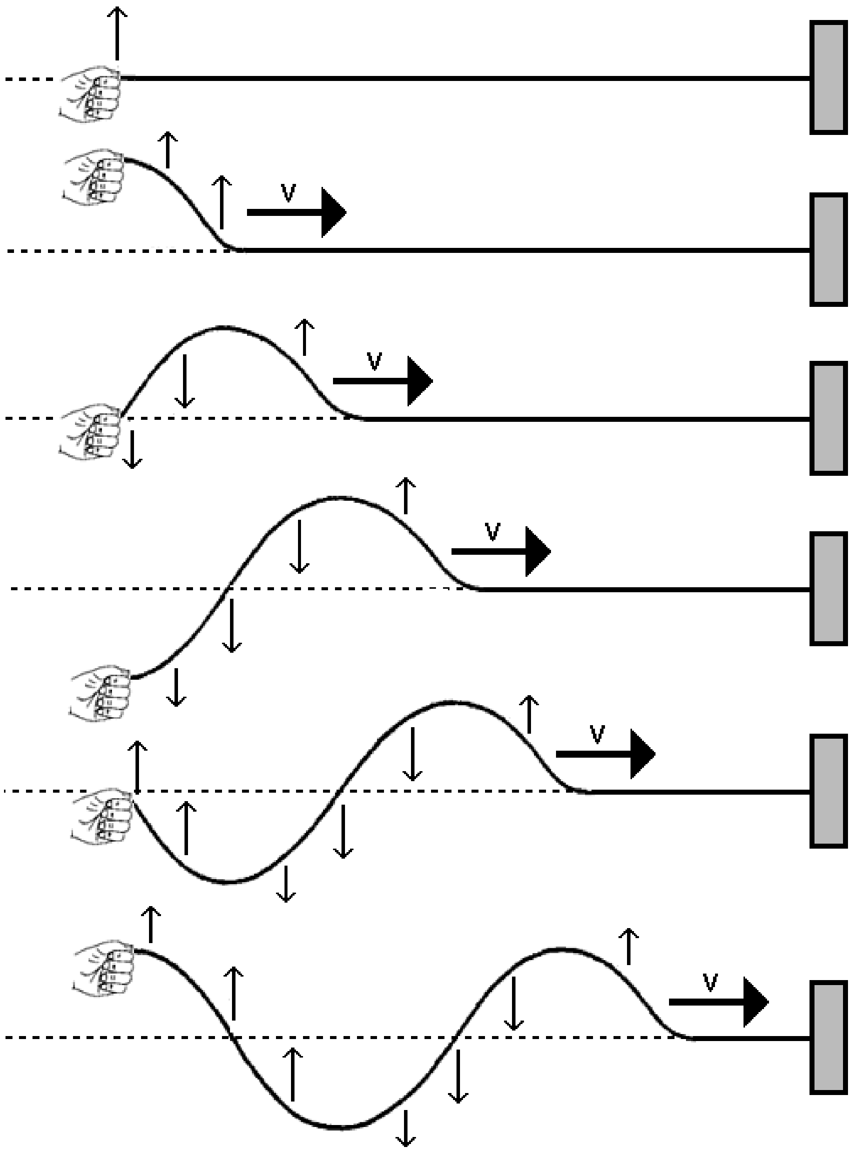
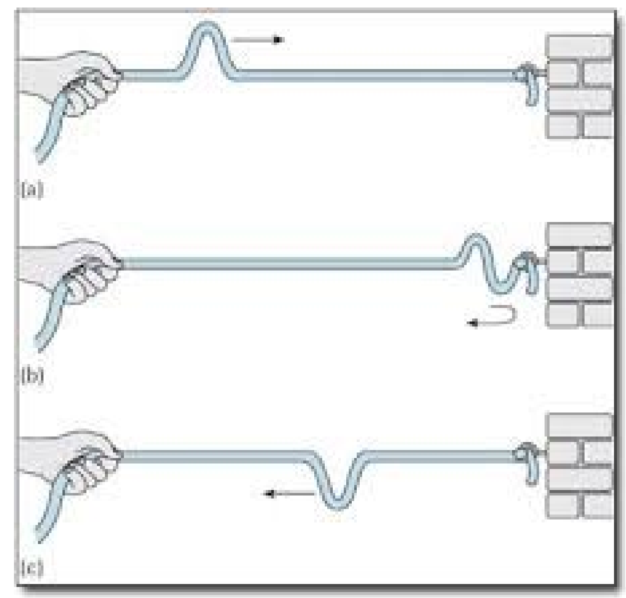
Traveling Waves

The one-dimensional (1D) case



The "perturbation" propagates on.

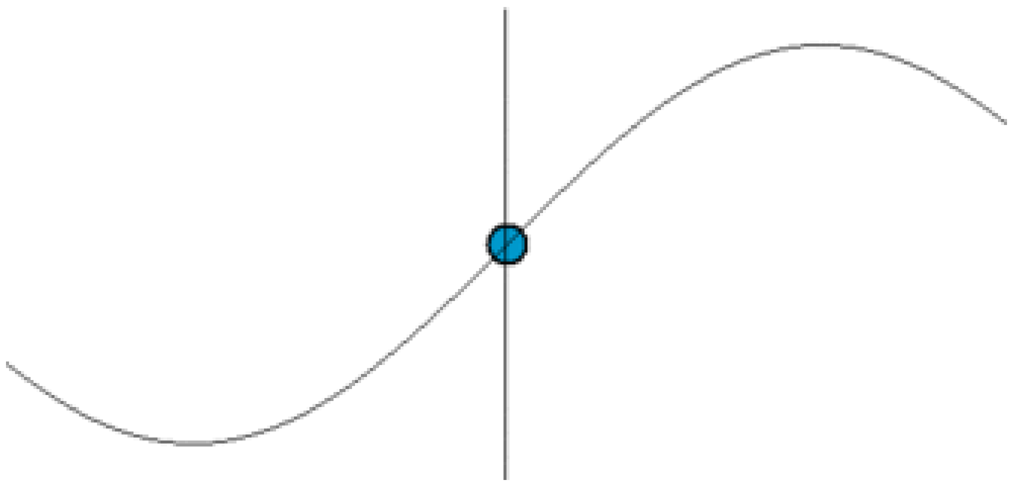
A traveling wave is the propagation of motion (disturbance) in a medium.



Reflection

Why is there reflection?

Visualization



https://zh.wikipedia.org/wiki/%E6%B3%A2#/media/File:Simple_harmonic_motion_animation.gif

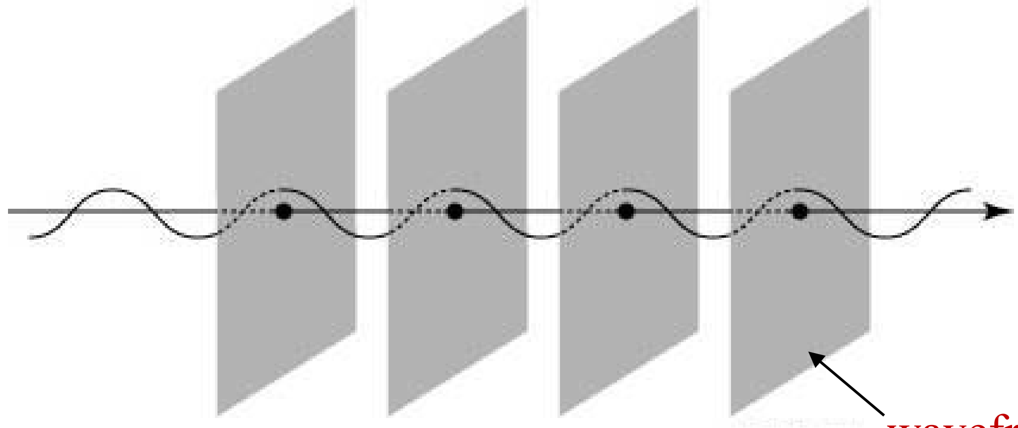
(Watch animation at the link)

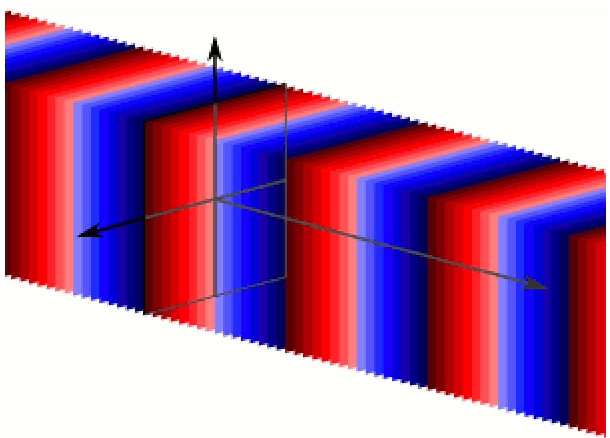
You can take a snapshot at a certain time.
You can trace its waveform at a certain location.

