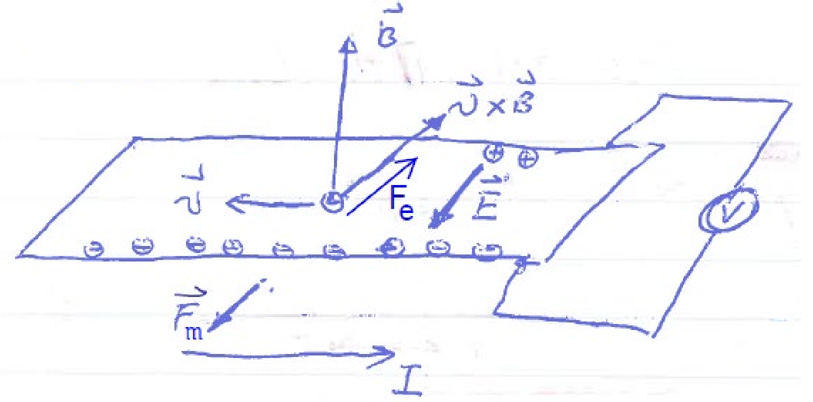
$$\text{emf} = \int (\vec{v}\times\vec{B})\cdot d\vec{\ell} = vB\ell$$

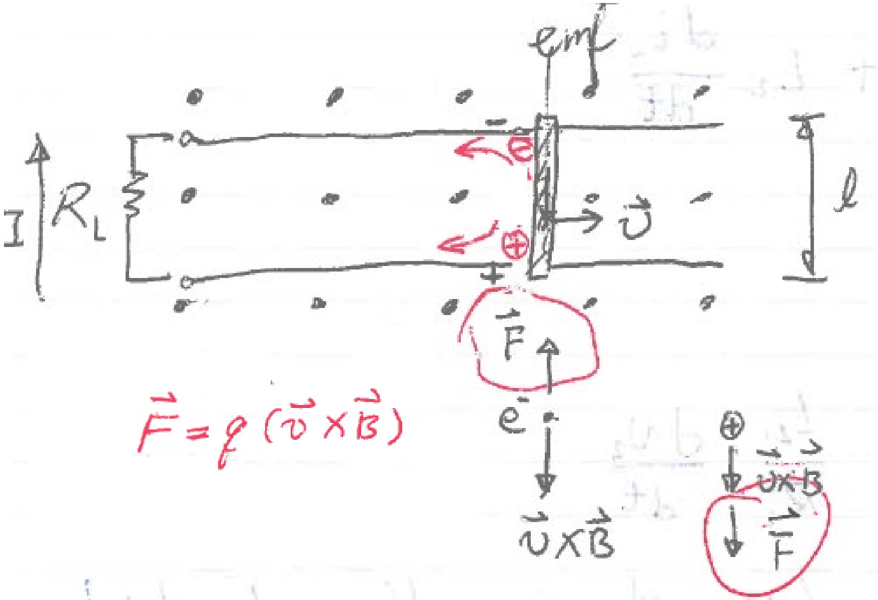
Recall that the magnetic force does not do work.
What provides the power?

Hint: Any resistance to the motion?
(see figure to the right)

We discussed Hall effect earlier

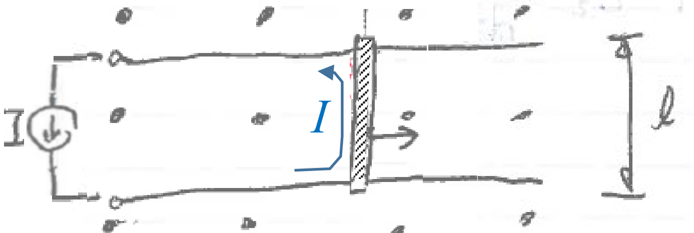






$\vec{F} = q(\vec{v} \times \vec{B})$

This a "generator".
It generates electric energy from mechanical motions.

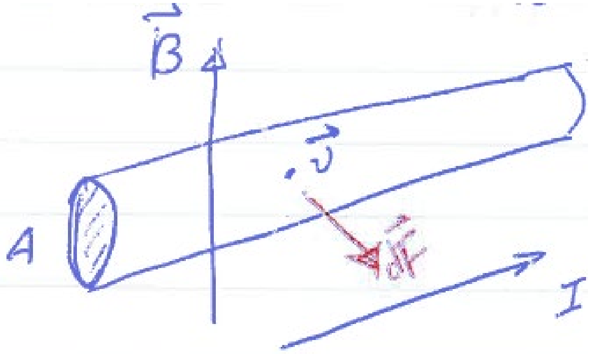


$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

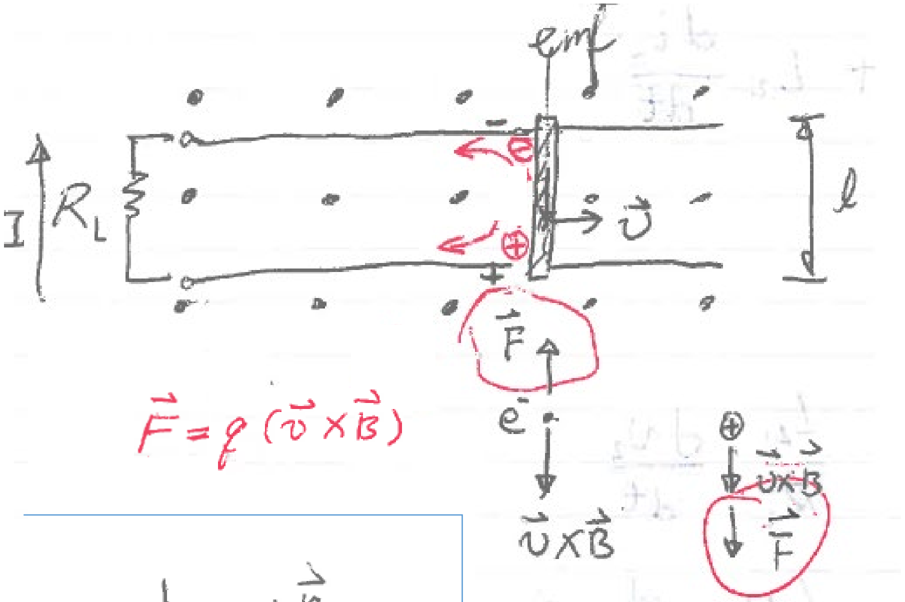
If we replace the load with a current source, the conductor bar will be pushed to move. A sort of "motor".

Again, the magnetic force does not do work. If the conductor bar drives a mechanical load, work is done. What does the work?

The bar is just like any wire:



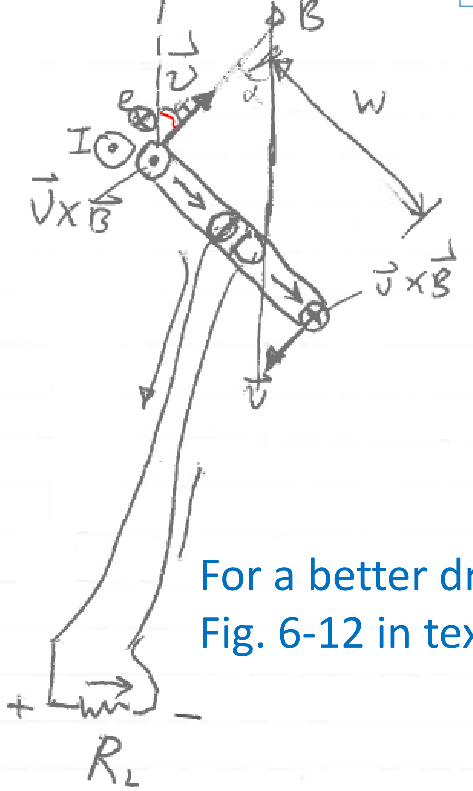
Hint: Let's assume the conductor bar is made of a perfect conductor. Without the magnetic field, there is no voltage drop on the bar, i.e., the bar consumes no power. When the magnetic field is on, will there be a voltage drop? Why?



This a “generator”.
It generates electric energy from mechanical motions.

Now we understand why generators convert mechanical energy into electric energy while the magnetic field does not do work.

Actual generators are more practical.
Rotation instead of translation.



For a better drawing, see Fig. 6-12 in textbook.

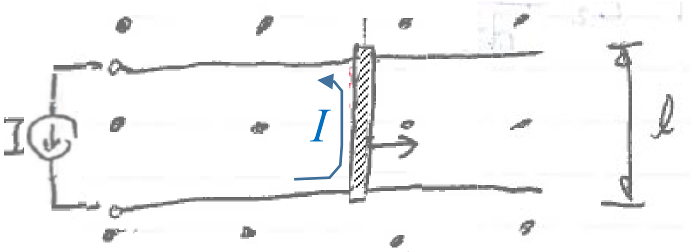
Let α be the angle between the coil normal and the magnetic field **B**.

$\alpha = \omega t + \alpha(0)$ and $v = \omega \frac{w}{2}$

$$\text{emf on one side} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l} = \omega \frac{w}{2} B \sin\alpha \cdot l$$

$$\text{total emf} = \omega w l B \sin\alpha = \omega A B \sin[\omega t + \alpha(0)]$$

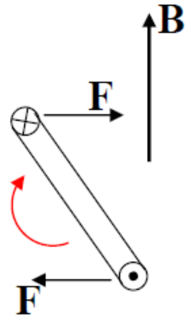
Area $A = wl$



$$d\vec{F} = I\, d\vec{\ell} \times \vec{B}$$

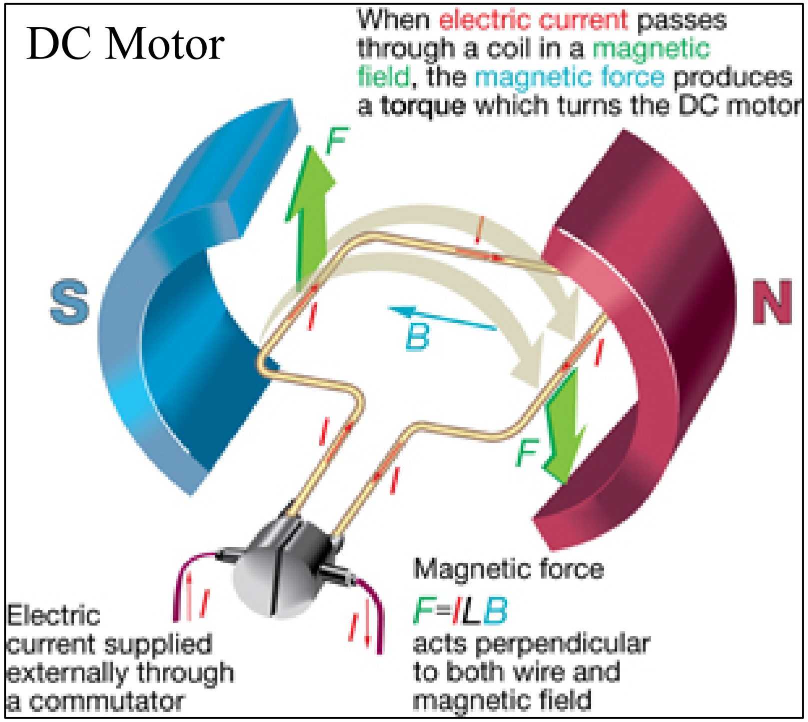
A sort of “motor”.
It converts electric energy to mechanical energy.

