A traveling wave is the propagation of motion (disturbance) in a medium.

The one-dimensional (1D) case

The “perturbation” propagates on.
Traveling Wave in Higher Dimensions

Plane waves in 3D

Example: sound waves

Watch animation: http://en.wikipedia.org/wiki/Plane_wave
A line source makes a cylindrical wave.

Cylindrical wave (3D; top view)

Water surface wave (2D) (Circular wave)

Make a cylindrical wave from a plane wave
A point source makes a spherical wave.

Intensity is energy carried per time per area, i.e., power delivered per area.

Conservation of energy:

\[ \frac{S}{4\pi r^2} = I \]

The energy twice as far from the source is spread over four times the area, hence one-fourth the intensity.
Somehow start with a changing electric field $E$, say $E \propto \sin \omega t$

The changing electric field induces a magnetic field, $B \propto \frac{\partial E}{\partial t} \propto \cos \omega t$

If the induced magnetic field is changing with time, it will in turn induce an electric field

$E \propto \frac{\partial B}{\partial t} \propto \sin \omega t$

And so on and so on....
Just as the mechanical wave on a string.
Mathematical Expression of the Traveling Wave

A traveling wave is the propagation of motion (disturbance) in a medium.

At time 0,
\[ y = f(x) \]

At time \( t \),
\[ y = f(x-\nu t) \]

This is the general expression of Traveling waves.

Questions:
What kind of wave does \( y = f(x+\nu t) \) stand for?
What about \( y = f(\nu t-x) \)?
What about $y = f(vt - x)$?

$$f(vt - x) = f[-(x - vt)]$$

Define $f(-x)$ your “new $f$”, or $g(x) \equiv f(-x)$, so it’s the same wave!

So, which way should I go? $f(x - vt)$ or $g(vt - x)$?
To your convenience!
No big deal. But this affects how we define our “sign conventions.”
People in different disciplines use different conventions.

If you are more concerned about seeing a waveform on an oscilloscope, you like $g(vt - x)$ better.
If you are more concerned about the spatial distribution of things, you like $f(x - vt)$ better.
We will talk later about how this choice affects ways to write the “same” (but apparently different) equations in different disciplines.
What about $y = f(vt-x)$?

$$f(vt-x) = f[v(t-x/v)]$$

At any $x$, you have a time-delayed version of $f(vt)$. The time delay is simply the time for the wave to travel a distance $x$, i.e., $x/v$.

For a single-wavelength, sinusoidal wave, this is always true. Because the wave travels at just one speed, $v$.

But, a general wave has components of different wavelengths/frequencies. The speeds of the different components may be different.

I am already talking about an important concept.

Then, a distance $x$ later, the “waveform” in time (as you see with an oscilloscope) will change. This is called “dispersion”
Let’s now look at the special case of the sinusoidal wave

\[ y(x, t) = A \cos (\omega t - \beta x + \phi_0) \]

\[ = A \cos \left[ \beta \left( \frac{\omega}{\beta} t - x + \frac{\phi_0}{\beta} \right) \right] \]

\[ = A \cos \left[ \beta (\nu t - x) + \phi_0 \right] \]

\[ = f(x - \nu t) = g(\nu t - x) \]

You can write this function, or group the terms, in so many ways.
It’s just about how you view them

\[ \lambda \text{ is the "spatial period";} \quad \frac{1}{\lambda} \text{ is the "spatial frequency".} \]

And, \( \beta \) is the spatial equivalent of \( \omega \).
Call it the wave vector or propagation constant.
\[ y(x, t) = A \cos (\omega t - \beta x + \phi_0) \]

\[ = A \cos \left[ \beta \left( \frac{\omega}{\beta} t - x + \frac{\phi_0}{\beta} \right) \right] \]

\[ = A \cos \left[ \beta (\omega t - x) + \phi_0 \right] \]

\[ = f(x - vt) = g(vt - x) \]

\[ v = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda} = f\lambda \]

\[ \leq \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T} \]

One wavelength traveled in one period.

For the free space (i.e. vacuum), \( v = c = \frac{\omega}{\beta} \), or \( \omega = c\beta \).

The relation between \( \beta \) and \( \omega \) for a wave traveling in a medium is a material property of the medium. We call it the “dispersion relation” or just “dispersion.”

Recall that we used the term to describe a phenomenon. Related.

In general the dispersion relation is not perfectly linear.

\( \frac{\omega}{\beta} \equiv v \) is not a constant. We call it the phase velocity, \( v_p \).

Thus the dispersion!
\[ y(x, t) = A \cos \left( \omega t - \beta x + \phi_0 \right) \]

\[
= A \cos \left[ \beta \left( \frac{\omega}{\beta} t - x + \frac{\phi_0}{\beta} \right) \right]
\]

\[
= A \cos \left[ \beta (\omega t - x) + \phi_0 \right]
\]

\( \phi_0 \) is the reference phase (the wave’s phase with time and space set to zero)

\[ y(x, t) = A \cos \left[ \omega \left( t + \frac{\phi_0}{\omega} \right) - \beta x \right] \]

\[
= A \cos \left[ \omega t - \beta \left( x - \frac{\phi_0}{\beta} \right) \right]
\]

Shift in time

Shift in position

Two ways to look at this.

Two ways to group the terms.
For the same wave,

\[ y(x, t) = A \cos \left( \frac{2\pi}{T} x + \phi_0 \right) = A \cos \left( \frac{2\pi}{T} x - \frac{2\pi}{\lambda} x + \phi_0 \right) \]

"Spatial period"

Take snapshots at different times

Convert phase to distance

\[ y(x, t) = A \cos \left( -\frac{2\pi}{\lambda} x + \phi_0 \right) \]

= \( A \cos \left[ -\frac{2\pi}{\lambda} x + \phi_0 + \frac{2\pi}{T} t \right] \)

For the same wave,

\[ y(x, t) = A \cos \left( \frac{2\pi}{T} t + \phi_0 \right) \]

Measure waveforms at different locations

Convert phase to time

\[ y(x, t) = A \cos \left( \frac{2\pi}{T} t + \phi_0 - \frac{2\pi}{\lambda} x \right) = A \cos \left[ \frac{2\pi}{T} t + \phi_0 - \frac{2\pi}{\lambda} x \right] \]
Waves carry information.

How much information does a sinusoidal wave carry?

Why do we study sinusoidal waves?
We want the wave to carry the “undistorted” information after it travels a distance $x$ to reach us.

We want its snapshots in space to be the same as at $t = 0$.
We want its waveform in time to be the same as at $x = 0$.

Recall that we talked about dispersion.

In most cases, the dispersion is not too bad.
The $\omega(\beta)$ is only slightly nonlinear.

The “signal” or “wave packet” or “envelope” travels at a different speed than $v_p$, which is different for different frequencies anyway.
That speed is the “group velocity” $v_g$.

Run the extra mile:
Find out the expression for $v_g$, given the dispersion $\omega(\beta)$. Derive it.
You’ll have a deep understanding about wave propagation.
Attenuation

In some cases, the amplitude $A$ decreases as the wave propagates.

For many types of waves, the power density $\propto A^2$.

For unit distance traveled, a fraction of power density is lost.

$$\frac{d A^2(x)}{A^2(x)} = -2\alpha dx$$

$$\frac{d A^2(x)}{dx} = -2\alpha A^2(x)$$

$$A^2(x) = A^2(0) e^{-2\alpha x}$$

$$A(x) = A(0) e^{-\alpha x}$$

$$y(x, t) = A_0 e^{-\alpha x} \cos(\omega t - \beta x + \phi_0)$$