Traveling Wave in Higher Dimensions

Plane waves in 3D

Example: sound waves

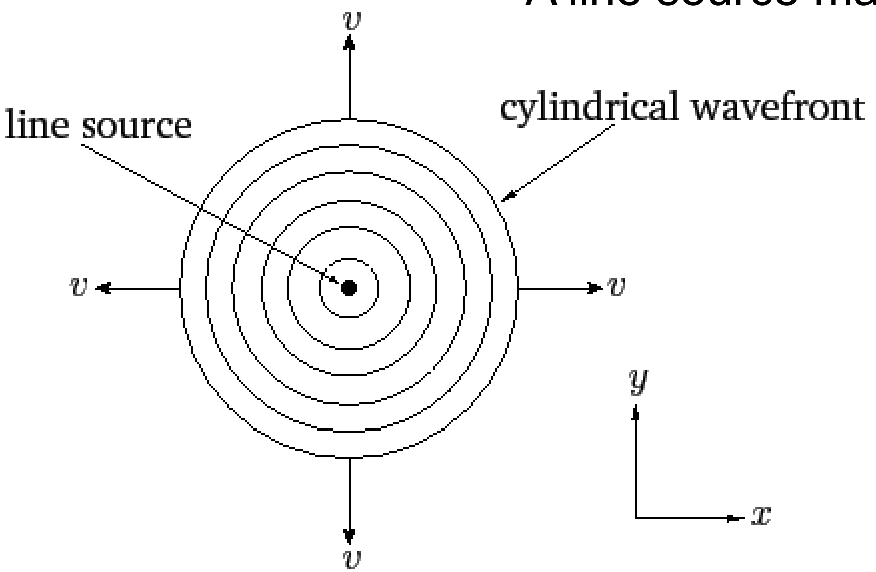




The plane wave is a 1D wave in 3D space:
No variation in the 2D plane of a wave front.
A wave front is a surface of equal phase.

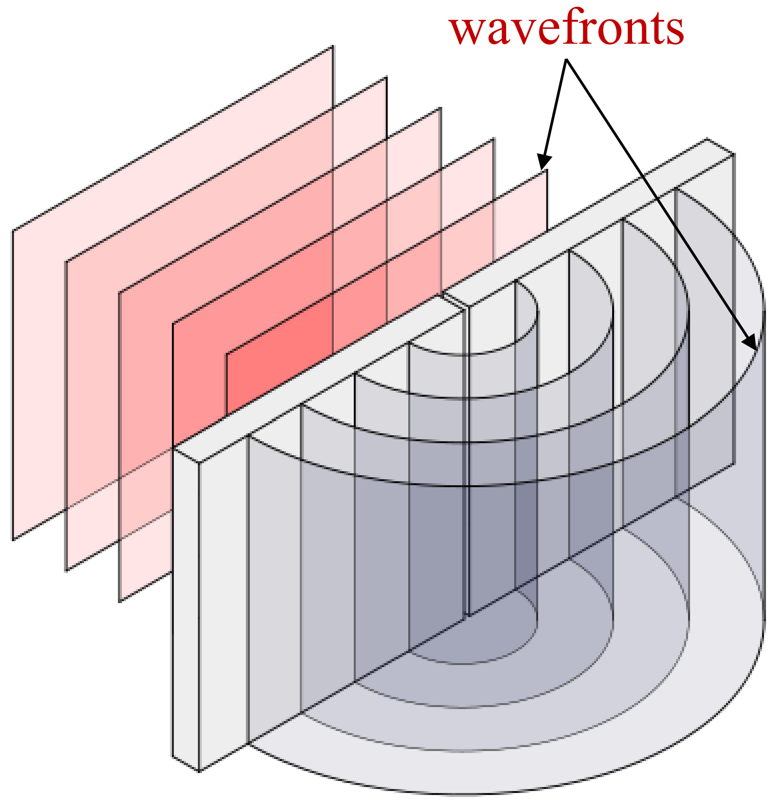
Watch animation: http://en.wikipedia.org/wiki/Plane_wave

A line source makes a cylindrical wave.



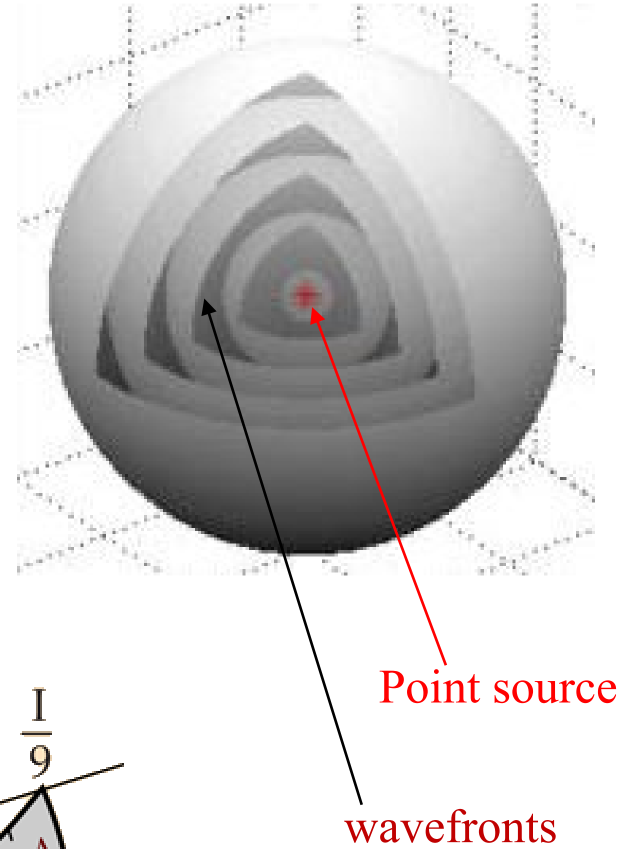
Cylindrical wave (3D; top view)

Water surface wave (2D)
(Circular wave)

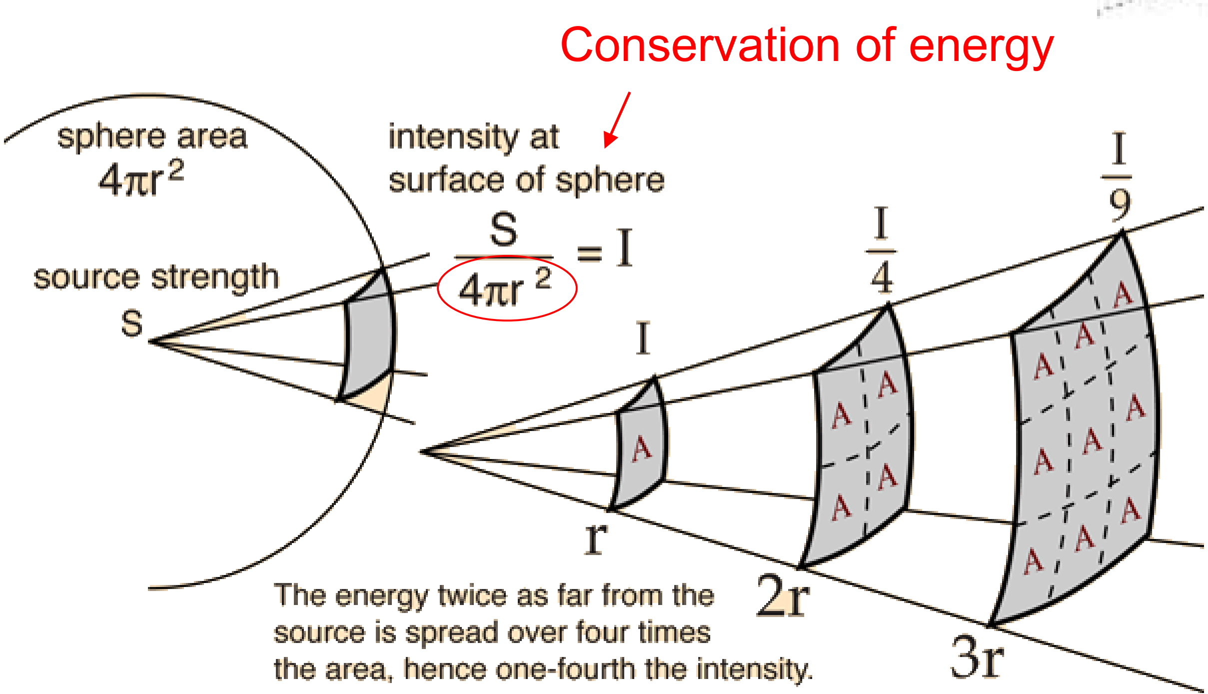


Make a cylindrical wave from a plane wave

A point source makes a spherical wave, the wave front of which are spherical surfaces.