Now we understand why motors convert electric energy into mechanical energy while the magnetic field does not do work.



Actual motors are more practical. Rotation instead of translation:

DC Motor



When electric current passes through a coil in a magnetic field, the magnetic force produces a torque which turns the DC motor

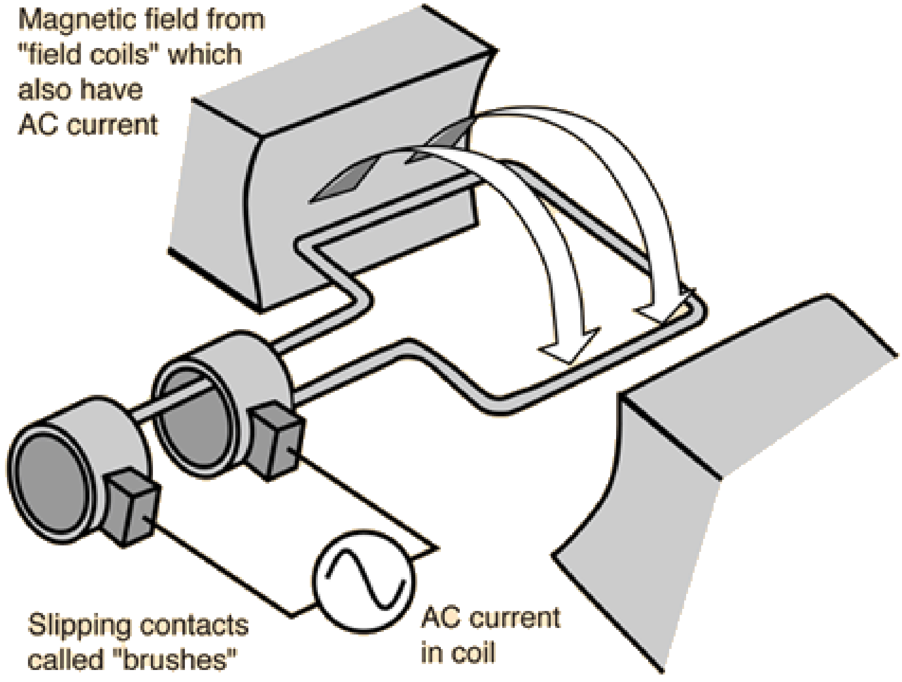
Electric current supplied externally through a commutator

Magnetic force $F=ILB$ acts perpendicular to both wire and magnetic field

http://resource.rockyview.ab.ca/rvlc/physics30_BU/Unit_B/m4/p30_m4_l03_p4.html

See also:
https://www.youtube.com/watch?v=Y-v27GPK8M4

AC Motor

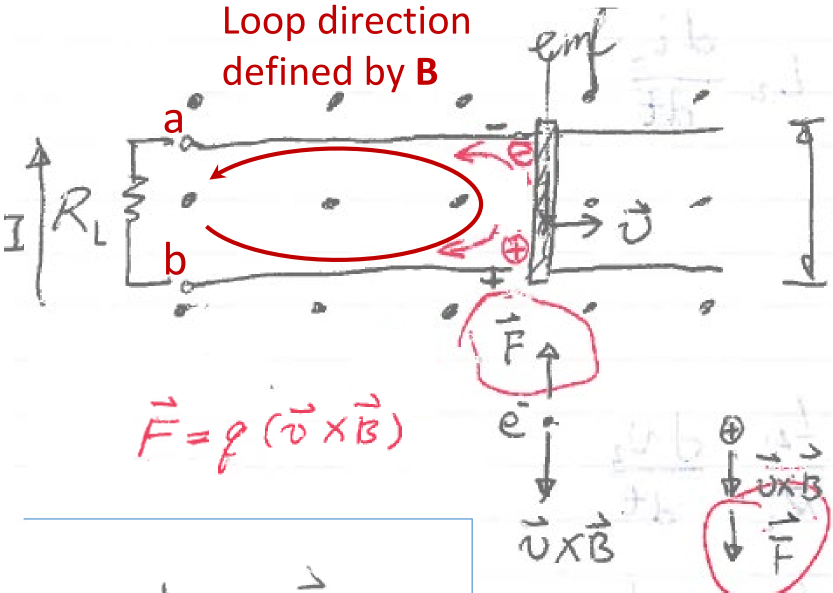


Magnetic field from "field coils" which also have AC current

Slipping contacts called "brushes"

AC current in coil

http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motorac.html



The "flux rule"

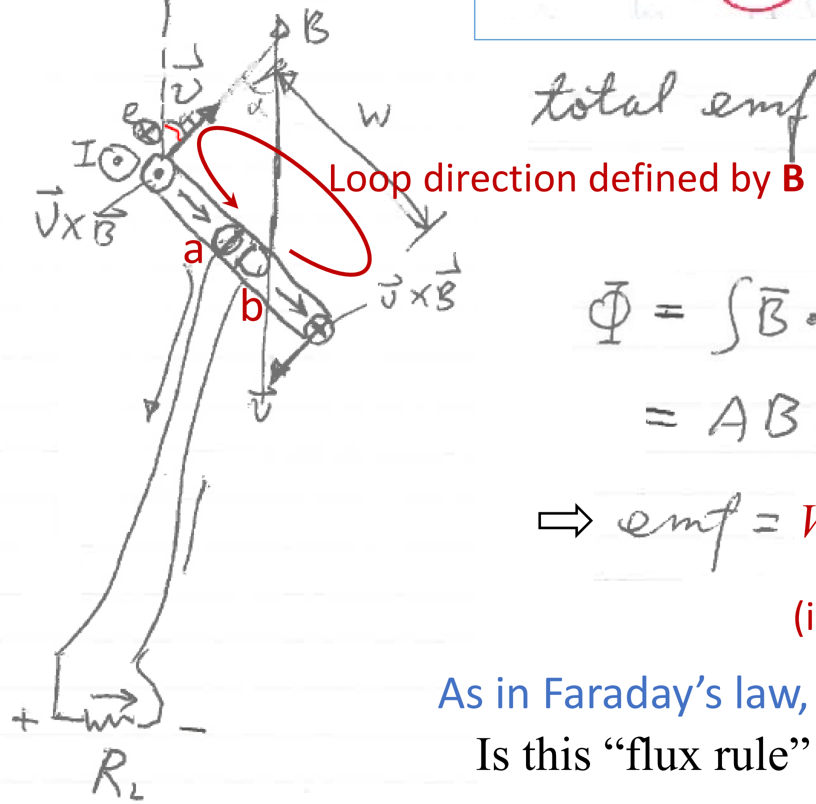
$$emf = \int (\vec{v}\times\vec{B})\cdot d\vec{l} = vBl$$

(if B is uniform)

Consider the increasing flux as the bar is moving: $\Phi = Bxl$ (with loop direction defined by **B**)

$$V_b - V_a = emf = \frac{d\Phi}{dt} = Bl\frac{dx}{dt} = vBl$$

As in Faraday's law, the induced emf is against the change of the flux.



$$total\ emf = \omega w l B \sin\alpha = \omega A B \sin[\omega t + \alpha(0)]$$

Area $A = wl$

$$\Phi = \int \vec{B}\cdot d\vec{S} = \vec{B}\cdot\hat{n}A = AB\cos\alpha$$

$$= AB\cos[\omega t + \alpha(0)]$$

$$\Rightarrow emf = V_b - V_a = \frac{d\Phi}{dt} = -\omega AB\sin[\omega t + \alpha(0)]$$

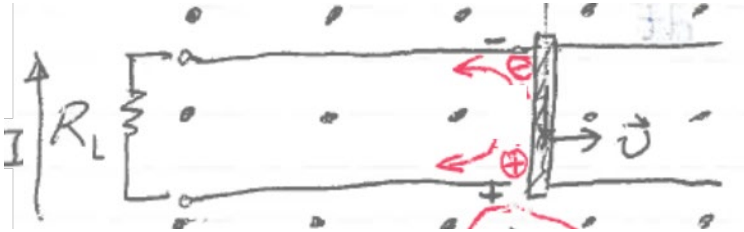
(in our sign convention with loop direction defined by **B**)

As in Faraday's law, the induced emf is against the change of the flux.

Is this "flux rule" related to Faraday's law?

The “flux rule” of moving conductor in static magnetic field

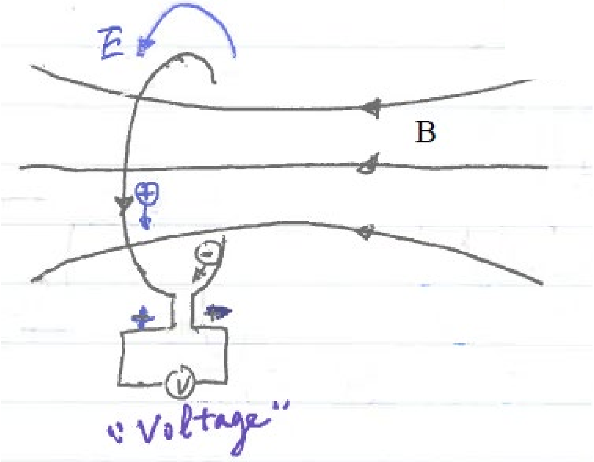
$$\text{emf} = \frac{d\Phi}{dt}$$



As in Faraday’s law, the induced emf is against the change of the flux.