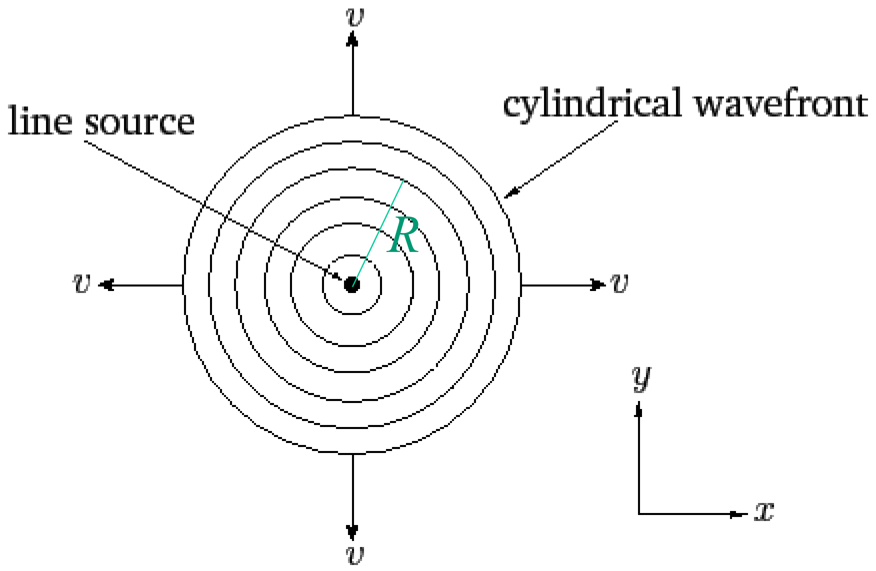
Intensity is energy carried per time per area, i.e., power delivered per area



Intensity $\propto 1/r^2$
Amplitude $\propto 1/r$

How does the intensity & amplitude of a cylindrical wave depend on R?

$$R^2 = x^2 + y^2$$



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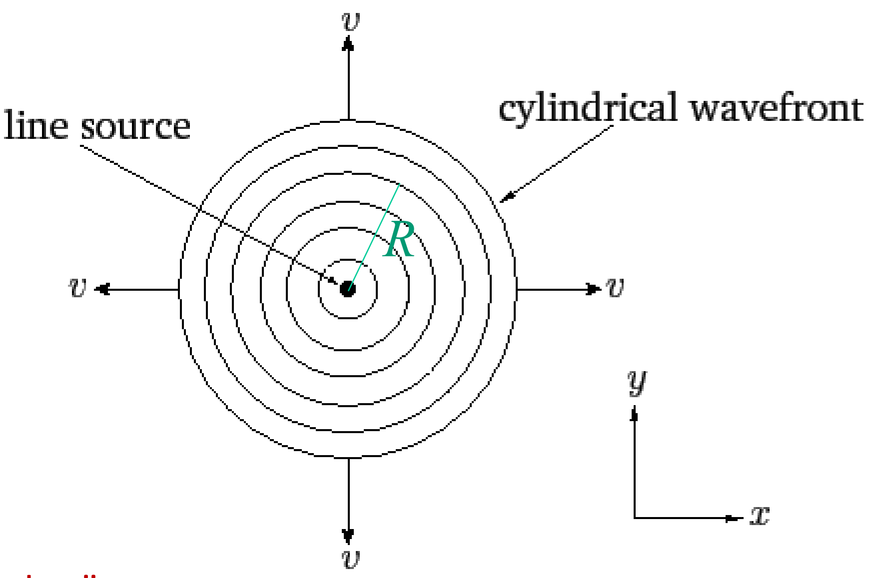


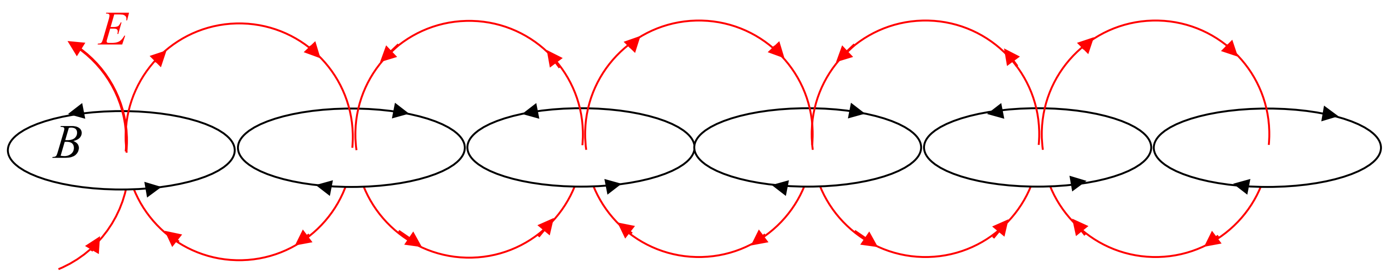
Circumference $\propto R$

symbol for "be proportional to"

Therefore,

Intensity $\propto 1/R$
Amplitude $\propto 1/(R^{1/2})$

Electromagnetic Wave



Somehow start with a changing electric field E, say $E \propto \sin\omega t$

The changing electric field induces a magnetic field, $B \propto \dfrac{\partial E}{\partial t} \propto \cos\omega t$

If the induced magnetic field is changing with time, it will in turn induce an electric field

$$E \propto -\frac{\partial B}{\partial t} \propto \sin\omega t$$

Notice that $\dfrac{d}{dt}\cos\omega t = -\omega\sin\omega t$

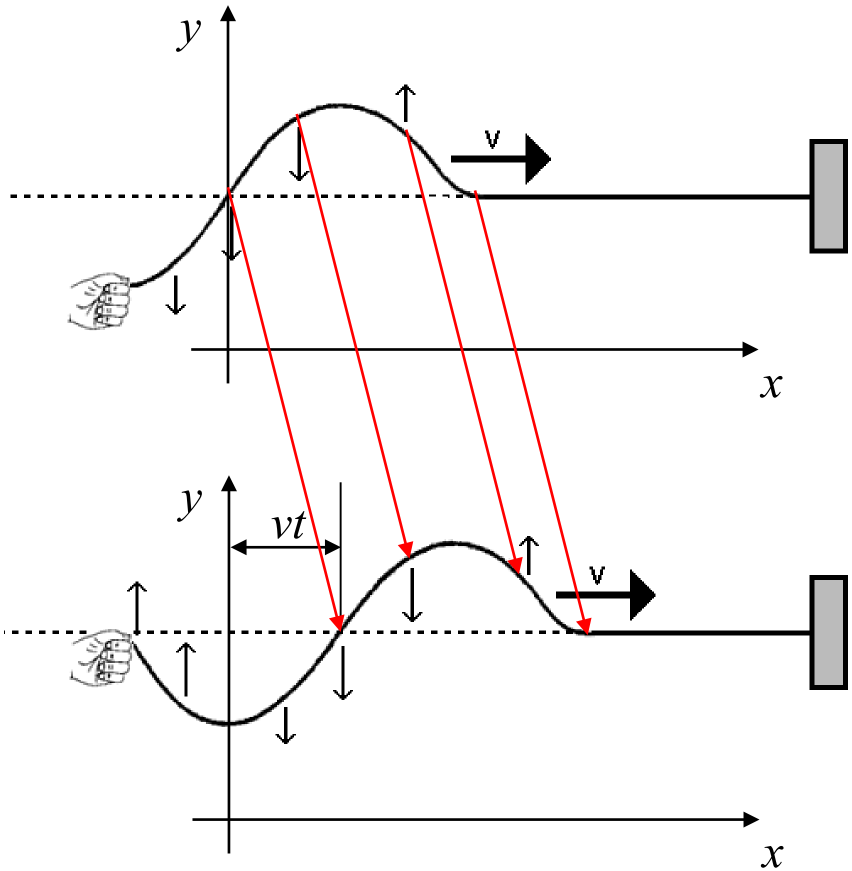
Negative signs cancel

And so on and so on....
Just as the mechanical wave on a string.

Note: The picture is NOT of a plane wave, but depicts a wave emitted by a source of limited size. Will revisit this picture later.

Mathematical Expression of the Traveling Wave

A traveling wave is the propagation of motion (disturbance) in a medium.



At time 0,

$$y = f(x)$$

At time t,

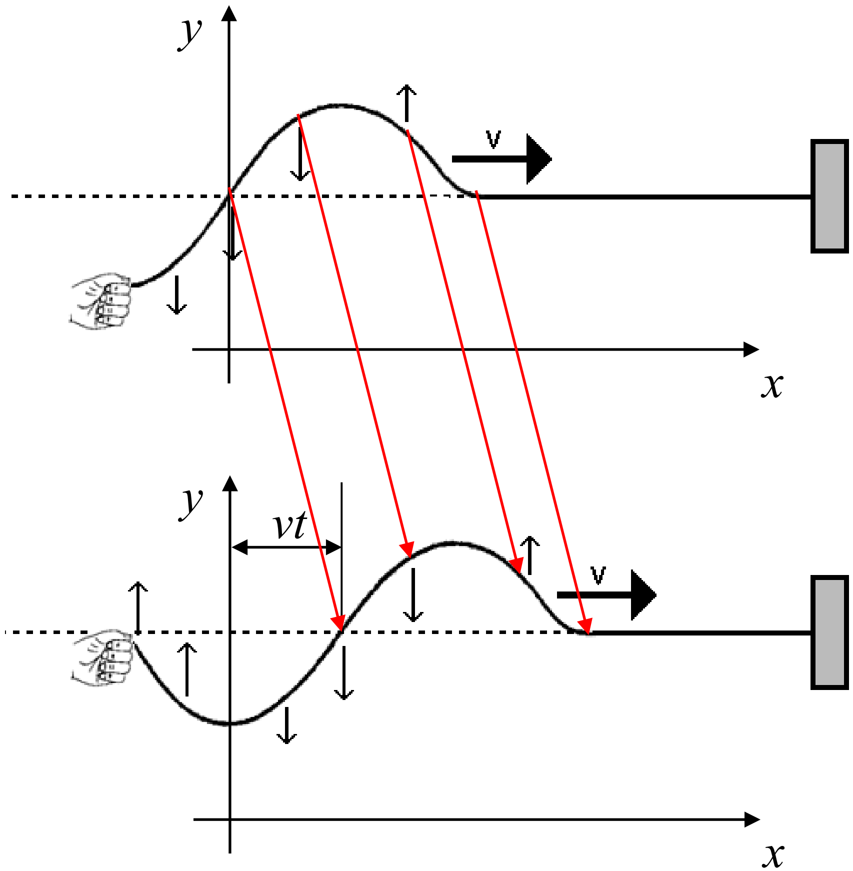
$$y = f(x - vt)$$

The wave form shifts vt in time t.