Faraday’s law of changing magnetic field

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$$



The induced electric field is against the change of the flux.

$$V_{\text{emf}} = \text{emf} = \int_a^b \mathbf{E}\cdot d\mathbf{l} \approx -\oint \mathbf{E}\cdot d\mathbf{l} = \int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} = \frac{d\Phi}{dt}$$

> "Usually such a beautiful generalization is found to stem from a single deep underlying principle. Nevertheless, in this case there does **not** appear to be any such profound implication. We have to understand the rule as the combined effects of **two quite separate phenomena**."

-- Richard Feynman

emf induced in moving conductor in magnetic field (no need for closed circuit)

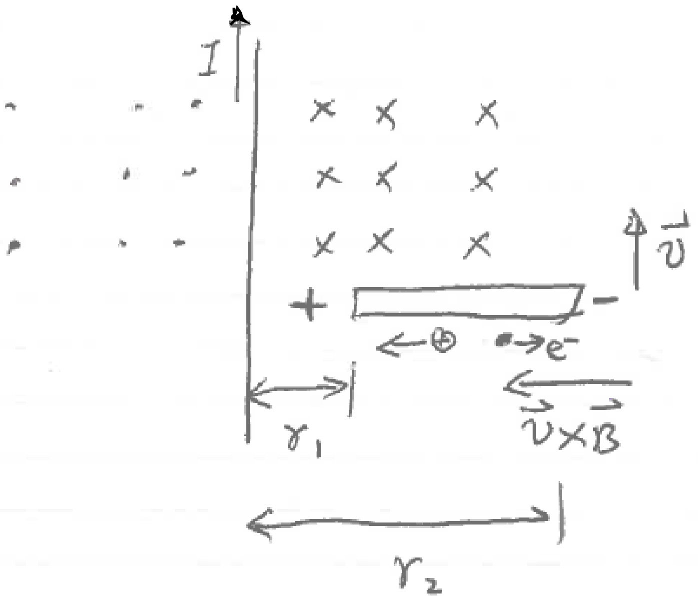
Example 4: emf induced by magnetic field in isolated moving conductor

$$V = \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

$$= v \int_{r_1}^{r_2} \frac{\mu_0 I}{2\pi r} \, dr$$

$$= \frac{\mu_0 I v}{2\pi} \int_{r_1}^{r_2} \frac{1}{r} \, dr$$

$$= \frac{\mu_0 I v}{2\pi} \ln \frac{r_2}{r_1}$$

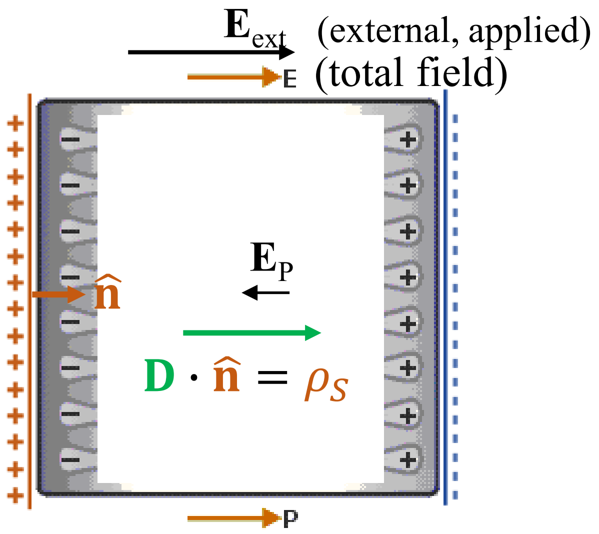


Review textbook Sections 6-4, 6-5.

Do Homework 11 Problems 5, 6, 8. Finish Homework 11.

Magnetism of materials

Recall the following for a dielectric in an external electric field:



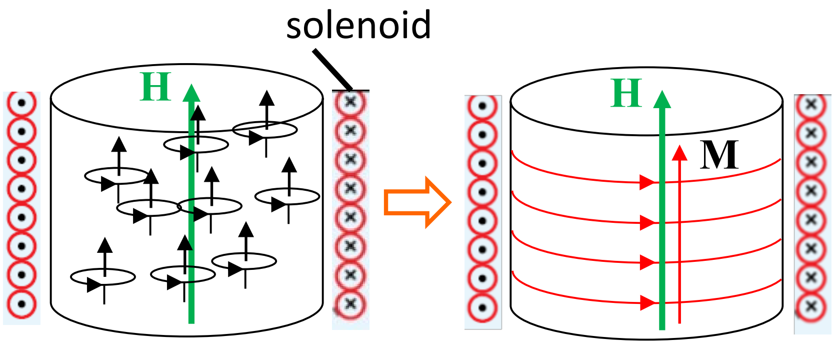
The polarization charge ρ_{sP} is opposite to the external charge ρ_s.
The polarization field $\mathbf{E}_\mathbf{P}$ is always against the externa field $\mathbf{E}_{\text{ext}}$. Therefore the name dielectric.
$\varepsilon_r = 1+\chi > 1$, $\varepsilon > \varepsilon_0$ (It takes more external charge than in free space to establish the same $\mathbf{E}_{\text{ext}}$.)

$$\nabla\cdot(\varepsilon_0 + \chi_e\varepsilon_0)\mathbf{E} \equiv \nabla\cdot\mathbf{D} = \rho\,,$$

$$\text{where } \varepsilon \equiv \varepsilon_0(1+\chi_e) \equiv \varepsilon_0\varepsilon_r\,,\quad \mathbf{D} \equiv \varepsilon_0\varepsilon_r\mathbf{E} \equiv \varepsilon\mathbf{E}$$

We lump the polarization effect of a dielectric material into a parameter ε, and substitute ε_0 (for free space) with ε (for the dielectric) in equations.

Similarly, in the presence of an external magnetic field, atomic magnetic moments line up.



magnetization

$$\mathbf{M} = \lim_{\Delta V\to 0}\frac{\sum_i \mathbf{m}_i}{\Delta V}$$

similar to

polarization

$$\mathbf{P} = \lim_{\Delta V\to 0}\frac{\sum_i \mathbf{p}_i}{\Delta V}$$

$$\mathbf{J}_M = \nabla\times\mathbf{M}$$

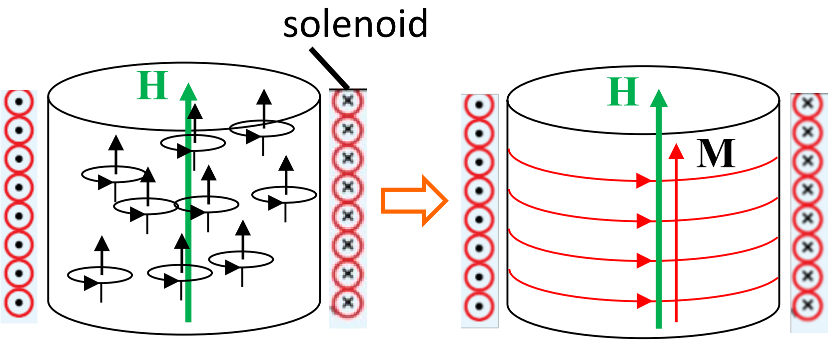
magnetization current

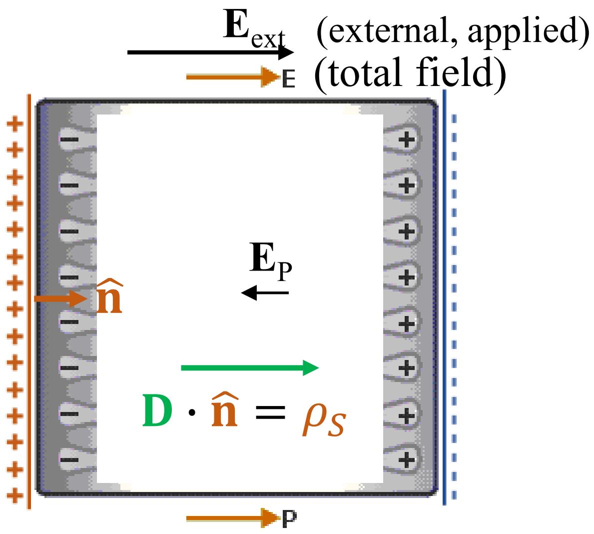
$$\rho_P = -\nabla\cdot\mathbf{P}$$

polarization charge

$$\mathbf{M} = \chi_m\mathbf{H}$$

$$\mathbf{P} = \chi_e\varepsilon_0\mathbf{E}$$





magnetization

$$\mathbf{M} = \lim_{\Delta V \to 0} \frac{\sum_i \mathbf{m}_i}{\Delta V}$$

similar to

polarization

$$\mathbf{P} = \lim_{\Delta V \to 0} \frac{\sum_i \mathbf{p}_i}{\Delta V}$$

$$\mathbf{J}_M = \nabla \times \mathbf{M}$$

magnetization current

$$\rho_P = -\nabla \cdot \mathbf{P}$$

polarization charge

Important difference

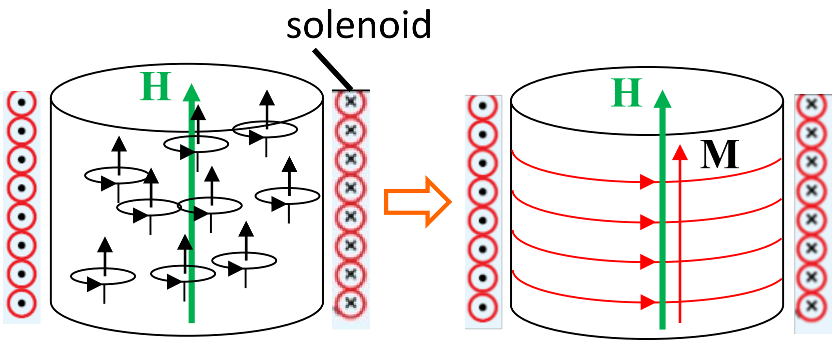
$$\mathbf{M} = \chi_m \mathbf{H}$$

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

External field due to external current

Total field

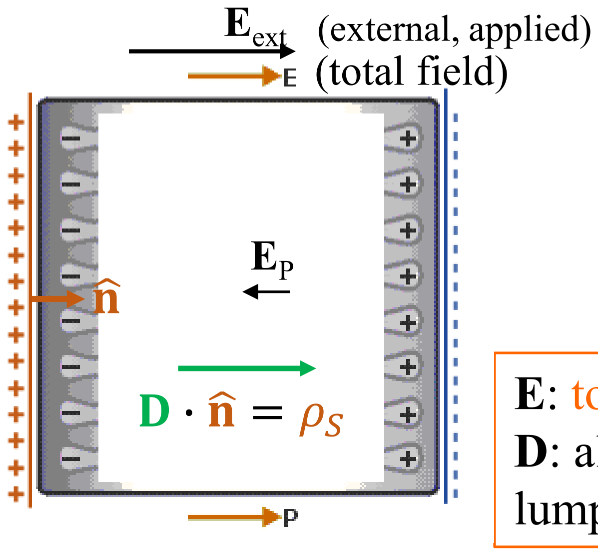
Notice the different “accounting” for magnetic and electric fields.