This is the general expression of traveling waves.

Mathematical Expression of the Traveling Wave

A traveling wave is the propagation of motion (disturbance) in a medium.



At time 0,

$y = f(x)$

At time t,

$y = f(x - vt)$

The wave form shifts vt in time t.

This is the general expression of traveling waves.

Questions:

What kind of wave does $y = f(x + vt)$ stand for?

What about $y = f(vt - x)$?

What about $y = f(vt - x)$?

$$f(vt - x) = f[-(x - vt)]$$

Define $f(-x)$ your "new f", or $g(x) \equiv f(-x)$, so it's the same wave!

So, which way should I go? $f(x - vt)$ or $g(vt - x)$?

To your convenience!

No big deal.

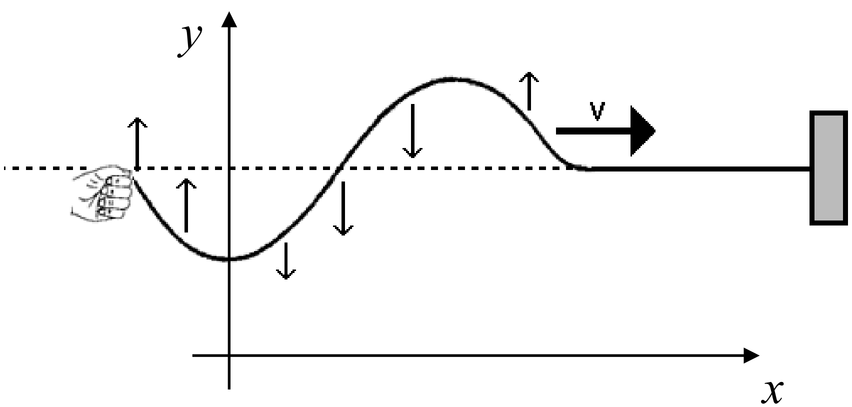
What about $y = f(vt - x)$?

$$f(vt - x) = f[-(x - vt)]$$

Define $f(-x)$ your "new f", or $g(x) \equiv f(-x)$, so it's the same wave!

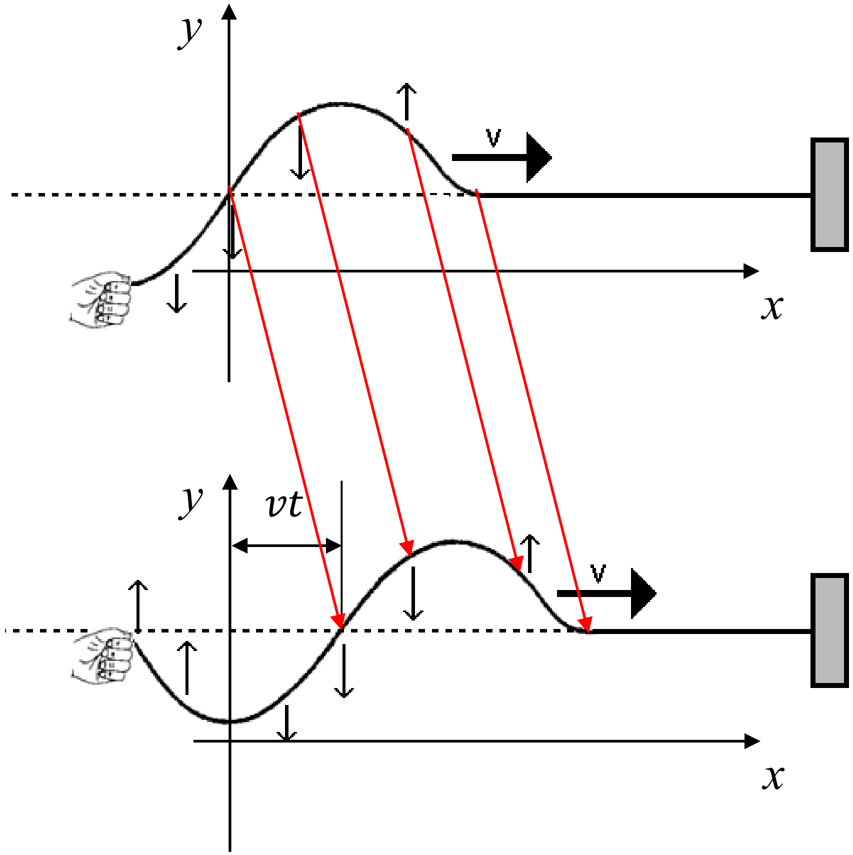
So, which way should I go? $f(x - vt)$ or $g(vt - x)$?
To your convenience!
No big deal. But this affects how we define our "sign conventions."
People in different disciplines use different conventions.

If you are more concerned about seeing waveforms on an oscilloscope at locations x, you like $g(vt - x)$ better.
If you are more concerned about seeing snapshots of the wave at times t, you like $f(x - vt)$ better.
We will talk later about how this choice affects the ways to write the "same" (but apparently different) equations in different disciplines (EE vs. physics).



A snapshot of a string

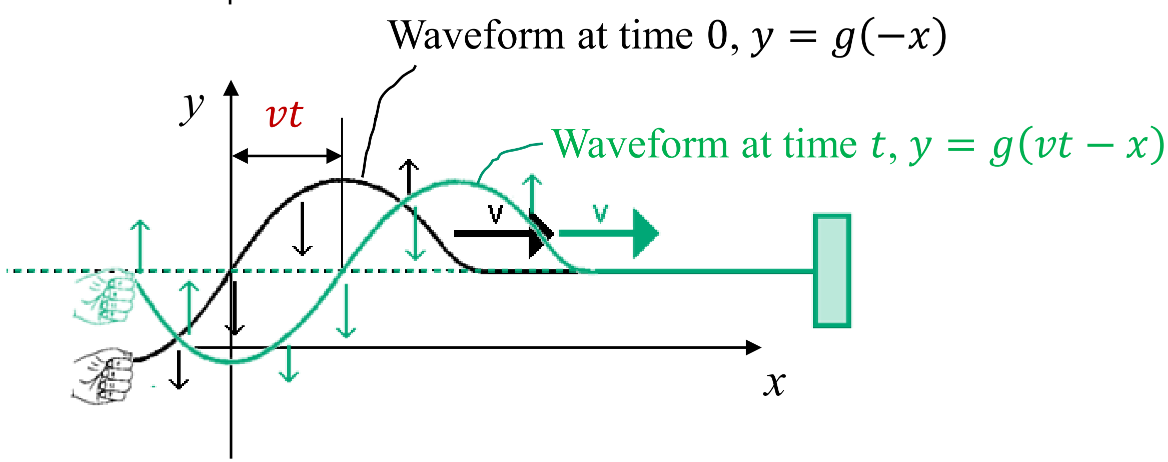
Snapshots of the string displacement y at different times



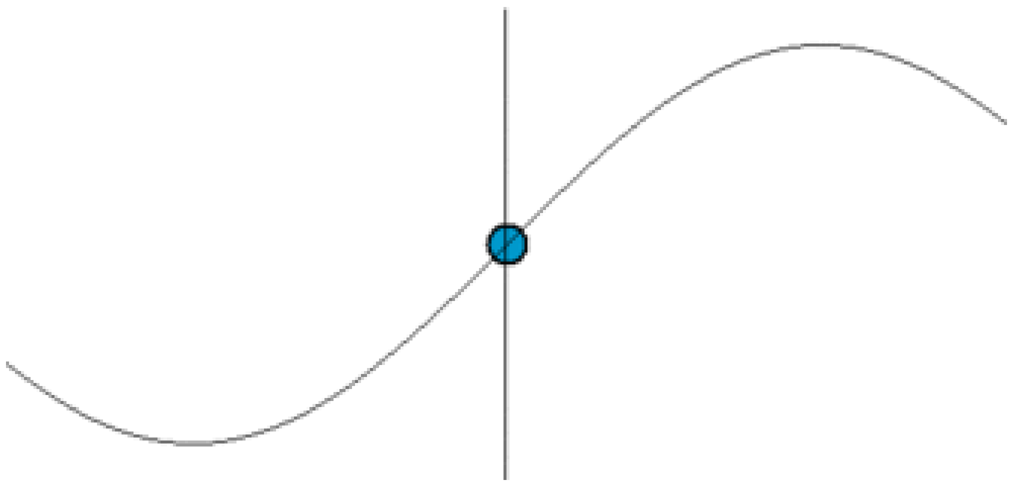
At time 0,
$y = g(-x)$

At time t,
$y = g(vt - x)$

The wave form shifts vt in time t.



Visualization



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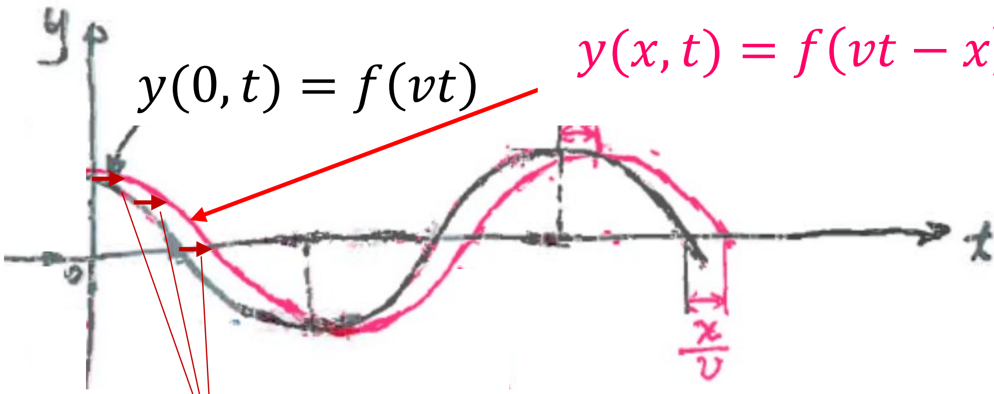
(Watch animation at the link)

You can take a snapshot at a certain time.
You can trace its waveform at a certain location.

Waveforms of the string displacement y at different locations

$$y(x,t) = f(vt - x) = f\left[v\left(t - \frac{x}{v}\right)\right]$$

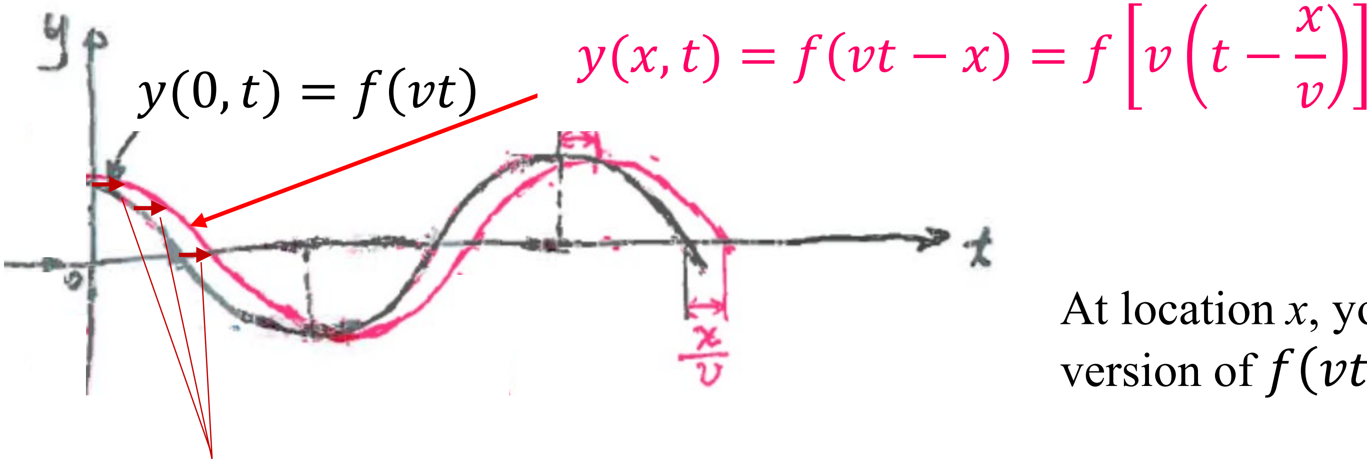
$$y(0,t) = f(vt)$$



At location x, you have a time-delayed version of $f(vt)$.

The time delay is simply the time for the wave to travel a distance x, i.e., $\frac{x}{v}$.

Waveforms of the string displacement y at different locations



At location x, you have a time-delayed version of $f(vt)$.

The time delay is simply the time for the wave to travel a distance x, i.e., $\frac{x}{v}$.

For a single-wavelength, sinusoidal wave,
the snapshot and waveform shift rigidly,
because the wave travels at just one speed, v.

(We have just touched on an important concept called "dispersion", to be discussed later.)

Let's now look at the special case of the sinusoidal wave

phase

What is the dimension (unit) of the phase?

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

Phase at $t = 0$ as a function of x

$$= A\cos\left[\beta\left(\frac{\omega}{\beta}t - x + \frac{\phi_0}{\beta}\right)\right]$$

$$= A\cos\left[\beta(vt - x) + \phi_0\right]$$

$$= f(vt - x)$$

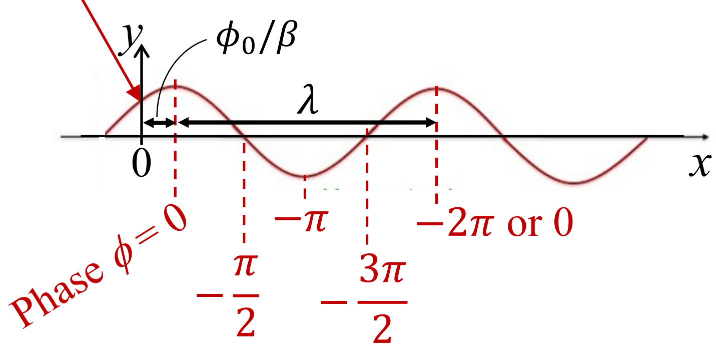
ω: angular frequency

β: propagation constant

You can write this function, or group the terms, in so many ways. It's just about how you view them.

"snapshot" at $t = 0$: $y(x,0) = A\cos(-\beta x + \phi_0)$

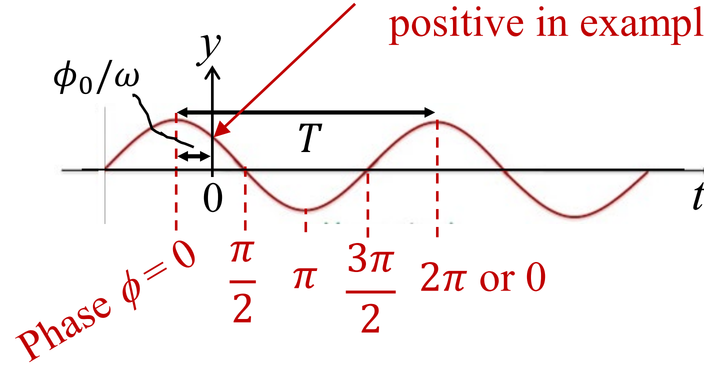
Phase at $t = 0$, $x = 0$ is ϕ_0, positive in example



What is the dimension (unit) of β?

"waveform" at $x = 0$: $y(0,t) = A\cos(\omega t + \phi_0)$

Phase at $t = 0$, $x = 0$ is ϕ_0, positive in example



What is the dimension (unit) of ω?

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

$$= A\cos\left[\beta\left(\frac{\omega}{\beta}t - x + \frac{\phi_0}{\beta}\right)\right]$$

$$= A\cos[\beta(v t - x) + \phi_0]$$

$$v = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$$

$$= \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T}$$

One wavelength λ traveled in one period T.

λ is the "spatial period"; $\frac{1}{\lambda}$ is the "spatial frequency".

And, β is the spatial equivalent of ω. Call it the wave vector or propagation constant.

$\frac{\omega}{\beta} \equiv v$ is called the phase velocity, thus also denoted v_p.

In general, v_p depends on frequency ("dispersion").

In free space (i.e. vacuum), $v = c = \frac{\omega}{\beta}$, or $\omega = c\beta$.

No "dispersion".
(c is a constant)

Revisit (offline) the reference phase ϕ_0

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

$$= A\cos\left[\beta\left(\frac{\omega}{\beta}t - x + \frac{\phi_0}{\beta}\right)\right]$$

$$= A\cos[\beta(vt - x) + \phi_0]$$

ϕ_0 is the reference phase (the wave's phase with time and space set to zero).

$$y(x,t) = A\cos\left[\omega\left(t + \frac{\phi_0}{\omega}\right) - \beta x\right]$$

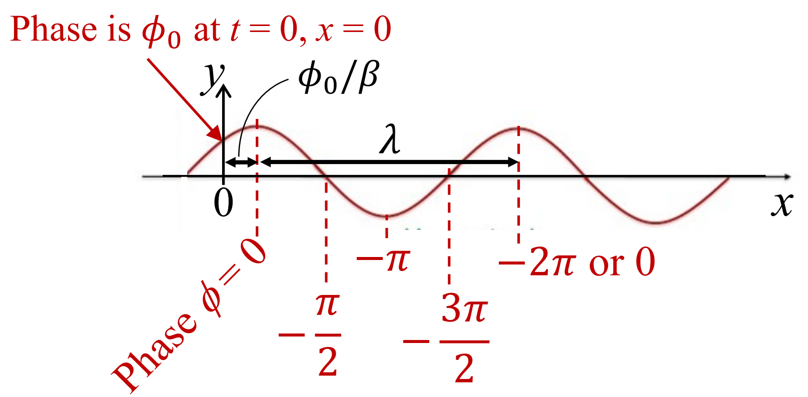
$$= A\cos\left[\omega t - \beta\left(x - \frac{\phi_0}{\beta}\right)\right]$$

$-\phi_0/\omega$ viewed as a shift in time
See right bottom figure.

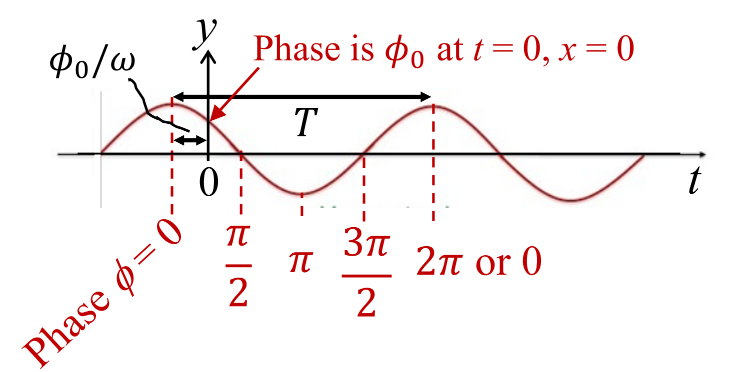
ϕ_0/β viewed as a shift in position
See left bottom figure.