$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_{\text{total}} = \mu_0 (\mathbf{J} + \mathbf{J}_M)$$

B: total effect of external & magnetization currents; felt by probe current.
H: allows us to consider external current only, with magnetization effects lumped into materials parameters. $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$

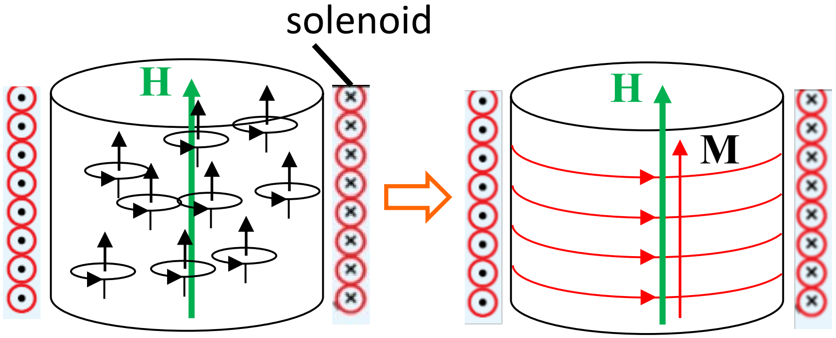
Lecture of Tue 11/22/2022 ends here.
Please view slides 31 (this one) through 35 offline before next class (Tue 11/29).



Compare with dielectric in external electric field:

$$\varepsilon_0 \nabla \cdot \mathbf{E} = \rho + \rho_P$$

E: total effect of external & polarization charges; felt by probe charge.
D: allows us to consider external charge only, with polarization effects lumped into materials parameters. $\varepsilon_0\mathbf{E} = \mathbf{D} - \mathbf{P}$



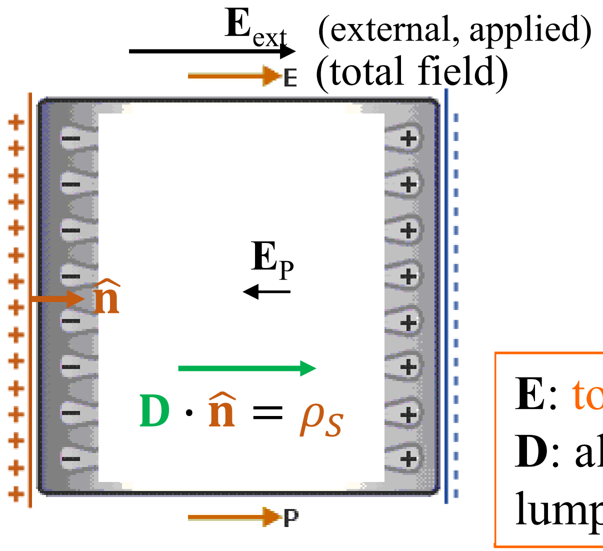
$$\left.\begin{aligned}\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}&=\mu_0(\mathbf{J}+\mathbf{J}_M)\\ \mathbf{J}_M&=\nabla\times\mathbf{M}\end{aligned}\right\}\Rightarrow$$

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}=\mu_0(\mathbf{J}+\mathbf{J}_M)=\mu_0\mathbf{J}+\mu_0\nabla\times\mathbf{M}$$

external current

B: total effect of external & magnetization currents; felt by probe current.
H: allows us to consider external current only, with magnetization effects lumped into materials parameters. $\mathbf{B}=\mu_0\mathbf{H}+\mu_0\mathbf{M}$

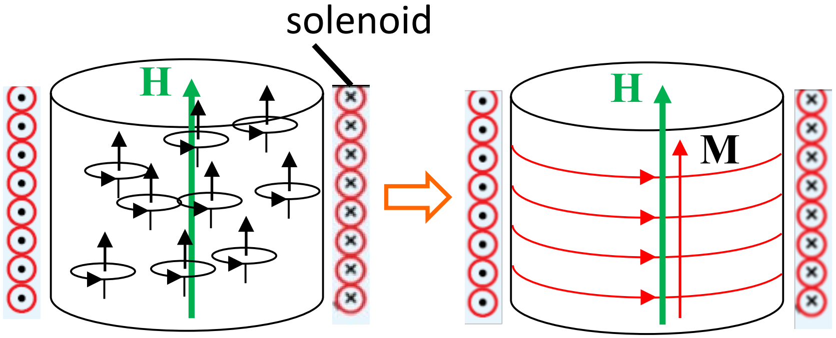
$$\left.\begin{aligned}\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}=\mu_0(\mathbf{J}+\mathbf{J}_M)&=\mu_0\mathbf{J}+\mu_0\nabla\times\mathbf{M}\\ &\boxed{\mathbf{J}=\nabla\times\mathbf{H}}\end{aligned}\right\}\Rightarrow\nabla\times\mathbf{B}=\mu_0\nabla\times(\mathbf{H}+\mathbf{M})$$



Compare with dielectric in external electric field:

$$\varepsilon_0\nabla\cdot\mathbf{E}=\rho+\rho_P \qquad \rho_P=-\nabla\cdot\mathbf{P}$$

E: total effect of external & polarization charges; felt by probe charge.
D: allows us to consider external charge only, with polarization effects lumped into materials parameters. $\varepsilon_0\mathbf{E}=\mathbf{D}-\mathbf{P}$



$$\left.\begin{aligned}\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}&=\mu_0(\mathbf{J}+\mathbf{J}_M)\\ \mathbf{J}_M&=\nabla\times\mathbf{M}\end{aligned}\right\}\Rightarrow$$

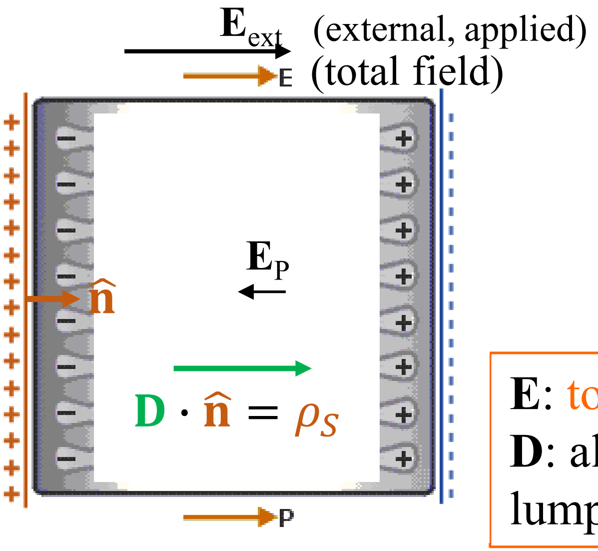
$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}=\mu_0(\mathbf{J}+\mathbf{J}_M)=\mu_0\mathbf{J}+\mu_0\nabla\times\mathbf{M}$$

external current

B: total effect of external & magnetization currents; felt by probe current.
H: allows us to consider external current only, with magnetization effects lumped into materials parameters. $\mathbf{B}=\mu_0\mathbf{H}+\mu_0\mathbf{M}$

$$\left.\begin{aligned}\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}=\mu_0(\mathbf{J}+\mathbf{J}_M)&=\mu_0\mathbf{J}+\mu_0\nabla\times\mathbf{M}\\ \boxed{\mathbf{J}=\nabla\times\mathbf{H}}\end{aligned}\right\}\Rightarrow\left.\begin{aligned}\nabla\times\mathbf{B}&=\mu_0\nabla\times(\mathbf{H}+\mathbf{M})\\ \mathbf{M}&=\chi_m\mathbf{H}\end{aligned}\right\}\Rightarrow$$

$$\nabla\times\mathbf{B}=\mu_0(1+\chi_m)\nabla\times\mathbf{H}$$

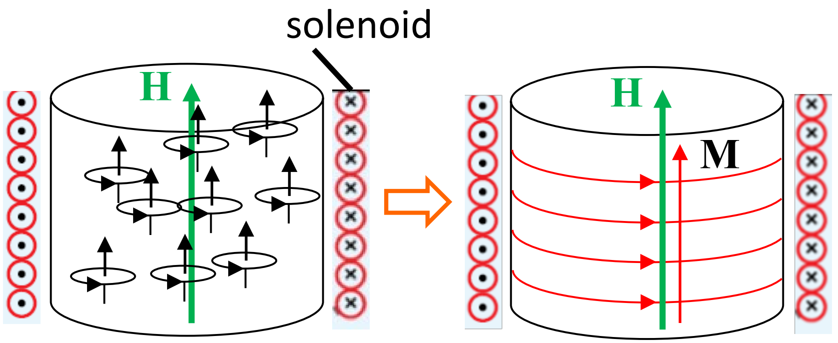


Compare with dielectric in external electric field:

$$\varepsilon_0\nabla\cdot\mathbf{E}=\rho+\rho_P \qquad \rho_P=-\nabla\cdot\mathbf{P} \qquad \mathbf{P}=\chi_e\varepsilon_0\mathbf{E},$$

$$\nabla\cdot(\varepsilon_0+\chi_e\varepsilon_0)\mathbf{E}\equiv\nabla\cdot\mathbf{D}=\rho$$

E: total effect of external & polarization charges; felt by probe charge.
D: allows us to consider external charge only, with polarization effects lumped into materials parameters. $\varepsilon_0\mathbf{E}=\mathbf{D}-\mathbf{P}$



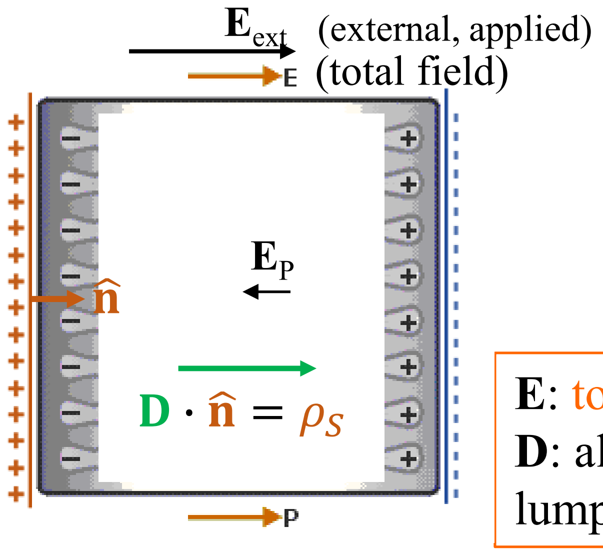
$$\left.\begin{aligned}\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}&=\mu_0(\mathbf{J}+\mathbf{J}_M)\\ \mathbf{J}_M&=\nabla\times\mathbf{M}\end{aligned}\right\}\Rightarrow$$

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}=\mu_0(\underset{\text{external current}}{\mathbf{J}}+\mathbf{J}_M)=\mu_0\mathbf{J}+\mu_0\nabla\times\mathbf{M}$$

B: total effect of external & magnetization currents; felt by probe current.
H: allows us to consider external current only, with magnetization effects lumped into materials parameters. $\mathbf{B}=\mu_0\mathbf{H}+\mu_0\mathbf{M}$

$$\left.\begin{aligned}\nabla\times\mathbf{B}=\mu_0\mathbf{J}_{\text{total}}=\mu_0(\mathbf{J}+\mathbf{J}_M)&=\mu_0\mathbf{J}+\mu_0\nabla\times\mathbf{M}\\ \boxed{\mathbf{J}=\nabla\times\mathbf{H}}\end{aligned}\right\}\Rightarrow\left.\begin{aligned}\nabla\times\mathbf{B}&=\mu_0\nabla\times(\mathbf{H}+\mathbf{M})\\ \mathbf{M}&=\chi_m\mathbf{H}\end{aligned}\right\}\Rightarrow$$

$$\nabla\times\mathbf{B}=\mu_0(1+\chi_m)\nabla\times\mathbf{H}\equiv\mu_0\mu_r\nabla\times\mathbf{H}\equiv\mu\nabla\times\mathbf{H},\ \text{where}\ \boxed{\begin{aligned}&\mu\equiv\mu_0(1+\chi_m)\equiv\mu_0\mu_r\\ &\mathbf{B}=\mu\mathbf{H}=\mu_r\mu_0\mathbf{H}=(1+\chi_m)\mu_0\mathbf{H}\end{aligned}}$$



Compare with dielectric in external electric field:

$$\varepsilon_0\nabla\cdot\mathbf{E}=\rho+\rho_P\qquad\rho_P=-\nabla\cdot\mathbf{P}\qquad\mathbf{P}=\chi_e\varepsilon_0\mathbf{E},$$

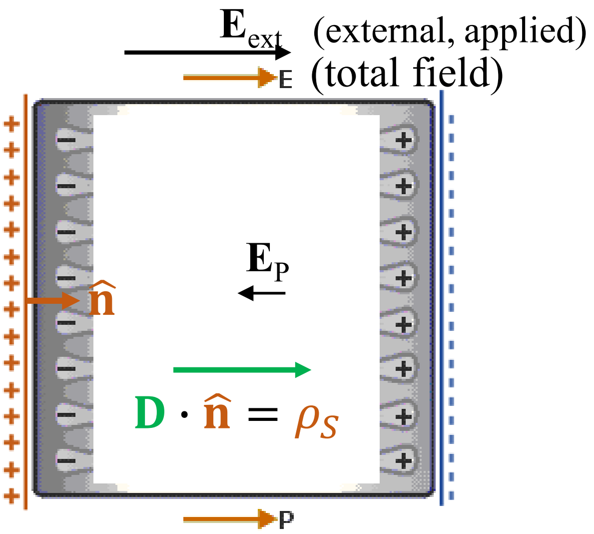
$$\nabla\cdot(\varepsilon_0+\chi_e\varepsilon_0)\mathbf{E}\equiv\nabla\cdot\mathbf{D}=\rho,$$

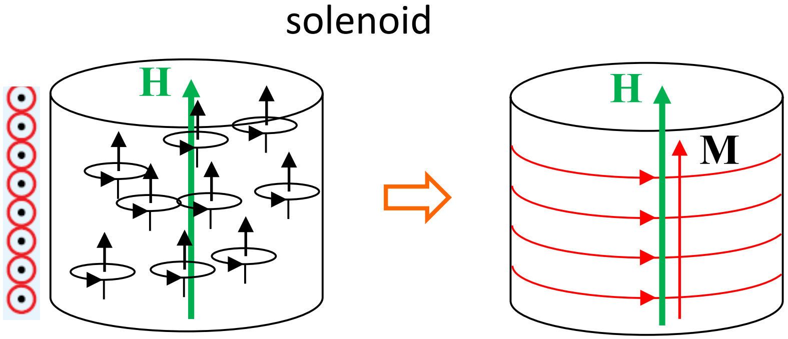
where $\varepsilon\equiv\varepsilon_0(1+\chi_e)\equiv\varepsilon_0\varepsilon_r$, $\mathbf{D}\equiv\varepsilon_0\varepsilon_r\mathbf{E}\equiv\varepsilon\mathbf{E}$

E: total effect of external & polarization charges; felt by probe charge.
D: allows us to consider external charge only, with polarization effects lumped into materials parameters. $\varepsilon_0\mathbf{E}=\mathbf{D}-\mathbf{P}$

One Way to Draw Analogy

Side Notes





polarization charge   polarization

$$\rho_P = -\nabla\cdot\mathbf{P}$$

magnetization current   magnetization

$$\mathbf{J}_M = \nabla\times\mathbf{M}$$

---

polarization field

$$\varepsilon_0\mathbf{E}_{\text{P}} = -\mathbf{P}$$

No equivalence, but let's define field due to magnetization

$$\mathbf{B}_{\text{M}} = \mu_0\mathbf{M} \text{ (and } \mathbf{B}_{\text{ext}} = \mu_0\mathbf{H}\text{)}$$

Total field $\mathbf{E} = \mathbf{E}_{\text{ext}} + \mathbf{E}_{\text{P}}$

$$\mathbf{B} = \mathbf{B}_{\text{ext}} + \mathbf{B}_{\text{M}} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$$

$$\varepsilon_0(\nabla\cdot\mathbf{E}_{\text{ext}} + \nabla\cdot\mathbf{E}_{\text{P}}) = \rho + \rho_P$$

$$\mu_0\nabla\times\mathbf{H} + \mu_0\nabla\times\mathbf{M} = \mu_0\mathbf{J} + \mu_0\mathbf{J}_M$$

$\varepsilon_0\nabla\cdot\mathbf{E}_{\text{ext}} = \rho$ external charge

$$\left.\begin{aligned}\Longrightarrow\ \varepsilon_0(\nabla\cdot\mathbf{E} - \nabla\cdot\mathbf{E}_{\text{P}}) &= \rho\\ \varepsilon_0\mathbf{E}_{\text{P}} &= -\mathbf{P}\end{aligned}\right\}$$

$$\left.\mathbf{P} = \chi_e\varepsilon_0\mathbf{E},\ \varepsilon \equiv \varepsilon_0(1+\chi_e) \equiv \varepsilon_0\varepsilon_r\right\} \Longrightarrow \varepsilon\nabla\cdot\mathbf{E} = \rho$$

$$\Longrightarrow \left.\begin{aligned}\varepsilon_0\nabla\cdot\mathbf{E} + \nabla\cdot\mathbf{P} &= \rho\\ \varepsilon_0\mathbf{E} + \mathbf{P} &= \mathbf{D}\end{aligned}\right\} \Longrightarrow \nabla\cdot\mathbf{D} = \rho$$

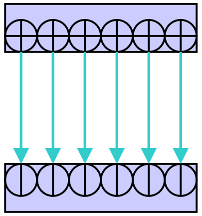
$\mu_0\nabla\times\mathbf{H} = \mu_0\mathbf{J}$ external current

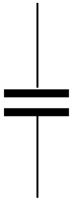
$$\left.\begin{aligned}\nabla\times\mathbf{B} &= \mu_0\nabla\times\mathbf{H} + \mu_0\nabla\times\mathbf{M}\\ \mathbf{M} &= \chi_m\mathbf{H}\\ \mu \equiv \mu_0(1+\chi_m) &\equiv \mu_0\mu_r\end{aligned}\right\} \Longrightarrow \nabla\times\mathbf{B} = \mu\mathbf{J}$$

$$\nabla\times\mathbf{H} = \mathbf{J}$$

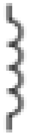
Side Notes

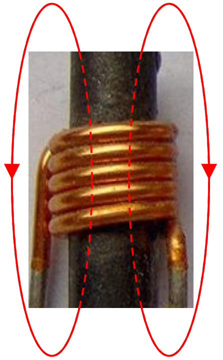
Another Way to Draw Analogy





In the context of circuit components, it makes sense to consider **D** as corresponding to **B** and **E** to **H** due to mathematical relations.