Two ways to look at this. Two ways to group the terms.

"snapshot" at $t = 0$: $y(x,0) = A\cos(-\beta x + \phi_0)$

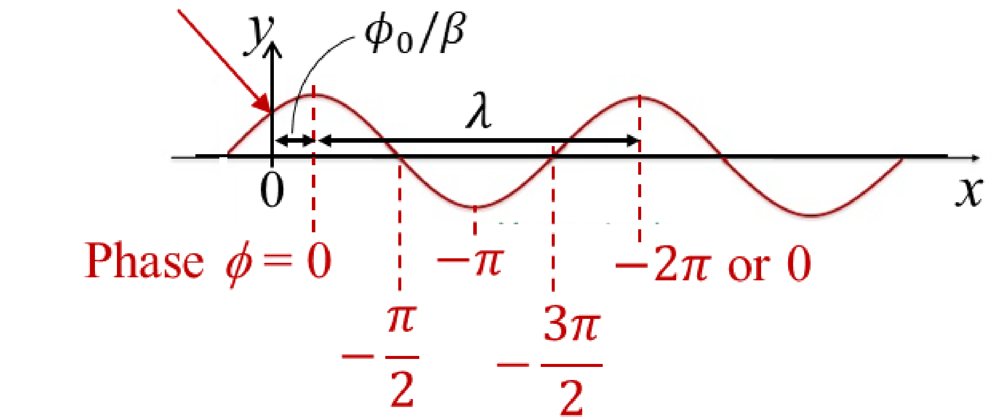


"waveform" at $x = 0$: $y(0,t) = A\cos(\omega t + \phi_0)$



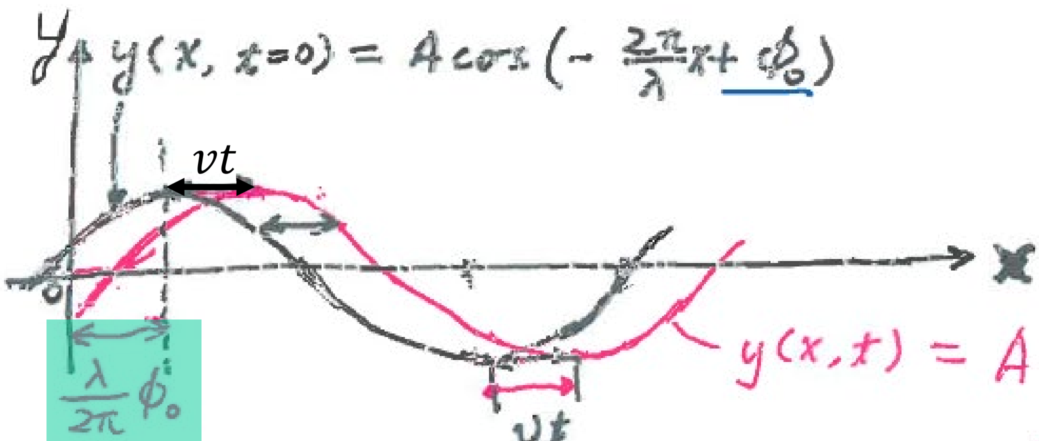
Revisit (offline) for harmonic waves: Snapshots at different times

"snapshot" at $t = 0$: $y(x, 0) = A\cos(-\beta x + \phi_0)$



$$y(x,t) = A\cos(\omega t - \beta x + \phi_0) = A\cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \phi_0\right)$$

"Spatial period"



Convert phase to distance

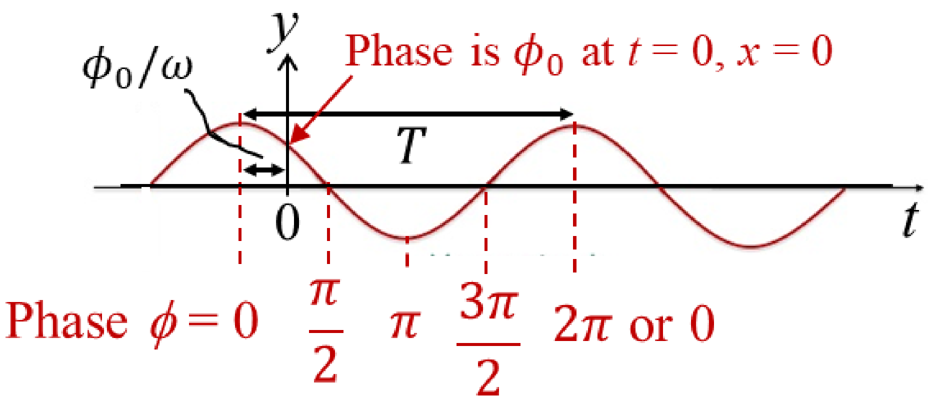
Take snapshots at different times

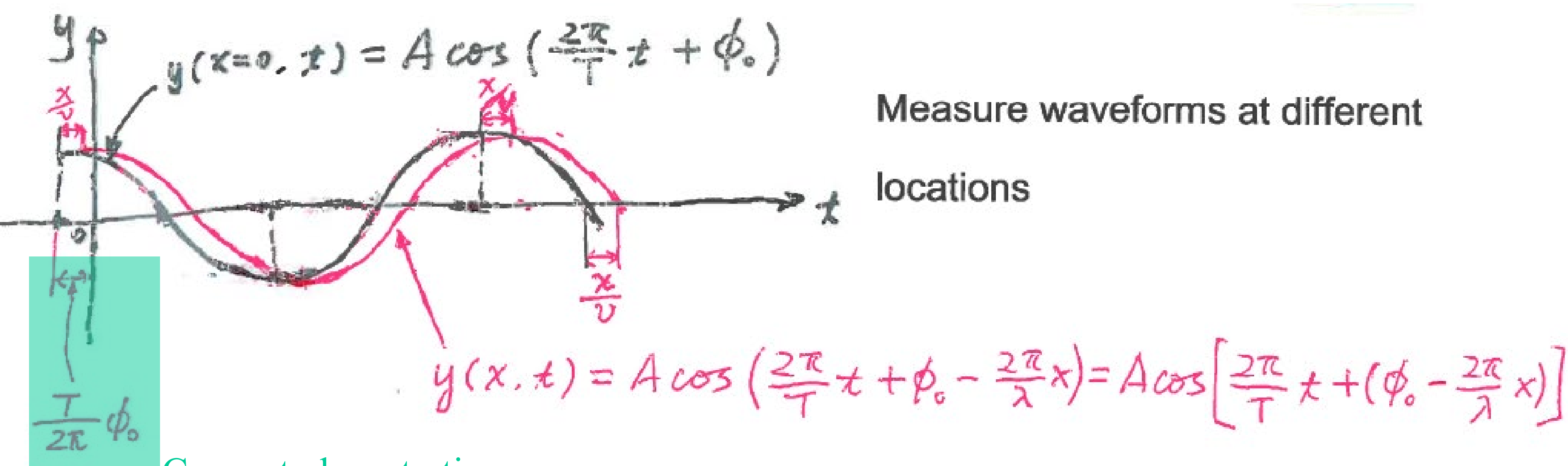
$$y(x,t) = A\cos\left[\left(-\frac{2\pi}{\lambda}x + \phi_0\right) + \frac{2\pi}{T}t\right]$$

$$= A\cos\left[-\frac{2\pi}{\lambda}x + \left(\phi_0 + \frac{2\pi}{T}t\right)\right]$$

Revisit (offline) for harmonic waves: Waveforms at different locations

"waveform" at $x = 0$: $y(0,t) = A\cos(\omega t + \phi_0)$





Measure waveforms at different locations

Convert phase to time

Read textbook Section 1-4 overview and 1-4.1, then work on HW1, P1 – P7.

Both the Homework & Answer sheet are online.
Review class notes and read the textbook, **then** do the homework, **then** check answers.

For your curiosity:
Next 3 slides about dispersion – phase velocity vs. group velocity

Waves carry information.

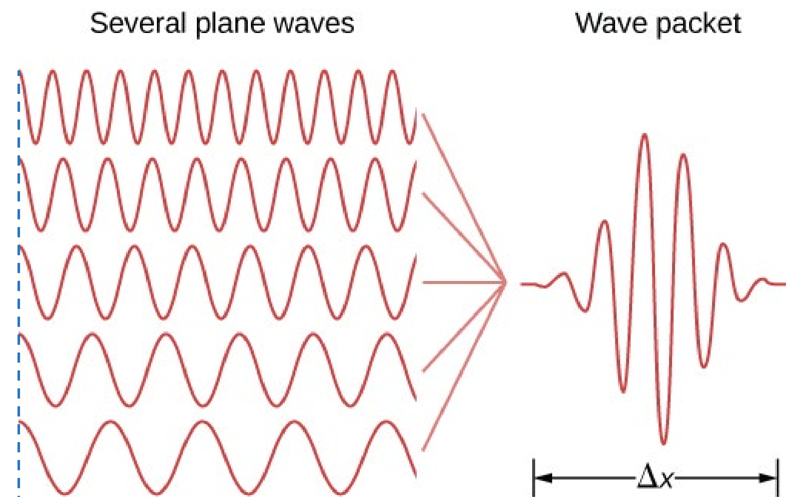
How much information does a sinusoidal (simple harmonic) wave carry?