$E \propto V, D \propto Q = \int I dt,$ V & I are measured.

$H \propto I\ , B \propto \Phi = \int V dt,$ I & V are measured.

Parallel-plate $D = \rho_S = Q/A$

General $$Q = \int \mathbf{D}\cdot d\mathbf{S}$$

Solenoid $B = \Phi/A$

General $$\Phi = \int \mathbf{B}\cdot d\mathbf{S}$$

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E} + \chi_e \varepsilon_0 \mathbf{E} = \varepsilon \mathbf{E}$$

$$\varepsilon \equiv \varepsilon_0 (1 + \chi_e) \equiv \varepsilon_0 \varepsilon_r$$

$$\mathbf{M} = \chi_m \mathbf{H}$$

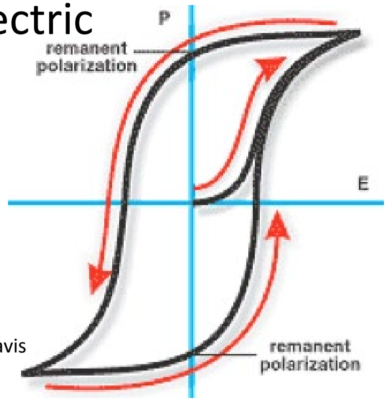
$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} = \mu_0 \mathbf{H} + \chi_m \mu_0 \mathbf{M} = \mu \mathbf{H}$$

$$\mu \equiv \mu_0 (1 + \chi_m) \equiv \mu_0 \mu_r$$

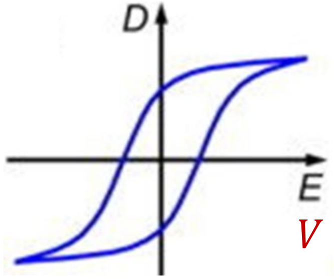
$$I = \frac{dQ}{dt} \qquad Q = CV = \int I dt$$

$$V = \frac{d\Phi}{dt} \qquad \Phi = LI = \int V dt$$

Ferroelectric

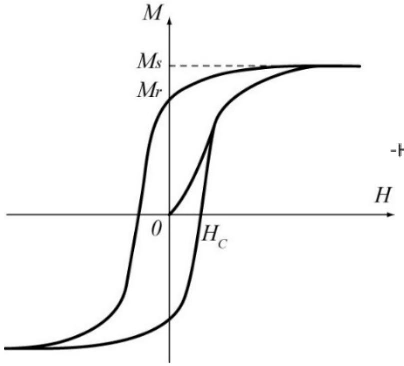


$Q = \int I dt$

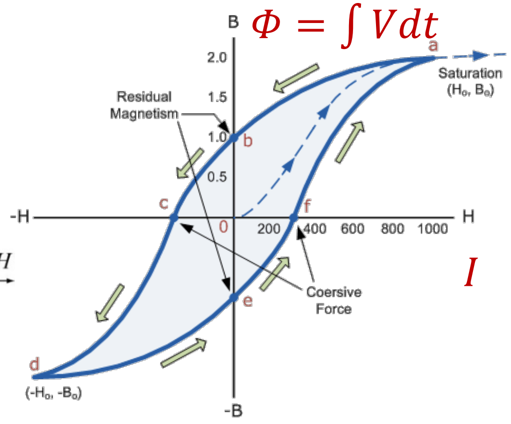


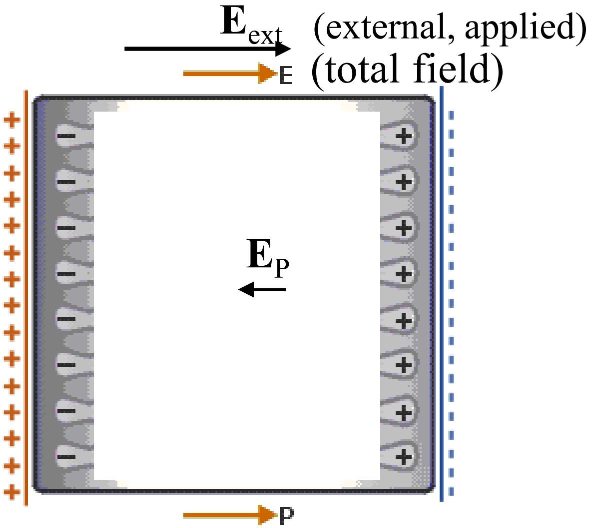
DOI: 10.1016/j.polymer.2013.01.035

Ferromagnetic



$\Phi = \int V dt$





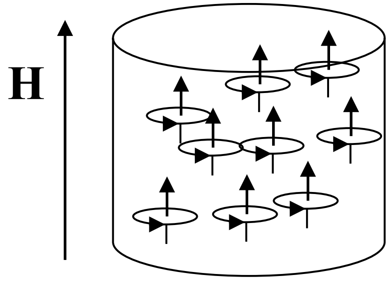
Dielectric polarization **P** always acts against external electric field.

The magnetization **M**, however, may be parallel or anti-parallel to the external magnetic field **H**.

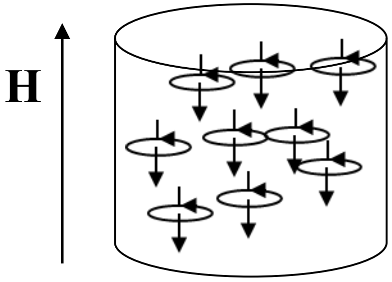
Paramagnetic: $\chi_m > 0, \mu_r = 1 + \chi_m > 1$

Diamagnetic: $\chi_m < 0, \mu_r = 1 + \chi_m < 1$

$$\boxed{\begin{array}{l}\mu_r \approx 1 \\ \mu \approx \mu_0\end{array}}$$



Paramagnetic



Diamagnetic

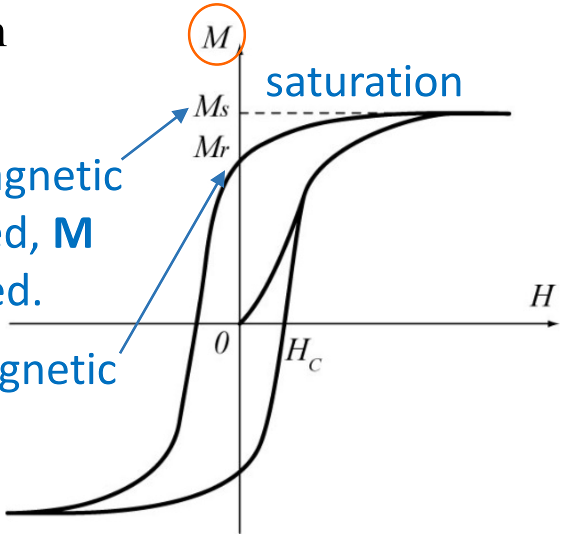
Ferromagnetic: Not due to dipole-dipole interaction!

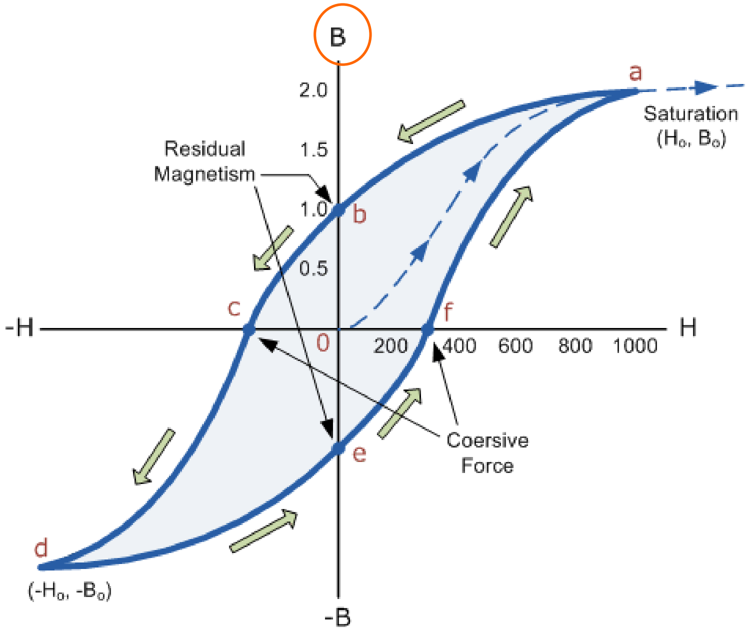
$\mu_r >> 1$, nonlinearity, hysteresis;

Spontaneous magnetization (w/o external field)

Intuitively, when all the magnetic moments have been aligned, **M** can not be further increased.

External field removed, magnetic moments still aligned → remnant/spontaneous magnetization





Ferromagnetic

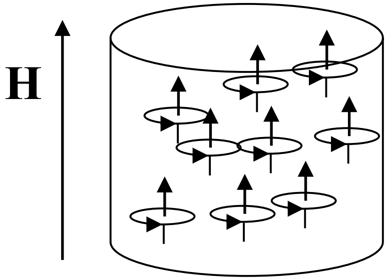
The description we give here is phenomenological – no real understanding. The explanation of paramagnetism, diamagnetism, and ferromagnetism are beyond the scope of this course.

> Now that we have tried to give you a qualitative explanation of diamagnetism and paramagnetism, we ***must*** correct ourselves and say that *it is not possible* to understand the magnetic effects of materials in any honest way from the point of view of classical physics. Such magnetic effects are a *completely quantum-mechanical phenomenon*.
>
> It is, however, possible to make some ***phoney*** classical arguments and to get some idea of what is going on.
>
> -- Richar Feynman

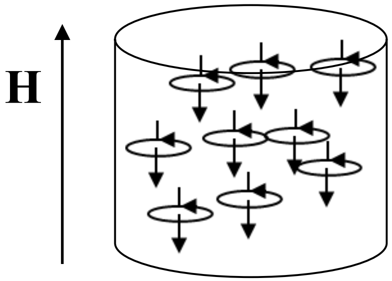
Other scientists would say "heuristic"

In the following, we try to give you some not-too-phoney explanations.