Why do we study sinusoidal waves?

Why do we study sinusoidal waves?

The concept of Fourier transformation also applies to the space domain and the "spatial frequency domain".
β is the spatial equivalent of ω.

☞



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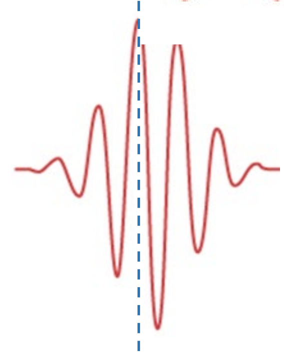


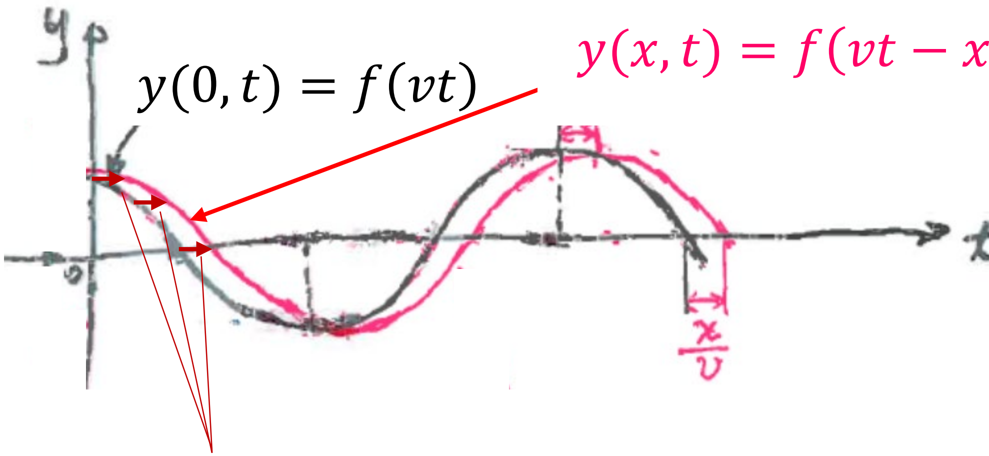
The peak of the wave packet (pulse envelope) is where all Fourier components happen to be in phase.

Special case:
various wavelengths, same propagation direction

Waveforms of the string displacement y at different locations

$$y(0,t) = f(vt)$$

$$y(x,t) = f(vt - x) = f\left[v\left(t - \frac{x}{v}\right)\right]$$



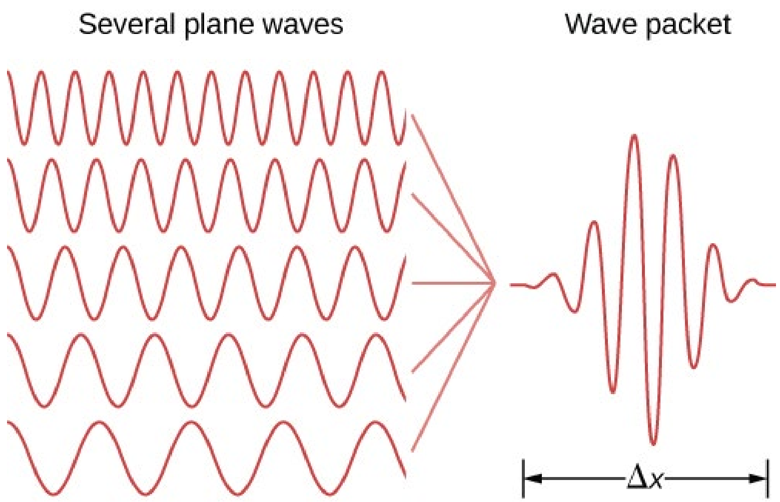
At location x, you have a time-delayed version of $f(vt)$.

The time delay is simply the time for the wave to travel a distance x, i.e., $\frac{x}{v}$.

For a single-wavelength, sinusoidal wave, the snapshot and waveform shift rigidly, because the wave travels at just one speed, v.

But, a general wave has components of different wavelengths/frequencies.
The speeds of the different components may be different.

Then, the shape of a "snapshot" taken some tine t later may have a different shape, and the "waveform" (as shown by an oscilloscope for a voltage) taken distance x out may also be different.
This is called "dispersion"



Watch animation

https://en.wikipedia.org/wiki/Dispersion_(optics)#/media/File:Optical_dispersion_dynamics.gif

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

$$= A\cos\left[\beta\left(\frac{\omega}{\beta}t - x + \frac{\phi_0}{\beta}\right)\right] \qquad v = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$$

$$= A\cos[\beta(vt - x) + \phi_0] \qquad = \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T}$$

One wavelength λ traveled in one period T.

$\frac{\omega}{\beta} \equiv v$ is called the phase velocity, thus also denoted v_p.

In general, v_p depends on frequency ("dispersion").

In free space (i.e. vacuum), $v = c = \frac{\omega}{\beta}$, or $\omega = c\beta$. No dispersion (c is a constant)

The relation between β and ω for a wave traveling in a medium is a material property of the medium. We call it the "dispersion relation" or just "dispersion."

We use the term to describe a phenomenon. Related.

In general the dispersion relation is not perfectly linear. Thus the dispersion!

$\frac{\omega}{\beta} \equiv v$ Is generlly not a constant. We call it the phase velocity, v_p.

We want the wave to carry the "undistorted" information after it travels a distance x to reach the receiver.

We want its snapshots in space to be the same as at $t = 0$ (with a shift).
We want its waveform in time to be the same as at $x = 0$ (with a delay).

There will be distortion due to dispersion. (Read the next slide offline)

The "signal" or "wave packet" or "envelope" travels at a different speed than v_p, which is different for different frequencies anyway.
That speed is the "group velocity" v_g.

In most cases, the dispersion is not too bad.
The $\omega(\beta)$ is only slightly nonlinear, i.e., $v_g \approx v_p$.

Run the extra mile:
Find out the expression for v_g, given the dispersion $\omega(\beta)$. Derive it.
You'll have a deep understanding about wave propagation.

(Read the next 2 slides offline)

Group velocity explained in simple math

As a simple case, we consider a signal with only two frequencies, carried by the wave

$$\cos\left(\omega_1 t - \frac{\omega_1}{v_1}x\right) + \cos\left(\omega_2 t - \frac{\omega_2}{v_2}x\right)$$

In general, the two phase velocities $v_1 \neq v_2$. First, we consider the special case $v_1 = v_2 = v$.

$$\cos\left(\omega_1 t - \frac{\omega_1}{v_1}x\right) + \cos\left(\omega_2 t - \frac{\omega_2}{v_2}x\right) = \cos\left[\omega_1\left(t - \frac{x}{v}\right)\right] + \cos\left[\omega_2\left(t - \frac{x}{v}\right)\right]$$

Define function $\quad f(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$

$f(t)$ is the wave at the transmitter end $x = 0$.
When the wave arrives at the receiver at distance x, it is described as $f(t-\tau)$, which is the same signal with a time delay $\tau = x/v$ but no distortion. The time delay is simply the time taken for the wave to propagate over the distance x.

In the general case $v_1 \neq v_2$, however, there is no way you can express the wave by $f(t-\tau)$ at an arbitrary distance x. In other words, you do not have the same signal at the receiver end!
Now, let's consider the usual case of small dispersion, i.e. small difference between v_1 and v_2.
Define $\beta_1 = \omega_1/v_1$ and $\beta_2 = \omega_2/v_2$, then

$$\cos\left(\omega_1 t - \frac{\omega_1}{v_1}x\right) + \cos\left(\omega_2 t - \frac{\omega_2}{v_2}x\right) = \cos(\omega_1 t - \beta_1 x) + \cos(\omega_2 t - \beta_2 x)$$

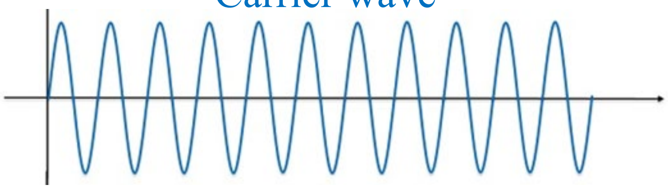
$$= \cos\left(\frac{\omega_1 + \omega_2}{2}t - \frac{\beta_1 + \beta_2}{2}x\right)\cos\left(\frac{\omega_1 - \omega_2}{2}t - \frac{\beta_1 - \beta_2}{2}x\right)$$

$$\cos\left(\omega_1 t - \frac{\omega_1}{v_1}x\right) + \cos\left(\omega_2 t - \frac{\omega_2}{v_2}x\right) = \cos(\omega_1 t - \beta_1 x) + \cos(\omega_2 t - \beta_2 x)$$

$$= \cos\left(\frac{\omega_1 + \omega_2}{2}t - \frac{\beta_1 + \beta_2}{2}x\right)\cos\left(\frac{\omega_1 - \omega_2}{2}t - \frac{\beta_1 - \beta_2}{2}x\right)$$