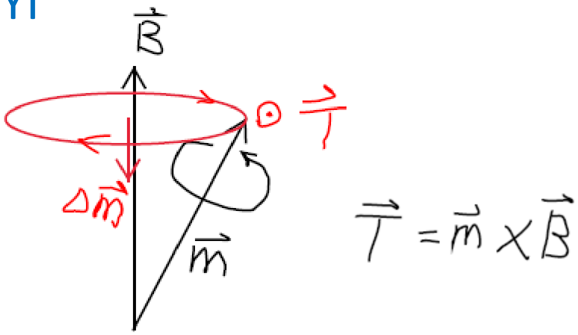
Paramagnetic: $\chi_m > 0$, $\mu_r = 1 + \chi_m > 1$
Material contains atoms with permanent magnetic moments, which are lined up by external magnetic field.



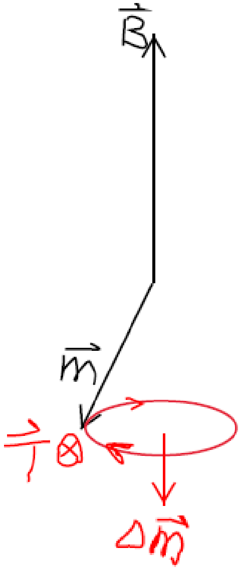
Diamagnetic: $\chi_m < 0$, $\mu_r = 1 + \chi_m < 1$
Exhibited by atoms without net permanent magnetic moments. Due to Larmor precession. Induced extra moment opposite to external magnetic field.



FYI



Electrons orbit and spin, each having an angular momentum **J**, and thus a magnetic moment **m** ∝ –**J**.



Each **m** precesses around **B** due to torque **T**, just like a gyro (or spin top): Lamor precession. The presession gives additional angular momentum Δ**J** and thus additional moment Δ**m**.

For an intuitive, easy-to-understand, classical analogy of Lamor precession, see precession of a gyro/spin top: https://en.wikipedia.org/wiki/Precession

If there is no net permanent magnetic moments, magnetic moments of electrons balance out. But, for opposite **m**, we have the same Δ**m**, always opposite to **B**: diamagnetic.

All materials have diamagnetism. In paramagnetic materials, paramagnetism dominates.

Ferromagnetic: Magnetic moments line up themselves without external field.
Should not exist had it not been for quantum mechanics.
Magnetic interaction among moments too weak even at 0.1 K temperature.

B: total effect of external & magnetization currents; felt by probe current.
H: allows us to consider external current only, with magnetization effects lumped into materials parameters χ_m, μ_r .

$$\boxed{\begin{array}{l}\mu \equiv \mu_0(1+\chi_m) \equiv \mu_0\mu_r \\ \mathbf{B} = \mu\mathbf{H} = \mu_r\mu_0\mathbf{H} = (1+\ \chi_m)\mu_0\mathbf{H}\end{array}}$$

External field due to external current

For most paramagnetic and diamagnetic materials: $\mu_r = 1 +\ \chi_m \approx 1$ for practical purposes.
For ferromagnetic materials, μ_r is large.

Compare this with:

E: total effect of external & polarization charges; felt by probe charge.
D: allows us to consider external charge only, with polarization effects lumped into materials parameters χ, ε_r. $\varepsilon_r = 1+\chi > 1$, $\varepsilon > \varepsilon_0$.

$$\boxed{\varepsilon \equiv \varepsilon_0(1+\chi_e) \equiv \varepsilon_0\varepsilon_r \quad \mathbf{D} \equiv \varepsilon_0\varepsilon_r\mathbf{E} \equiv \varepsilon\mathbf{E}}$$

Total field

$\varepsilon_r = 1$ for air. ε_r between 2 and 3 for plastics. $\varepsilon_r = 3.9$ for SiO_2. $\varepsilon_r \sim 10$ or more for high-k dielectrics

Notice the different "accounting" for magnetic and electric fields.

$\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$ is equivalent to $\varepsilon_0\mathbf{E} = \mathbf{D} - \mathbf{P}$ or $\mathbf{E} = \frac{1}{\varepsilon_0}\mathbf{D} - \frac{1}{\varepsilon_0}\mathbf{P}$.

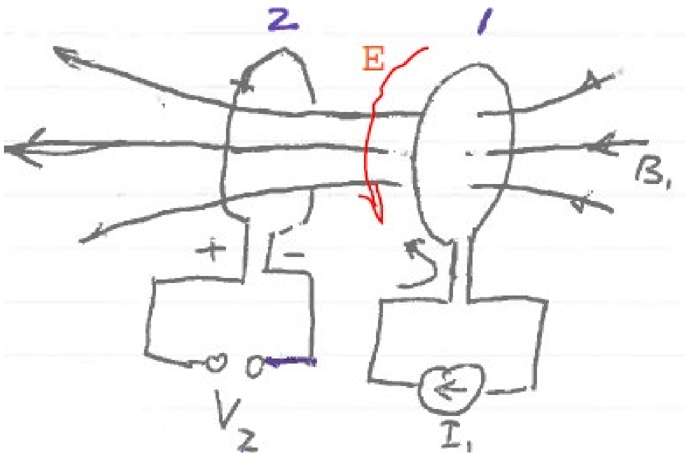
Here is the coupling between two loops/coils

Feed a current to loop 1, which induces $\mathbf{B}_1$.

Part of the flux, Φ_{12}, goes through loop 2.

$\mathbf{B}_1$ increases as I_1 increases. So does Φ_{12}.

The changing Φ_{12} induces an emf in loop 2.



Make sure you get the directions/polarities right.

Here is the coupling between two loops/coils

Feed a current to loop 1, which induces $\mathbf{B}_1$.

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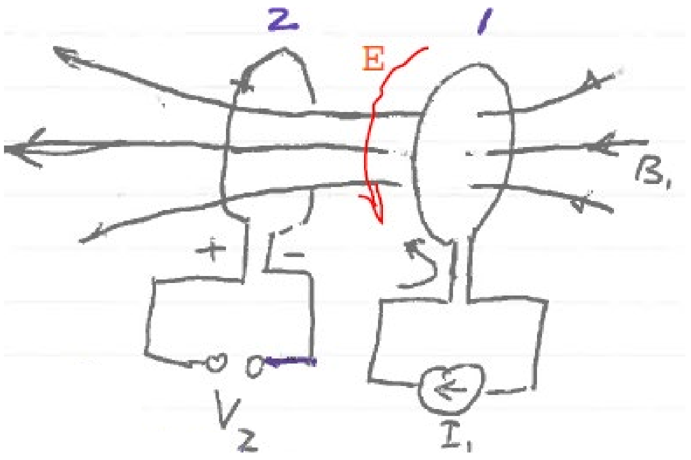
$\mathbf{B}_1$ increases as I_1 increases. So does Φ_{12}.

The changing Φ_{12} induces an emf in loop 2.

$$\Phi_{12} = \oint_{S_2} \vec{B}_1 \cdot d\vec{S}_2 \propto I_1$$

$$\Lambda_{12} = N_2 \Phi_{12} \equiv L_{12} I_1$$

$$V_2 = \frac{d\Lambda_{12}}{dt} = L_{12} \frac{dI_1}{dt}$$



Make sure you get the directions/polarities right.

Question:

If I_1 is sinusoidal, what is the phase difference between V_2 and I_1?

Here is the coupling between two loops/coils

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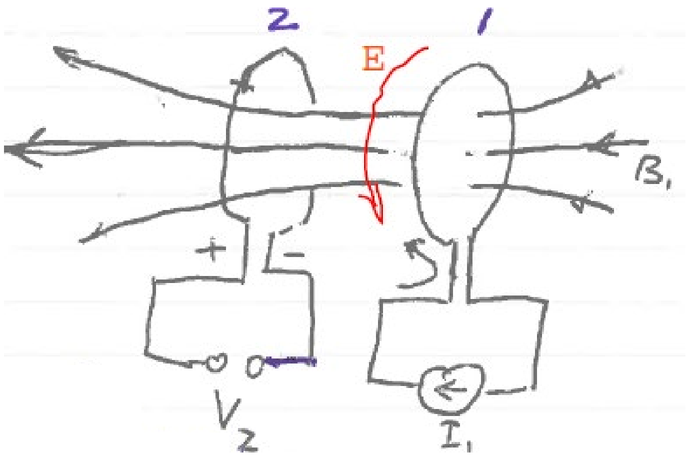


Make sure you get the directions/polarities right.

$$\Phi_{12} = \oint_{S_2} \vec{B}_1 \cdot d\vec{S}_2 \propto I_1$$

$$\Lambda_{12} = N_2 \Phi_{12} \equiv L_{12} I_1$$

$$V_2 = \frac{d\Lambda_{12}}{dt} = L_{12} \frac{dI_1}{dt}$$

Question:

If I_1 is sinusoidal, what is the phase difference between V_2 and I_1?

Similarly, $V_1 = \frac{d\Lambda_{21}}{dt} = L_{21} \frac{dI_2}{dt}$

It is mathematically shown that $L_{12} = L_{21}$

More generally, a changing current induces an emf in a nearby circuit/conductor.

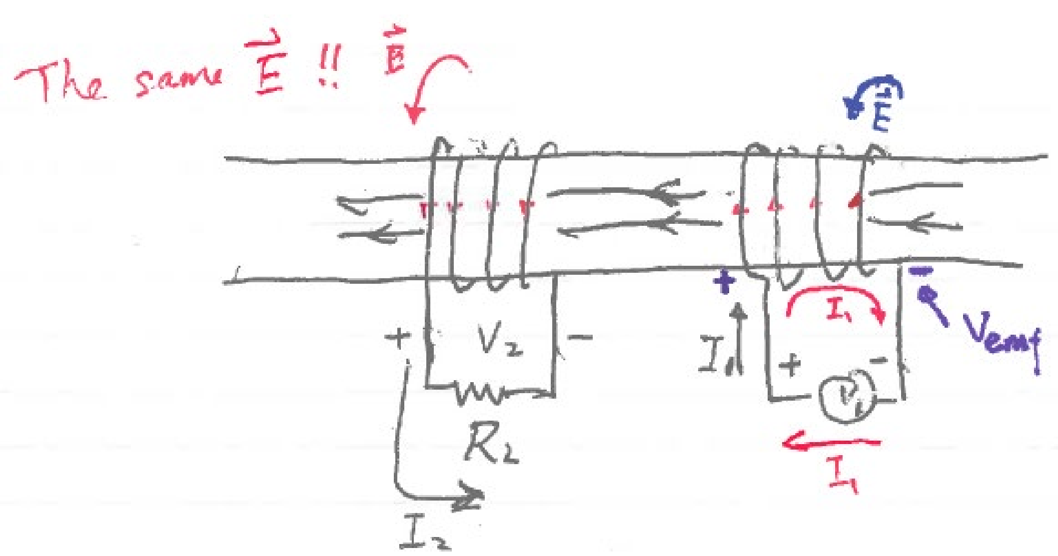
What if we wind the two coils around a magnetic material with very high μ?