Carrier wave

Envelope



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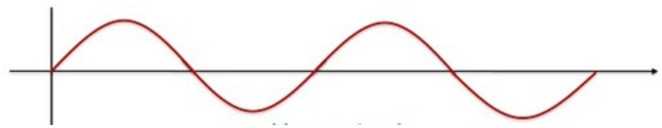
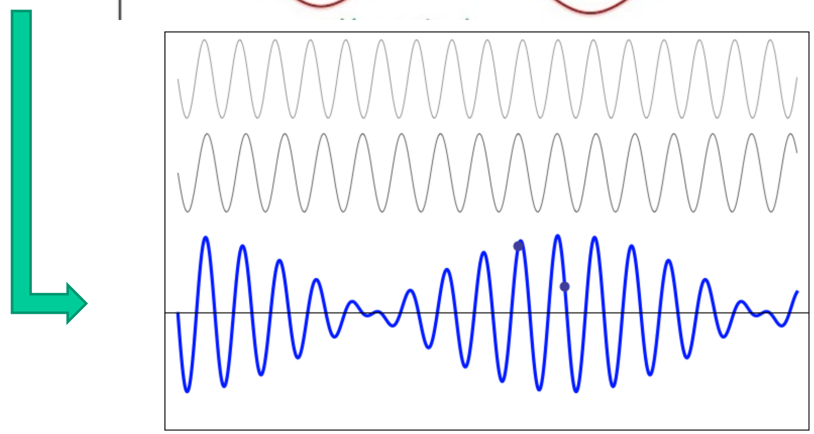




The carrier shifts at a speed

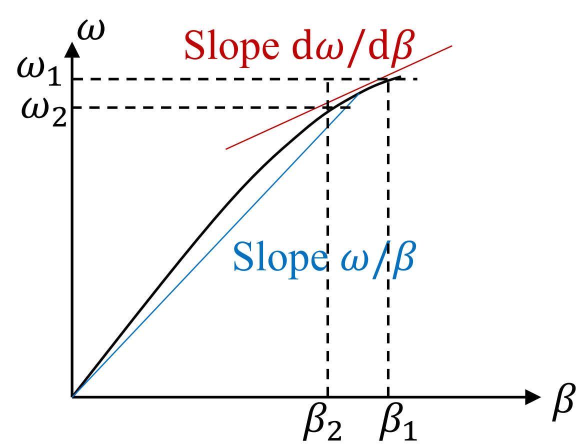
$$v_p = \frac{\frac{\omega_1 + \omega_2}{2}}{\frac{\beta_1 + \beta_2}{2}}$$

This is the "average phase velocity" of the two single-tone waves.

https://www.acs.psu.edu/drussell/demos/superposition/superposition.html

The envelope shifts at a speed $v_g = \frac{\frac{\omega_1 - \omega_2}{2}}{\frac{\beta_1 - \beta_2}{2}} = \frac{\omega_1 - \omega_2}{\beta_1 - \beta_2} = \frac{d\omega}{d\beta}$

This is the **group velocity**.



We have discussed the simplest case of two tones.
In the general case, the wave has a spectral bandwidth.
The group velocity $v_g = d\omega/d\beta$.

Attenuation

In some cases, the amplitude A decreases as the wave propagates.

For many types of waves, the power density $\propto A^2$

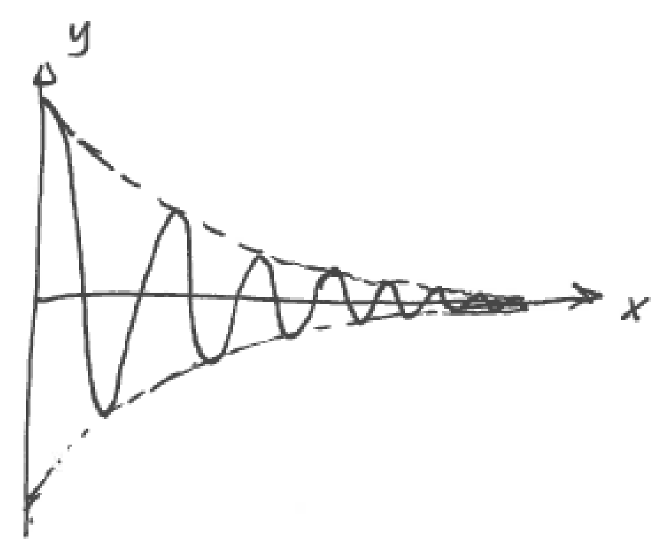
For unit distance traveled, a certain fraction of power density is lost:

$$\frac{d\,A^2(x)}{A^2(x)} = -2\alpha\,dx$$

$$\frac{d\,A^2(x)}{dx} = -2\alpha\,A^2(x)$$

$$A^2(x) = A^2(0)\,e^{-2\alpha x}$$

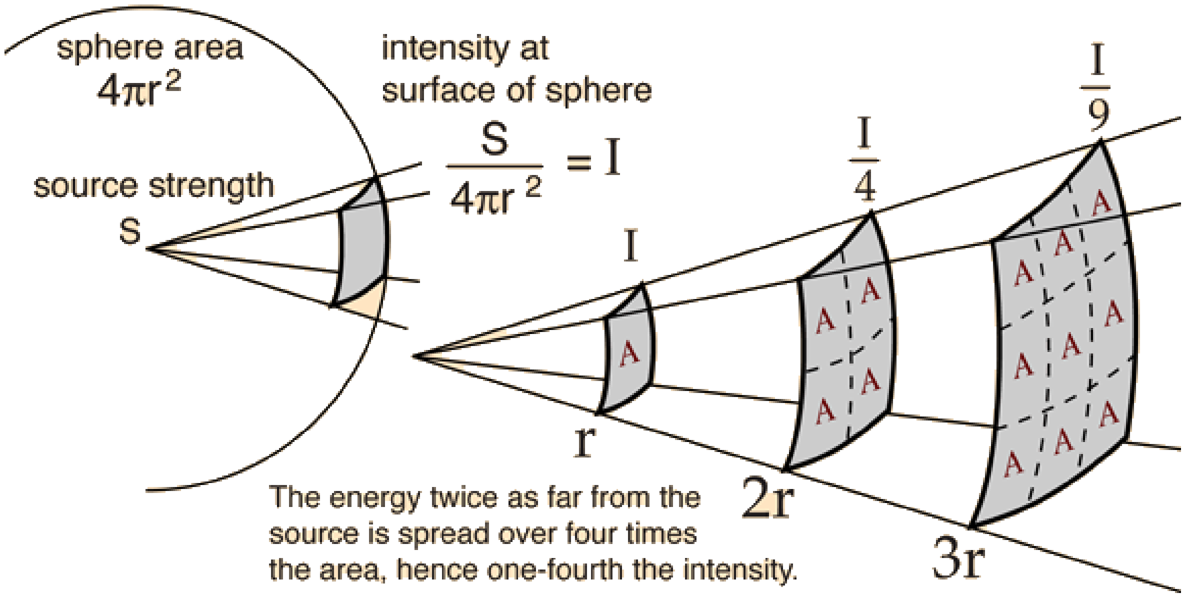
$$A(x) = A(0)\,e^{-\alpha x}$$



$$y(x,t) = A_0 e^{-\alpha x} \cos(\omega t - \beta x + \phi_0)$$

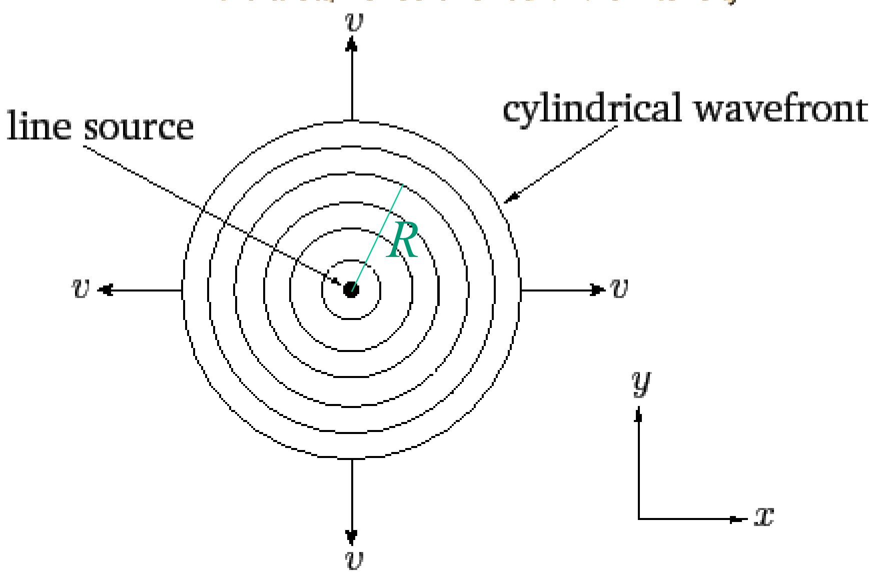
$$y(x,t) = A_0 e^{-\alpha x} \cos(\omega t - \beta x + \phi_0)$$

The exponential attenuation of the plane wave is due to loss, not to be confused with the amplitude reduction due to “spreading”.



$$y(r,t) = A_0(1/r)\cos(\omega t + \beta r + \phi_0)$$

Intensity $\propto 1/r^2$
Amplitude $\propto 1/r$



$$y(r,t) = A_0(1/R^{1/2})\cos(\omega t + \beta R + \phi_0)$$

Intensity $\propto 1/R$
Amplitude $\propto 1/(R^{1/2})$

Read textbook Section 1-4.2, and do HW1 up to P9.

Both the Homework & Answer sheet are online.
Review class notes and read the textbook, **then** do the homework, **then** check answers.

We finished this lecture on Tue 8/30/2022.