Recall that magnetic materials ($\mu_r >> 1$) confine the magnetic field.

Say, $\mu = \infty$. There will be no flux leakage.
All magnetic flux Φ generated by coil 1 goes through coil 2.

When applied a voltage V_1, coil 1 has to develop an emf exactly countering it.

$$V_1 = N_1 \frac{d\Phi}{dt}.$$



The same Φ goes through coil 2.

$$V_2 = N_2 \frac{d\Phi}{dt}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

By energy conservation,

$$V_1 I_1 = V_2 I_2 \Rightarrow \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Make sure you get the directions/polarities right.

The input impedance of coil 1 is

Notice that coil 1 is a load to the voltage source, while coil 2 is giving power to the load resistor.

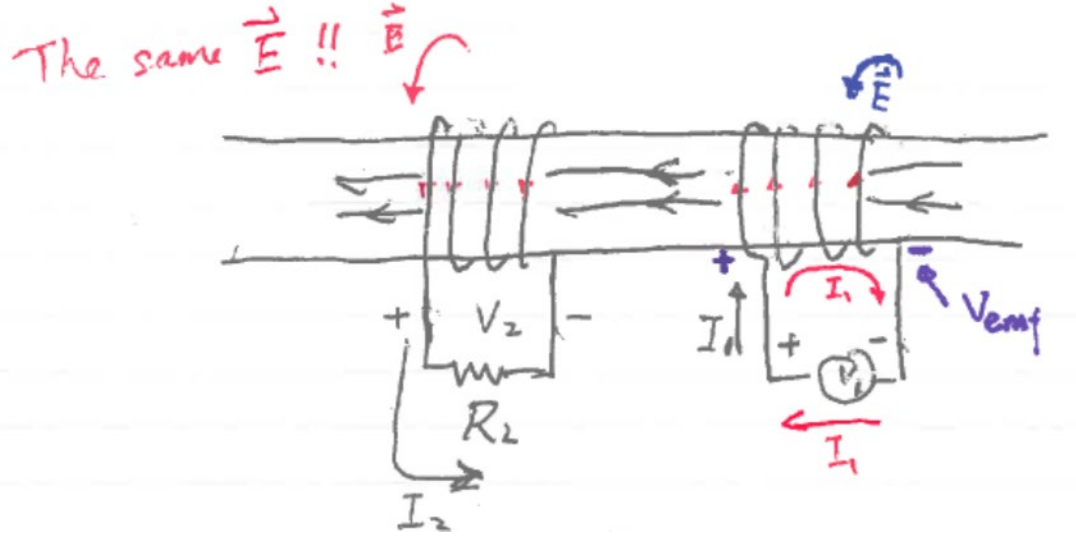
$$R_{in} = \frac{V_1}{I_1} = \frac{\frac{N_1}{N_2} V_2}{\frac{N_2}{N_1} I_2} = \left(\frac{N_1}{N_2}\right)^2 R_2$$

(used for impedance matching for amplifiers)

What is this that we are talking about?

This is the **ideal** transformer.

Assuming $\mu = \infty$ for the magnetic core.



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$R_{in} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

The input impedance of coil 1 is resistive. Actually, V_1, V_2, I_1, and I_2 are all in phase.

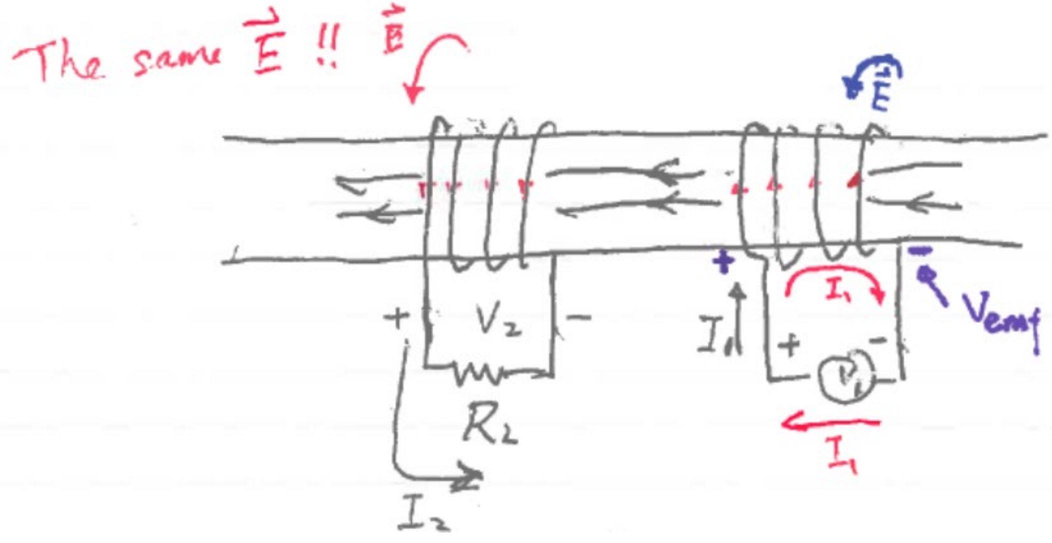
Wait a minute, is this right?

Should the input impedance of coil 1 be inductive?

I_1 depends on R_L. Given V_1, Φ is determined. This means that no matter what I_1 is (depending on R_L), we always have the same Φ. But should Φ depend on I_1?

This is the ideal transformer.

Assuming $\mu = \infty$ for the magnetic core.



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

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Wait a minute, is this right?
Should the input impedance of coil 1 be inductive?
I_1 depends on R_L. Given V_1, Φ is determined. This means that no matter what I_1 is (depending on R_L), we always have the same Φ. But should Φ depend on I_1?

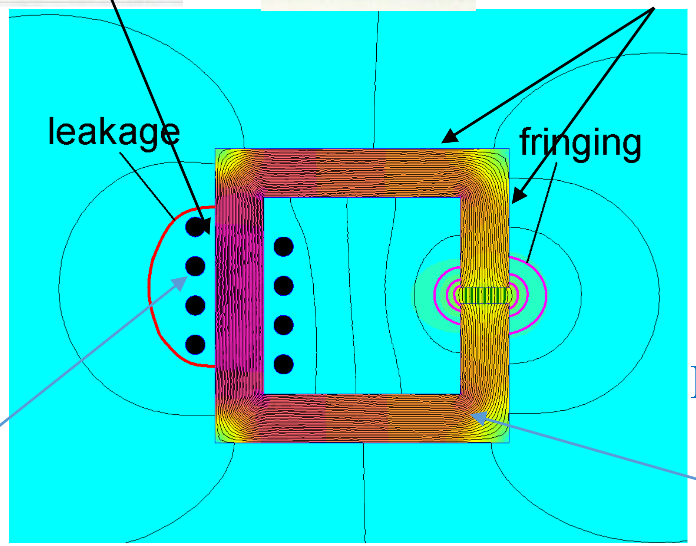
Hint:

- Keep in mind that we assume $\mu = \infty$ for the magnetic core. (Ideal!)
- In absence of coil 2, what would be I_1? What would be the input impedance of coil 1?
- If R_L is replaced with an open circuit, answer the above questions.
- Now, we have a finite R_L. Therefore a finite I_2, which induces a finite H field in the core. The corresponding B field is infinite since $\mu = \infty$! But don't worry. Figure out its direction. This H field will be exactly canceled by that generated by I_1. $N_1 I_1 = N_2 I_2$.

Magnetic materials ($\mu_r >> 1$) confine the magnetic field

Recall magnetic boundary conditions.

$$H_{2t} - H_{1t} = J_s \qquad B_{1n} = B_{2n} \qquad H_{1t} = H_{2t}$$



B field distribution

http://www.encyclopedia-magnetica.com/doku.php/flux_fringing

1: air

2: magnetic core material

$$\left.\begin{array}{l} H_{2t} = H_{1t} \\ B = \mu H \\ \mu_2 >> \mu_1 \end{array}\right\} \Rightarrow B_{2t} >> B_{1t}$$

Magnetic materials ($\mu_r >> 1$) also give you a lot more B field out of the same I

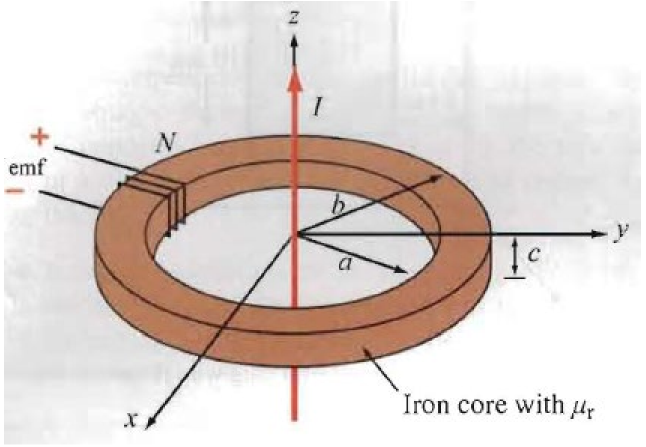
The clamp meter is a great tool to measure an AC current.

$$\oint \vec{H} \cdot d\vec{\ell} = I$$

$$B = \mu H$$

$$\text{emf} = N \int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

In general, a changing current induces an emf in a nearby circuit/conductor.



Review textbook Sections 5.5, 5.7-3, 6-3.
Notice that we discuss topics in a different sequence than in the book, for better understanding. Review the notes, think about the questions.